IPNTJ Summer School on GNSS

Theory of Precise Positioning (2)
RTKLIB Practice
Advanced Topics

Tokyo Univ. of Marine Science and Technology
Tomoji TAKASU

2014-07-28 ~ 2014-08-02 @Tokyo, Japan

Timetable

B-1	**RTKLIB Introduction**	July 29	8:30- 9:50
B-2	**RTKLIB Practice (1)**		10:10-11:30
B-3	**Theory of Precise Positioning (1)**		12:30-13:50
	JAXA Activities & QZSS Intro by JAXA		14:10-15:30
	QZSS-Demo & RTKLIB Practice (2)		15:50-17:10
B-4	**Theory of Precise Positioning (2)**	July 30	8:30- 9:50
B-5	**Theory of RTK and RTKLIB Practice (3)**		10:10-11:30
	What is AIS? by JRC		12:30-12:50
	Port Cruise Demo (G-1) & RTKLIB Practice (4)		12:55-14:20
	Port Cruise Demo (G-2) & RTKLIB Practice (4)		14:15-15:40
B-6	**Advanced Topics**		15:50-17:10

B-4
Theory of Precise Positioning (2)

Time Systems

- **Time Systems**
  - TAI: International Atomic Time
  - UTC: Coordinated Universal Time
  - Local Time (JST, EDT, ...)
  - UT0, UT1, UT2: Universal Time
  - GMST: Greenwich Mean Sidereal Time
  - GPS Time
  - GLONASS Time
  - ...

Time System Conversion

TAI to UTC:

$$t_{UTC} = t_{TAI} + (\underline{UTC - TAI})$$

UTC-TAI (s)			
-25	1990/1/1-	-30	1996/1/1-
-26	1991/1/1-	-31	1997/7/1-
-27	1992/7/1-	-32	1999/1/1-
-28	1993/7/1-	-33	2006/1/1-
-29	1994/7/1-	-34	2009/1/1-

UTC to UT1:

$$t_{UT1} = t_{UTC} + (\underline{UT1 - UTC})$$

UT1 to GMST:

$$GMST_{0h\,UT1} = 24110.54841 + 8640184.812866\,T'_u + 0.093104\,T'^{\,2}_u - 6.2 \times 10^{-6}\,T'^{\,3}_u$$

$$GMST = GMST_{0h\,UT1} + r(t_{UT1} - t_{0h\,UT1})$$

$$r = 1.002737909350795 + 5.9006 \times 10^{-11}\,T'_u - 5.9 \times 10^{-15}\,T'^{\,2}_u$$

$T'_u = d'_u / 36525$ $\quad d'_u$: number of days elapsed since 2000 Jan 1, 12h UT1

GPS Time to TAI:

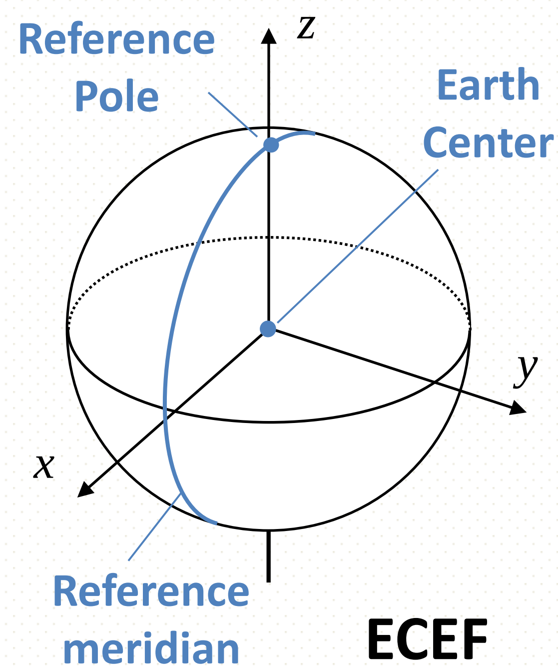
$$t_{TAI} \approx t_{GPST} + 19\,s$$

GPS Time to UTC:

$$t_{UTC} = t_{GPST} - (\underline{\Delta t_{LS}} + \underline{A_0} + \underline{A_1}(t_{GPST} - \underline{t_{ot}}))$$

Coordinate Systems

- **ECEF: Earth-Centered Earth-Fixed**
  - ITRF
  - WGS 84: US (GPS)
  - PZ90: Russia (GLONASS), ...
- **ECI: Earth-Centered Inertial**
  - ICRF: International Celestial Reference Frame
- **ECI-ECEF Connection**
  - Precession/Nutation Model
  - EOP: Earth Orientation Parameters

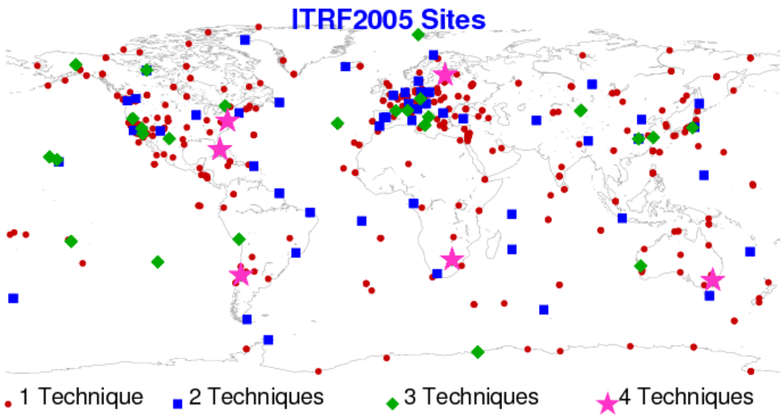


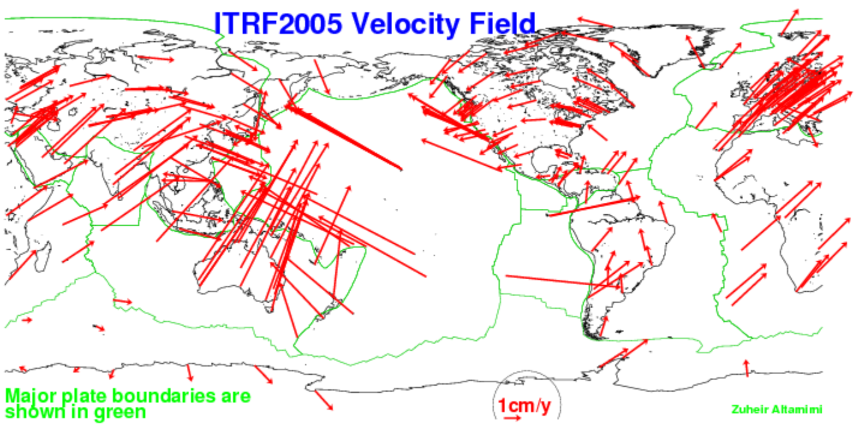
ITRF

- **International Terrestrial Reference Frame**
  - A "Realization" of Maintained by IERS
  - GPS, VLBI, SLR, DORIS Site Position/Velocity List
  - ITRF2005, ITRF2000, ITRF97, ITRF96, ...

