Helmert 変換係数の推定

Helmert 変換

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = (1+\Delta S)\begin{pmatrix} 1 & -R_z & R_y \\ R_z & 1 & R_x \\ -R_y & R_x & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

x, y, z ： 変換前座標，x', y', z'： 変換後座標

$\boldsymbol{X} = (\Delta x, \Delta y, \Delta z, R_x, R_y, R_z, \Delta S)^T$ Helmert 変換係数

最小二乗法による Helmert 変換係数の推定

$$\boldsymbol{Y} = \boldsymbol{Y}_0 + \boldsymbol{A}(\boldsymbol{X} - \boldsymbol{X}_0) + \boldsymbol{\varepsilon}$$

$$\boldsymbol{X} = \boldsymbol{X}_0 + (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T(\boldsymbol{Y} - \boldsymbol{Y}_0)$$

$$\boldsymbol{X}_0 = (0,0,0,0,0,0,0)^T$$

$$\boldsymbol{Y}_0 = (\boldsymbol{r}_1^T, \boldsymbol{r}_2^T, \ldots, \boldsymbol{r}_n^T)^T = (x_1, y_1, z_1, x_2, y_2, z_2, \ldots, x_n, y_n, z_n)^T$$

$$\boldsymbol{Y} = (\boldsymbol{r}'^T_1, \boldsymbol{r}'^T_2, \ldots, \boldsymbol{r}'^T_n) = (x'_1, y'_1, z'_1, x'_2, y'_2, z'_2, \ldots, x_n, y_n, z_n)^T$$

$$\boldsymbol{A} = \frac{\partial \boldsymbol{Y}}{\partial \boldsymbol{X}}\bigg|_{\boldsymbol{X}=\boldsymbol{X}_0} = \begin{pmatrix} \frac{\partial \boldsymbol{r}_1}{\partial \boldsymbol{X}} \\ \frac{\partial \boldsymbol{r}_2}{\partial \boldsymbol{X}} \\ \vdots \\ \frac{\partial \boldsymbol{r}_n}{\partial \boldsymbol{X}} \end{pmatrix}$$

$$\frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{X}} = \begin{pmatrix} \frac{\partial \boldsymbol{r}_i}{\partial \Delta x} & \frac{\partial \boldsymbol{r}_i}{\partial \Delta y} & \frac{\partial \boldsymbol{r}_i}{\partial \Delta z} & \frac{\partial \boldsymbol{r}_i}{\partial R_x} & \frac{\partial \boldsymbol{r}_i}{\partial R_y} & \frac{\partial \boldsymbol{r}_i}{\partial R_z} & \frac{\partial \boldsymbol{r}_i}{\partial \Delta S} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & z_i & -y_i & x_i \\ 0 & 1 & 0 & -z_i & 0 & x_i & y_i \\ 0 & 0 & 1 & x_i & -x_i & 0 & z_i \end{pmatrix}$$