VLBI: Very Long Baseline Interferometry
SLR: Satellite Laser Ranging
DORIS: Doppler Orbit determination and Radiopositioning Integrated on Satellite

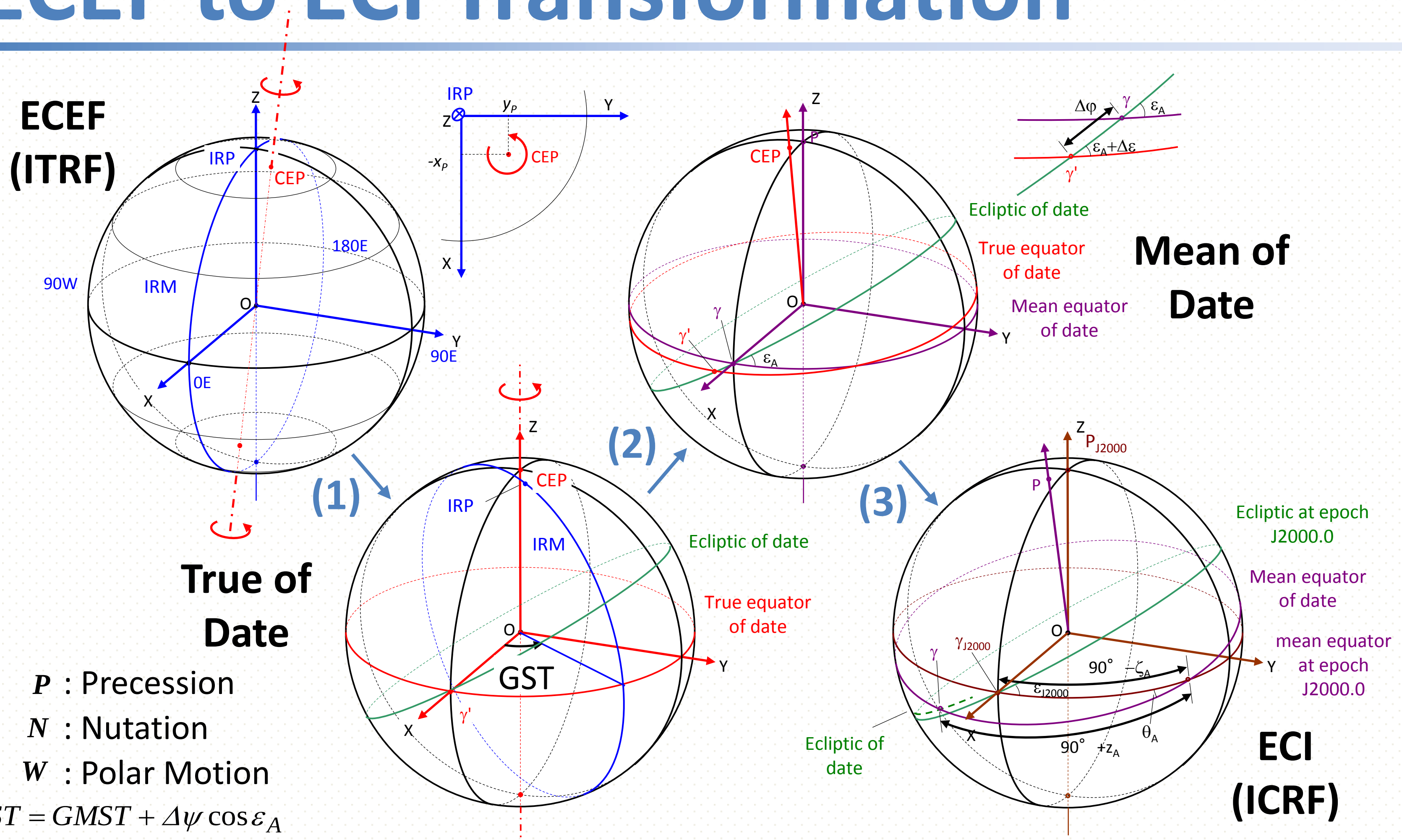
ITRS: International Terrestrial Reference System
IERS: International Earth Rotation Service





http://itrf.ensg.ign.fr/ITRF_solutions/2005/ITRF2005.php

ECEF to ECI Transformation



$\boldsymbol{P}$: Precession

$\boldsymbol{N}$: Nutation

$\boldsymbol{W}$: Polar Motion

$$GST = GMST + \Delta\psi\cos\varepsilon_A + 0''.00264\sin\Omega + 0''.000063\sin 2\Omega$$

$$\boldsymbol{r}_{eci} = \boldsymbol{P}\boldsymbol{N}\boldsymbol{R}_z(-GST)\boldsymbol{W}\boldsymbol{r}_{ecef}$$

$$= \underbrace{\boldsymbol{R}_Z(\zeta_A)\boldsymbol{R}_Y(-\theta_A)\boldsymbol{R}_Z(z_A)}_{(3)}\underbrace{\boldsymbol{R}_X(-\varepsilon_A)\boldsymbol{R}_Z(\Delta\varphi)\boldsymbol{R}_X(\varepsilon_A+\Delta\varepsilon)}_{(2)}\underbrace{\boldsymbol{R}_Z(-GST)}\underbrace{\boldsymbol{R}_X(y_P)\boldsymbol{R}_Y(x_P)}_{(1)}\boldsymbol{r}_{ecef}$$

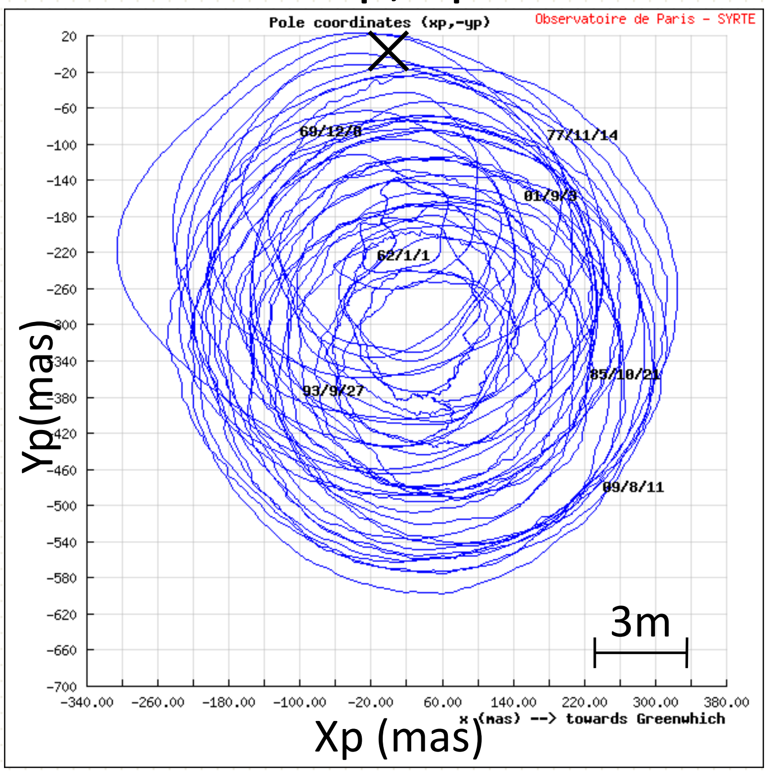
EOP: Earth Orientation Parameters

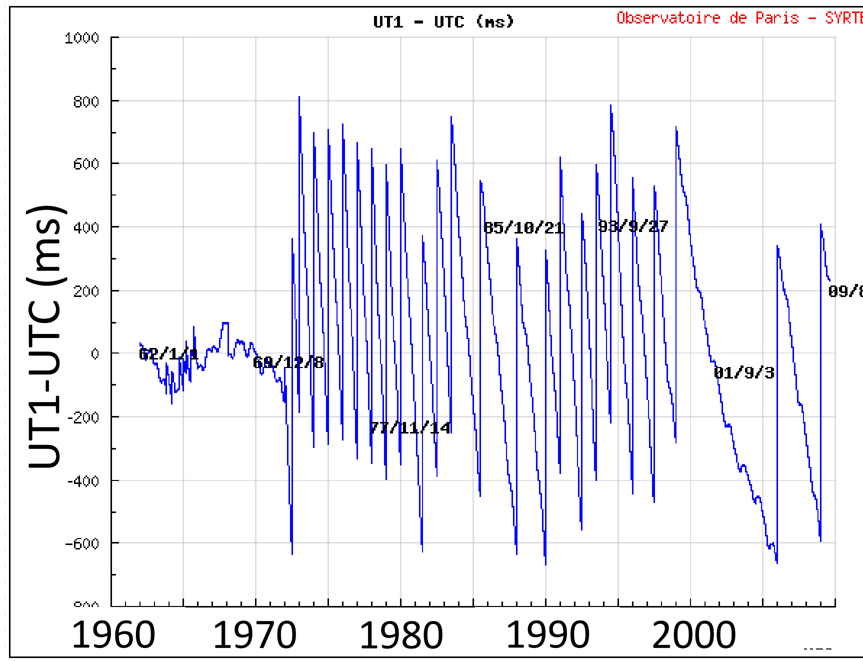
Polar Motion:

Xp, Yp

Earth Rotation Angle:

UT1-UTC

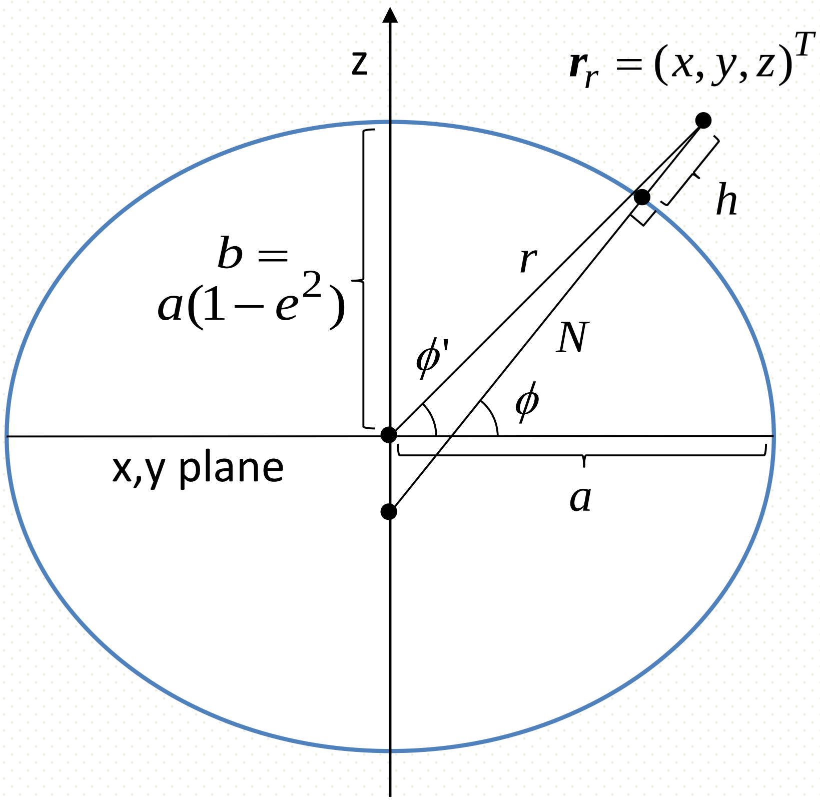




IERS C04 Series (1962/1/1-2009/8/11)

Ellipsoid and Datum

Ellipsoid:



ϕ' : Geocentric Latitude

ϕ : Geodetic Latitude

λ : Longitude

h : Ellipsoidal Height

	GRS 80	WGS 84
a (m)	6378137	6378137
f	1/298.257222101	1/298.257223563
GM (m^3/s^2)	3986005.000 x 10^8	3986004.418 x 10^8

Lat/Lon/Height to ECEF:

$$e^2 = f(2-f)$$

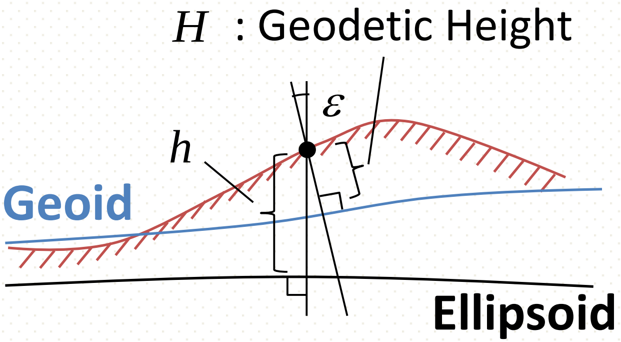
$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

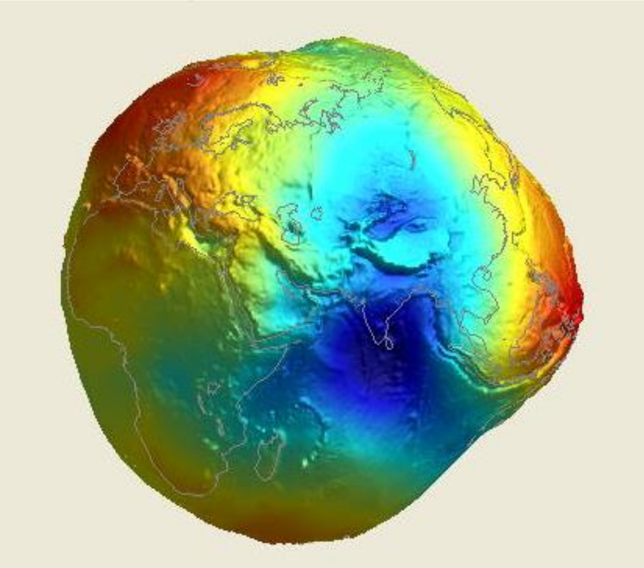
$$\boldsymbol{r}_r = \begin{pmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ (N(1+e^2)+h)\sin\phi \end{pmatrix}$$

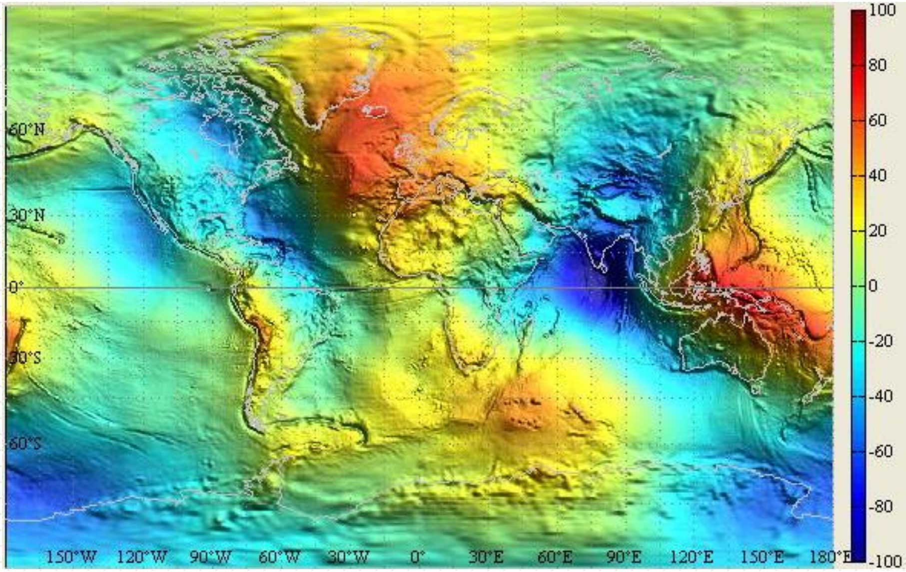
Geoid

Geopotential:

$$V(r,\phi',\lambda) = \frac{GM}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n}(\bar{C}_{nm}Y_{nmc} + \bar{S}_{nm}Y_{nms})\right\}$$







EGM96 Geoid Model

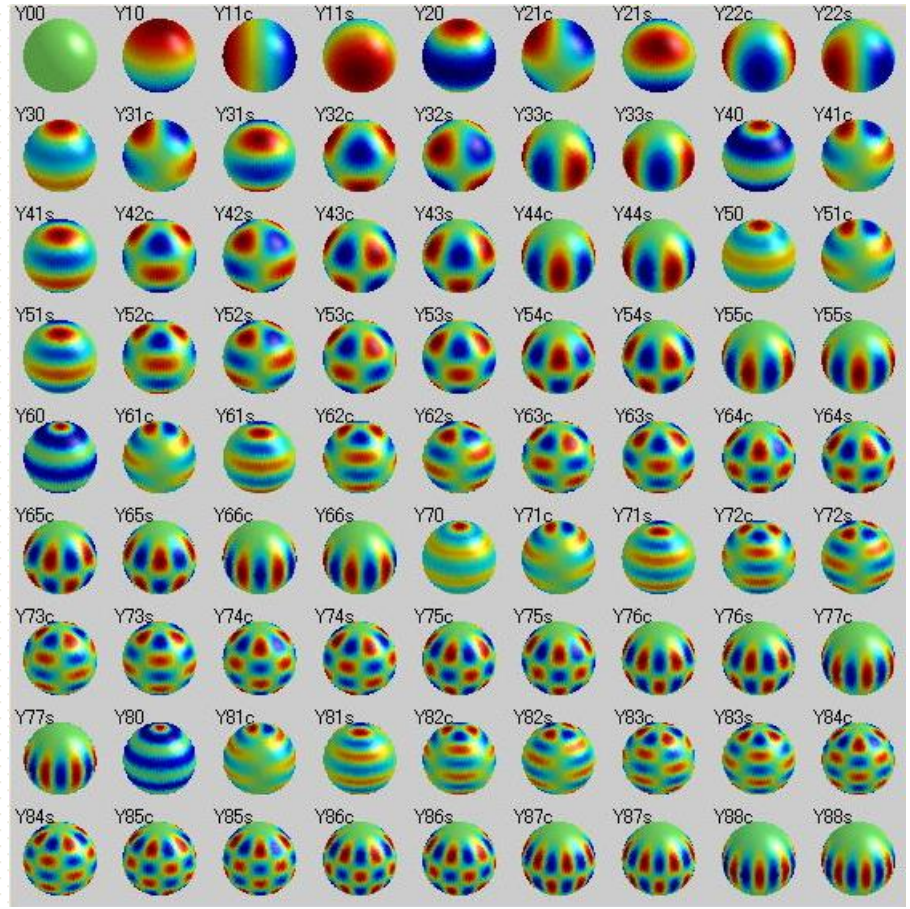
Spherical Harmonics

Spherical harmonic functions:

$$Y_{n0} = Y_{n0c}$$

$$Y_{nmc} = \overline{P}_{nm}(\sin\phi')\cos m\lambda$$

$$Y_{nms} = \overline{P}_{nm}(\sin\phi')\sin m\lambda$$



Legendre function:

$$\overline{P}_{nm} = N_{nm}P_{nm},\ P_{00}(x) = 1, P_{10}(x) = x$$

$$P_{n-1,n}(x) = 0,$$

$$P_{nn}(x) = (2n-1)(1-x^2)^{1/2}P_{n-1,n-1}(x)$$

$$P_{nm}(x) = \frac{(2n-1)xP_{n-1,m}(x) - (n+m-1)P_{n-2,m}(x)}{n-m}$$

$$N_{nm} = \begin{cases} \sqrt{2n+1} & (m=0) \\ \sqrt{\dfrac{2(2n+1)(n-m)!}{(n+m)!}} & (m>0) \end{cases}$$

Coordinates Transformation

Helmert Transformation (A to B):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_B = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + (1+D)\begin{pmatrix} 1 & -R_3 & R_2 \\ R_3 & 1 & -R_1 \\ -R_2 & R_1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_A$$

- T1, T2, T3 : Translation along coordinate axis
- D : Scale factor
- R1, R2, R3 : Rotation of coordinate axis

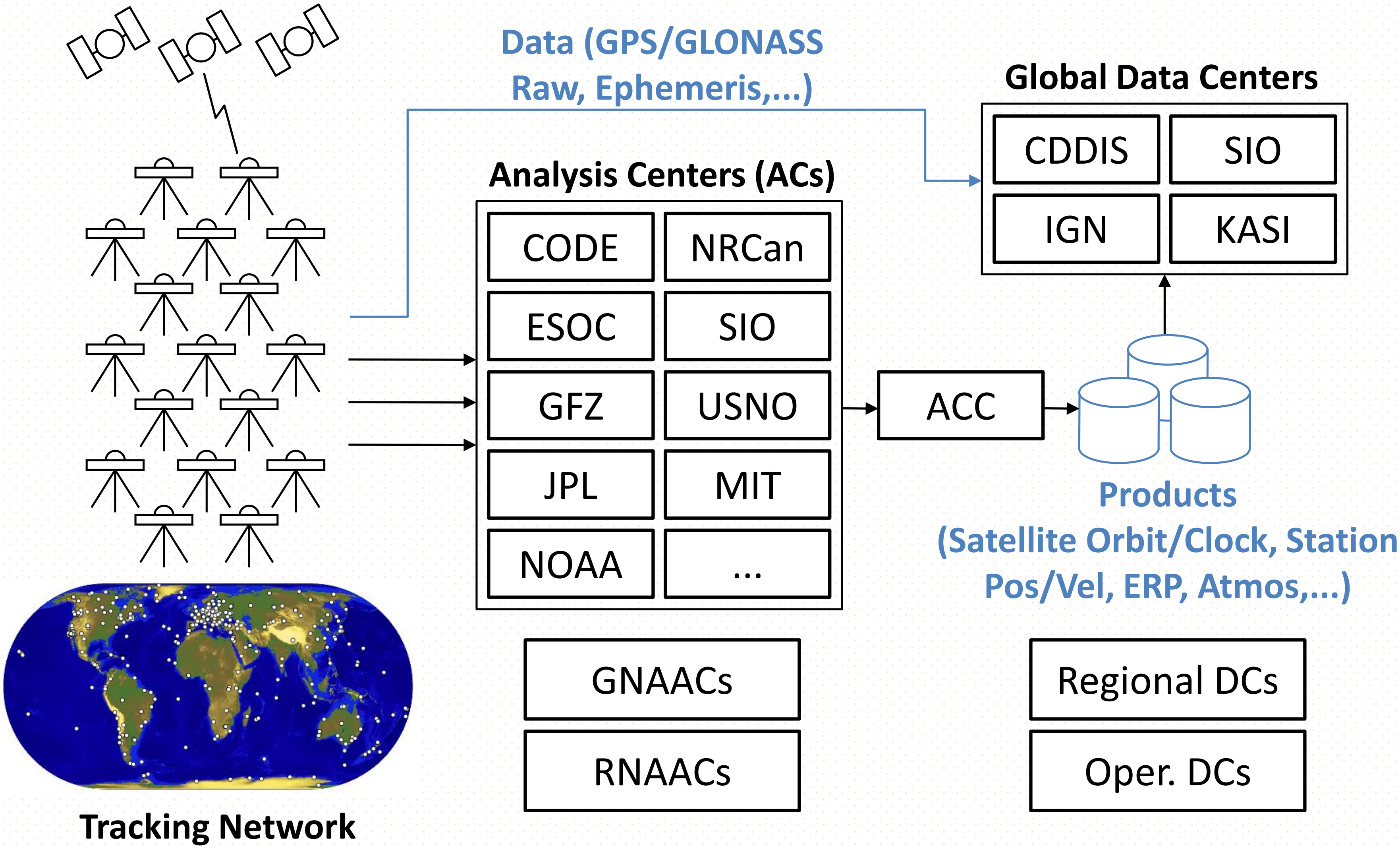
Coordinates		T1 (mm)	T2 (mm)	T3 (mm)	D (10^{-9})	R1 (mas)	R2 (mas)	R3 (mas)
A	B							
ITRF2005	ITRF2000	0.1	-0.8	-5.8	0.40	0.00	0.00	0.00
		-0.2/y	0.1/y	-1.8/y	0.08/y	0.00/y	0.00/y	0.00/y

(Epoch 2000.0)

Precise Ephemeris

- **Precise Satellite Orbit and Clock**
  - By Post-Processing or in Real-time
  - Observation Data of Tracking Stations World-Wide
- **Format:**
  - Orbit: NGS SP3
  - Clock: NGS SP3 or RINEX Clock Extension
- **Contents:**
  - Orbit: ECEF-Positions of Satellite Mass Center
  - Clock: Clock-biases wrt Time Scale Aligned to GPS Time

IGS: International GNSS Service



IGS Products

		Final (IGS)	Rapid (IGR)	Ultra-Rapid (IGU)		Broadcast
				Observed	Predicted	
Accuracy	Orbit	~2.5cm	~2.5cm	~3cm	~5cm	~100cm
	Clock	~75ps RMS ~20ps STD	~75ps RMS ~25ps STD	~150ps RMS ~50ps STD	~3ns RMS ~1.5ns STD	~5ns RMS ~2.5ns STD
Latency		12-18 days	17-41 hours	3-9 hours	realtime	realtime
Updates		every Thursday	at 17 UTC daily	at 03, 09, 15, 21 UTC	at 03, 09, 15, 21 UTC	-
Sample Interval	Orbit	15min	15min	15min	15min	daily
	Clock	Sat: 30s Stn: 5min	5min	15min	15min	daily

(2009/8, http://igscb.jpl.nasa.gov/)

Interpolation of Satellite Orbit

Lagrange Interpolation:

$$\boldsymbol{r}^s(t) = \frac{(t-t_2)(t-t_3)\ldots(t-t_{n+1})}{(t_1-t_2)(t_1-t_3)\ldots(t_1-t_{n+1})}\boldsymbol{r}^s(t_1) + \frac{(t-t_1)(t-t_3)\ldots(t-t_{n+1})}{(t_2-t_1)(t_2-t_3)\ldots(t_2-t_{n+1})}\boldsymbol{r}^s(t_2)$$

$$+\frac{(t-t_1)(t-t_2)\ldots(t-t_{n+1})}{(t_3-t_1)(t_3-t_2)\ldots(t_3-t_{n+1})}\boldsymbol{r}^s(t_3) + \ldots + \frac{(t-t_1)(t-t_2)\ldots(t-t_n)}{(t_{n+1}-t_1)(t_{n+1}-t_2)\ldots(t_{n+1}-t_n)}\boldsymbol{r}^s(t_{n+1})$$

Interpolation Error of 15-min Sample Orbit

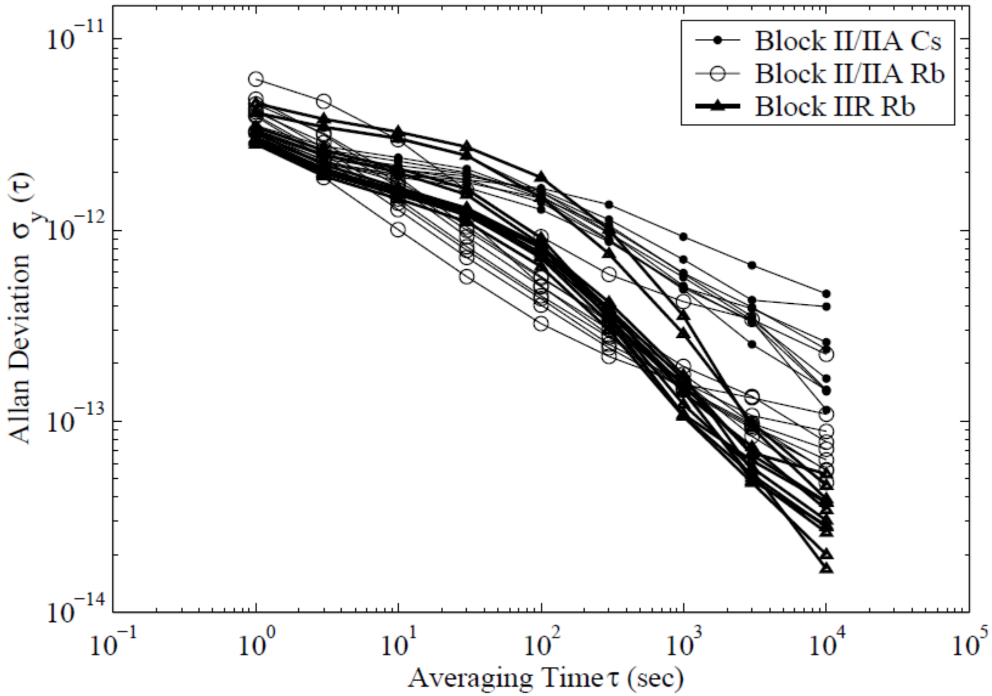
Degree of Polynomial	Position RMS Error (cm)			Velocity RMS Error (cm/s)		
	Radial	Along-Trk	Cross-Trk	Radial	Along-Trk	Cross-Trk
n=5	72.10	73.84	57.48	0.253	0.260	0.202
n=6	7.31	6.89	5.75	0.032	0.031	0.025
n=7	0.63	0.63	0.50	0.017	0.019	0.014
n=8	0.08	0.11	0.08	0.017	0.018	0.013
n=9	0.05	0.11	0.05	0.017	0.018	0.013
n=10	0.05	0.10	0.06	0.017	0.018	0.013
n=11	0.05	0.12	0.06	0.017	0.018	0.013

Interpolation of Satellite Clock

Linear Interpolation:

$$dT^s(t) = \frac{(t_2 - t)\,dT^s(t_1) + (t - t_1)\,dT^s(t_2)}{t_2 - t_1} + \Delta t_{rel}$$

Satellite Clock Stability:



Ionospheric Delay

Ionospheric Delay Model:

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{{Y_T}^2}{2(1-X-iZ)} \pm \sqrt{\frac{{Y_T}^4}{4(1-X-iZ)^2} + {Y_L}^2}} \approx 1 - X = 1 - f_N^2 / f^2 \ (L-band)$$

: Appleton-Hartree Formula

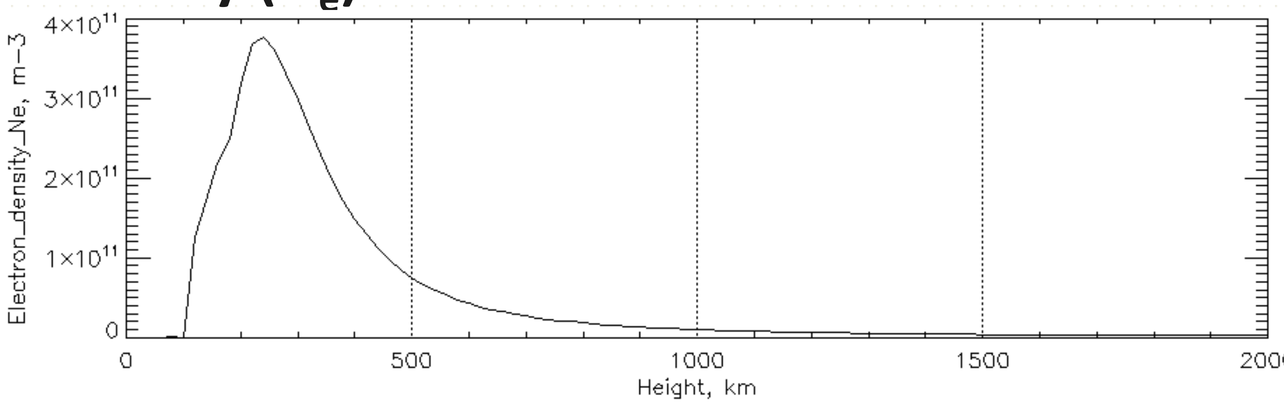
$$n = \sqrt{1 - f_N^2 / f^2} \approx 1 - f_N^2 / 2f^2 = 1 - 40.30 N_e / f^2$$

$$f_N^2 = \frac{N_e e^2}{4\pi^2 \varepsilon_0 m_e}$$: plasma frequency

$$I_r^s \approx \int 40.30 N_e / f^2 dl = 40.30 \times 10^{16} TEC / f^2$$

TEC: Total Electron Content

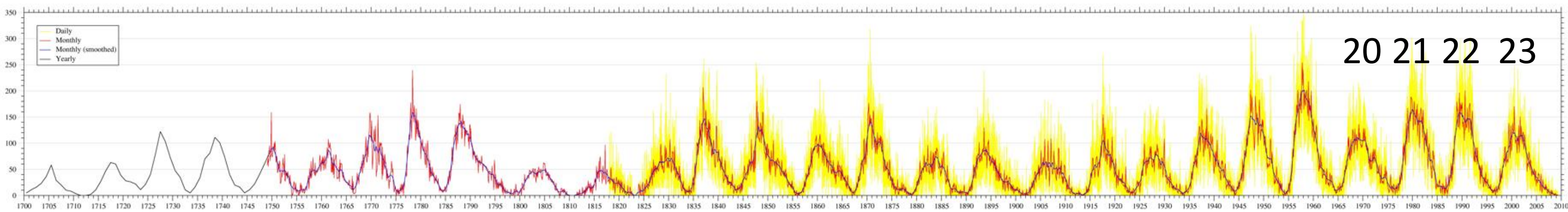
Electron Density (N_e):



IRI-2007 model: 2009/7/31 0:00 Tokyo (http://modelweb.gsfc.nasa.gov/models/iri.html)

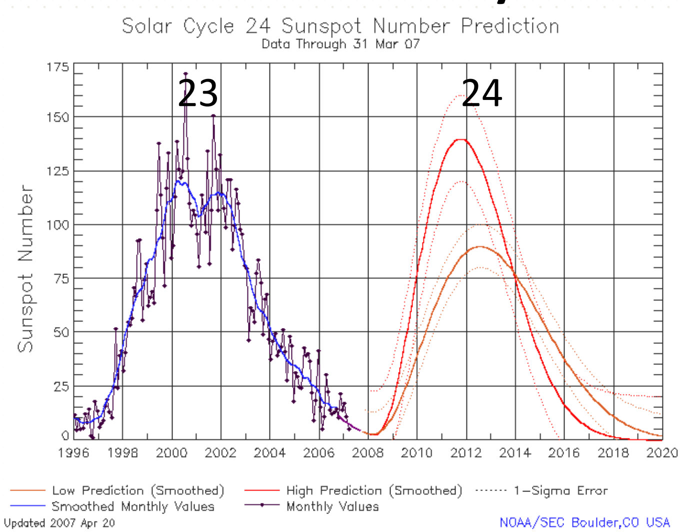
Solar Cycle

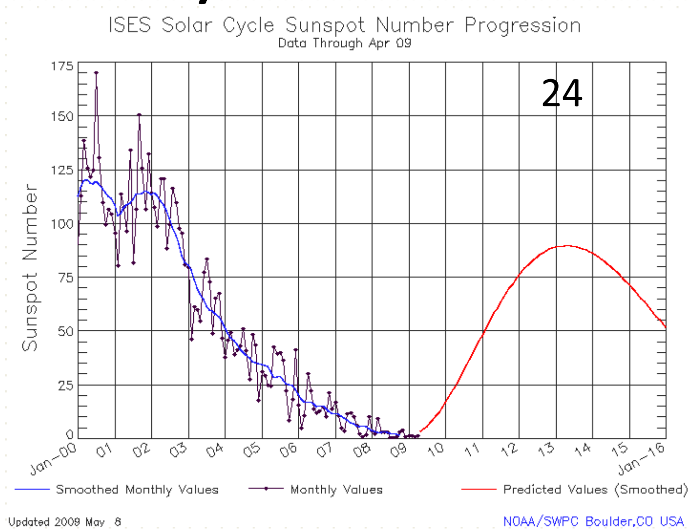
International Sunspot Number (ISN): 1700-2009



by SIDC (Solar Influences Data Analysis Center) in Belglum (http://sidc.oma.be)

Solar Cycle Prediction: Cycle 24





by NOAA SWPC (Space Weather Prediction Center) (http://www.swpc.noaa.gov/SolarCycle)

LC: Linear Combination

$$C = a\Phi_1 + b\Phi_2 + cP_1 + dP_2 (\Phi_1 = \lambda_1\phi_1,\ \Phi_2 = \lambda_2\phi_2)$$

	LC	Coefficients				Wave Length (cm)	Ionos Effect wrt L1	Typical Noise (cm)
		a	**b**	**c**	**d**			
L1	L1 Carrier-Phase	1	0	0	0	19.0	1.0	0.3
L2	L2 Carrier-Phase	0	1	0	0	24.4	1.6	0.3
LC/L3	Iono-Free Phase	C_1	C_2	0	0	-	0.0	0.9
LG/L4	Geometry-Free Phase	1	-1	0	0	-	0.6	0.4
WL	Wide-Lane Phase	λ_W / λ_1	$-\lambda_W / \lambda_2$	0	0	86.2	1.3	1.7
NL	Narrow-Lane Phase	λ_N / λ_1	λ_N / λ_2	0	0	10.7	1.3	1.7
MW	Melbourne-Wübbena	λ_W / λ_1	$-\lambda_W / \lambda_2$	λ_N / λ_1	λ_N / λ_2	86.2	0.0	21
MP1	L1-Multipath	$2C_2 - 1$	$-2C_2$	1	0	-	0.0	30
MP2	L2-Multipath	$-2C_1$	$2C_1 - 1$	0	1	-	0.0	30

$$C_1 = f_1^2/(f_1^2 - f_2^2),\ C_2 = -f_2^2/(f_1^2 - f_2^2),\ \lambda_W = 1/(1/\lambda_1 - 1/\lambda_2),\ \lambda_N = 1/(1/\lambda_1 + 1/\lambda_2)$$

Single Layer Model

Ionospheric Delay Model:

$$I = \frac{40.30 \times 10^{16}}{f^2} TEC \approx \frac{1}{\cos z'} \frac{40.30 \times 10^{16}}{f^2} \times VTEC(t, \phi_{pp}, \lambda_{pp})$$

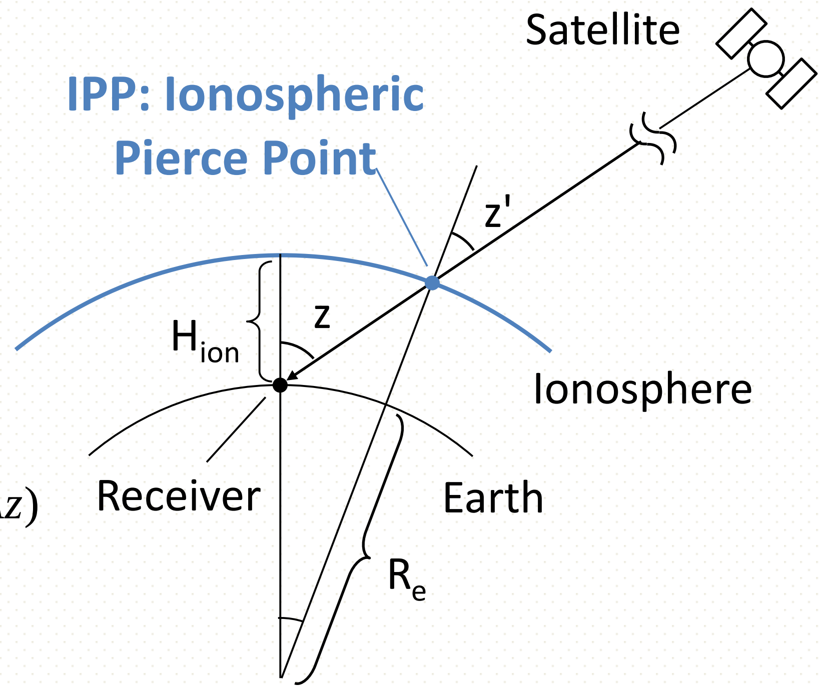
IPP Position/Slant Factor:

$$z = \pi/2 - El$$

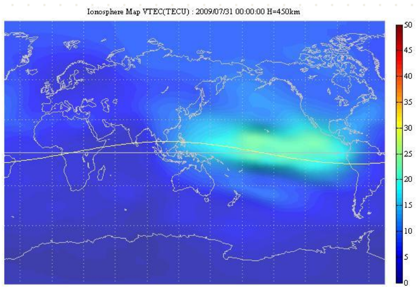
$$z' = \arcsin \frac{R_e \sin z}{R_e + H_{ion}}, \alpha = z - z'$$

$$\phi_{pp} = \arcsin(\cos\alpha \sin\phi + \sin\alpha \cos\phi \cos Az)$$

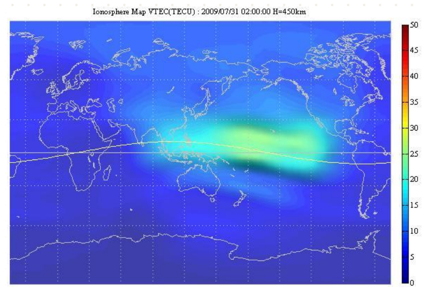
$$\lambda_{pp} = \lambda + \arcsin \frac{\sin\alpha \sin Az}{\phi_{pp}}$$



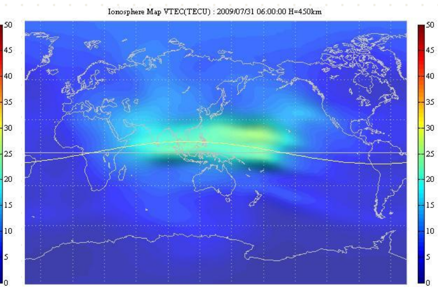
Ionospheric TEC Grid



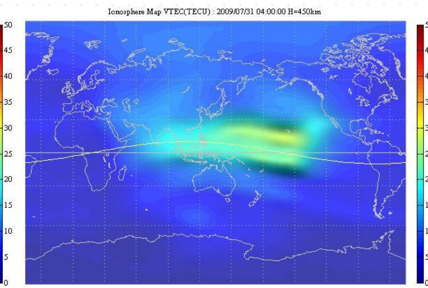
2009/7/31 0:00



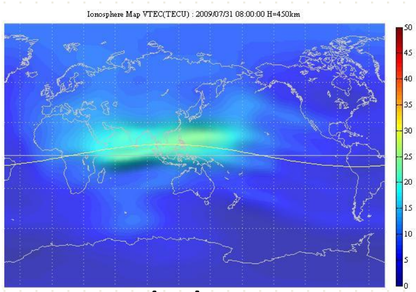
2009/7/31 2:00



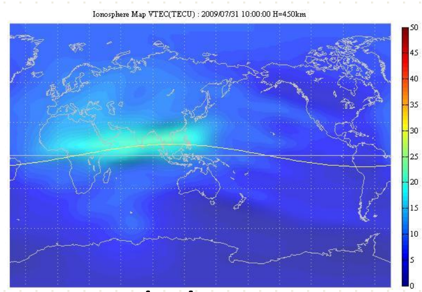
2009/7/31 4:00



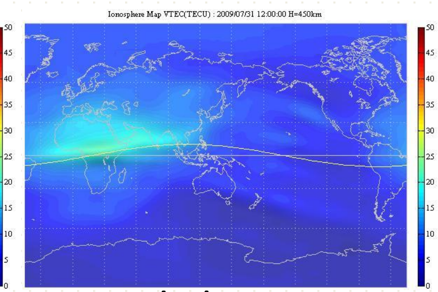
2009/7/31 6:00



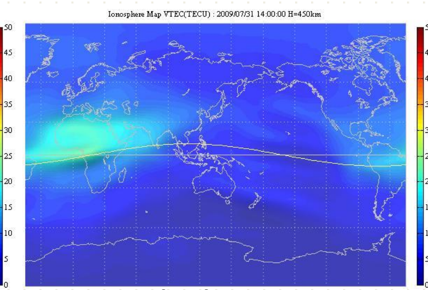
2009/7/31 8:00



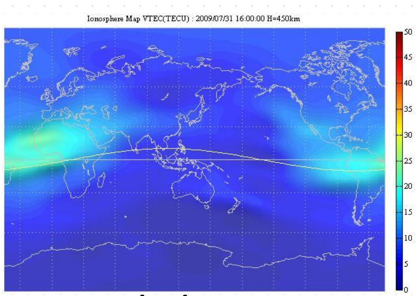
2009/7/31 10:00



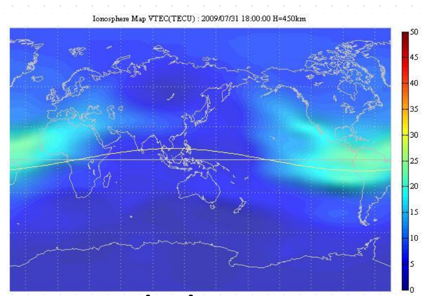
2009/7/31 12:00



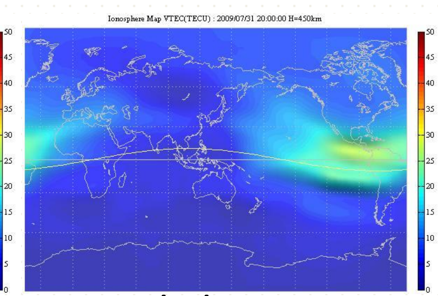
2009/7/31 14:00



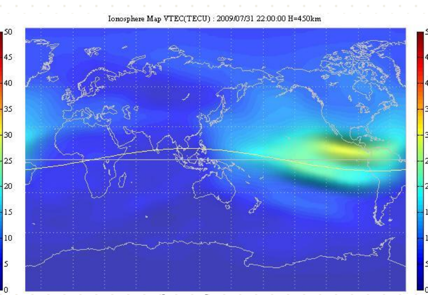
2009/7/31 16:00



2009/7/31 18:00



2009/7/31 20:00



2009/7/31 22:00

(IGS TEC Final, GPS Time)

Tropospheric Delay

Tropospheric Delay Model:

$$T = m_h(El)ZHD + m_w(El)ZWD$$

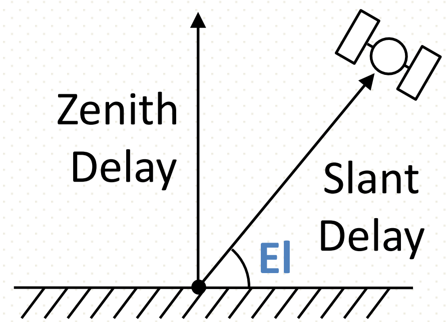
$$ZHD = \frac{0.0022768\,p}{1 - 0.00266\cos 2\phi - 2.8\times10^{-7}H}$$

: Zenith Hydrostatic Delay (m)

ZWD : Zenith Wet Delay (m)

$m_h(El)$: Hydrostatic Mapping Function

$m_w(El)$: Wet Mapping Function



ZWD to PWV (Precipitable Water Vapor):

$$T_m = 70.2 + 0.72T$$

$$PWV = \frac{1\times10^5}{R_v\left(k_2 - k_1\dfrac{m_v}{m_d} + \dfrac{k_3}{T_m}\right)}ZWD$$

$R_v = 461, k_1 = 77.6,$

$k_2 = 71.98, k_3 = 3.754\times10^5$

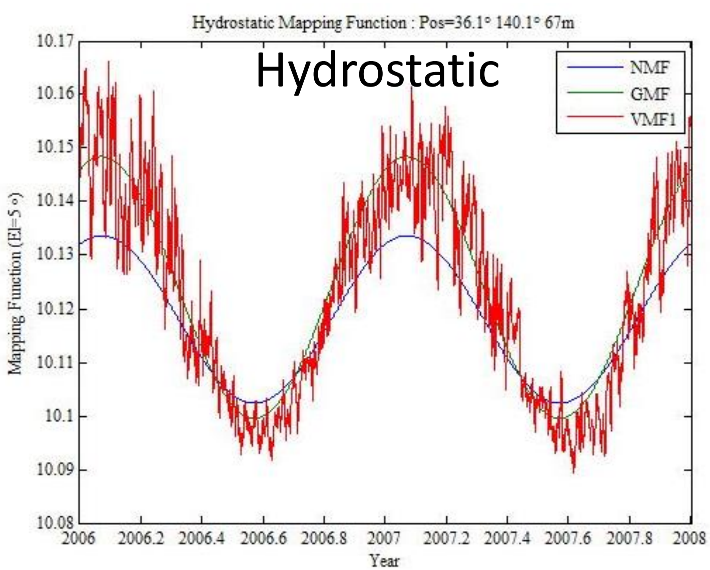
$m_v = 18.0152, m_d = 28.9644$

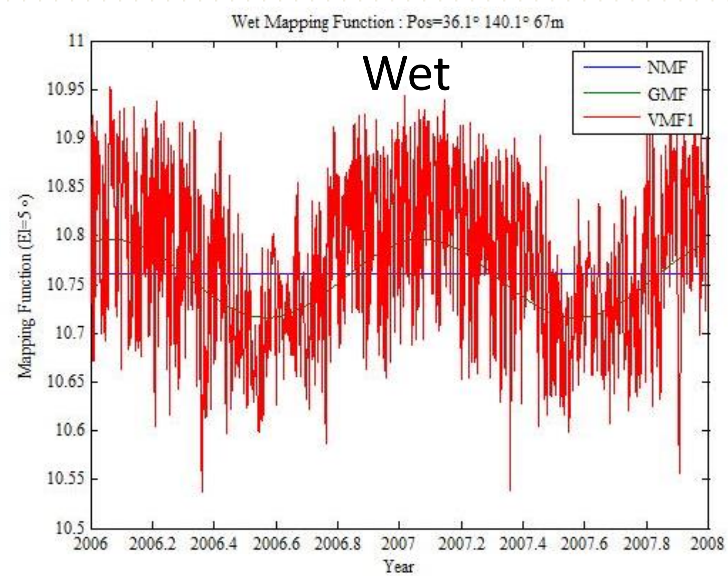
Mapping Function

$$m(El) = \frac{1 + \dfrac{a}{1 + \dfrac{b}{1 + c}}}{\sin(El) + \dfrac{a}{\sin(El) + \dfrac{b}{\sin(El) + c}}}$$

a, b, c : Mapping Function Coefficients

NMF, GMF, VMF1





(2006/1/1-2007/12/31, TSKB, El=5deg)

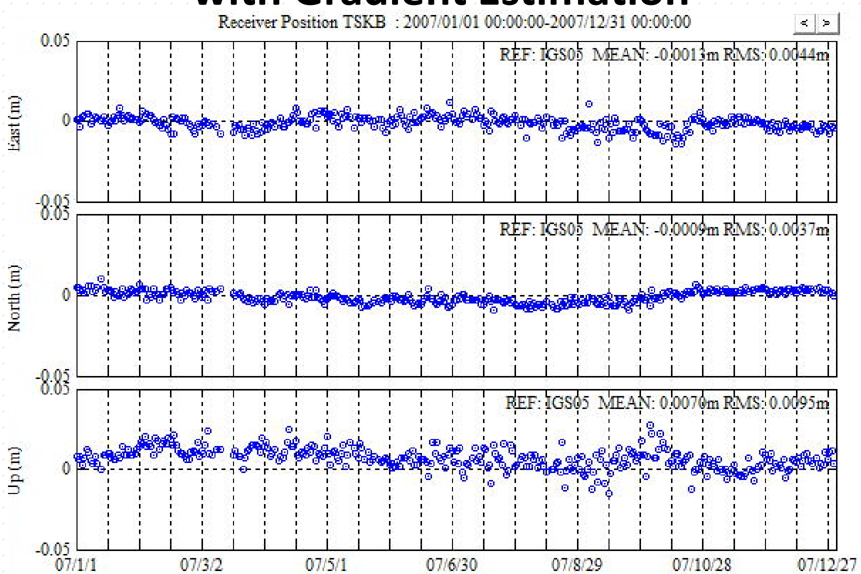
Tropospheric Gradient

Mapping Function with Horizontal Gradient:

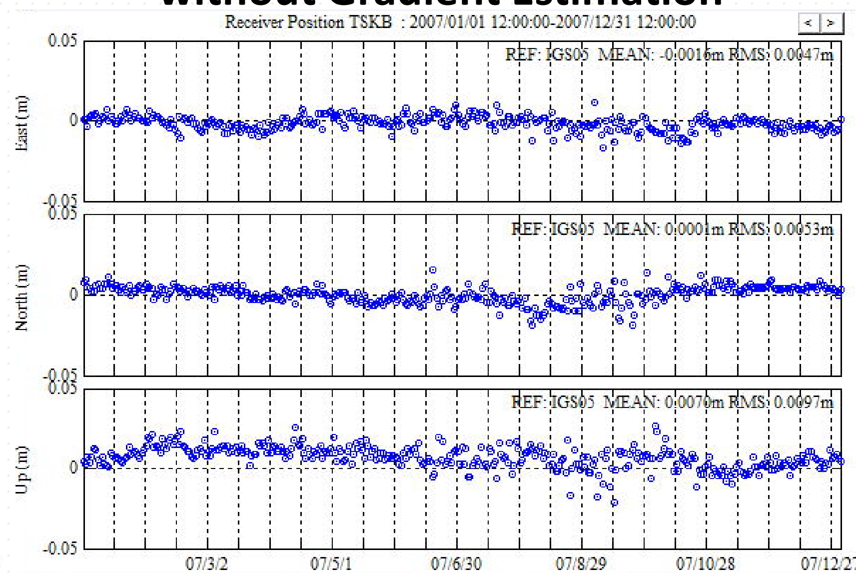
$$m(El, Az) = m_0(El) + m_0(El)\cot(El)\big(G_N \cos(Az) + G_E \sin(Az)\big)$$

G_N, G_E : North/East Gradient Parameters

PPP Solutions with Gradient Estimation



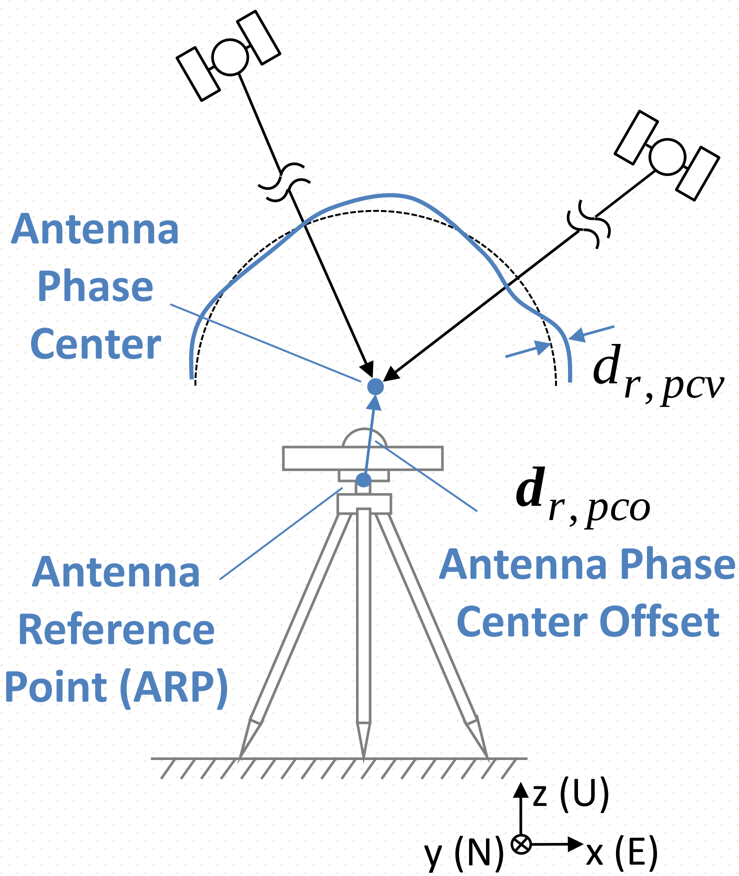
PPP Solutions without Gradient Estimation



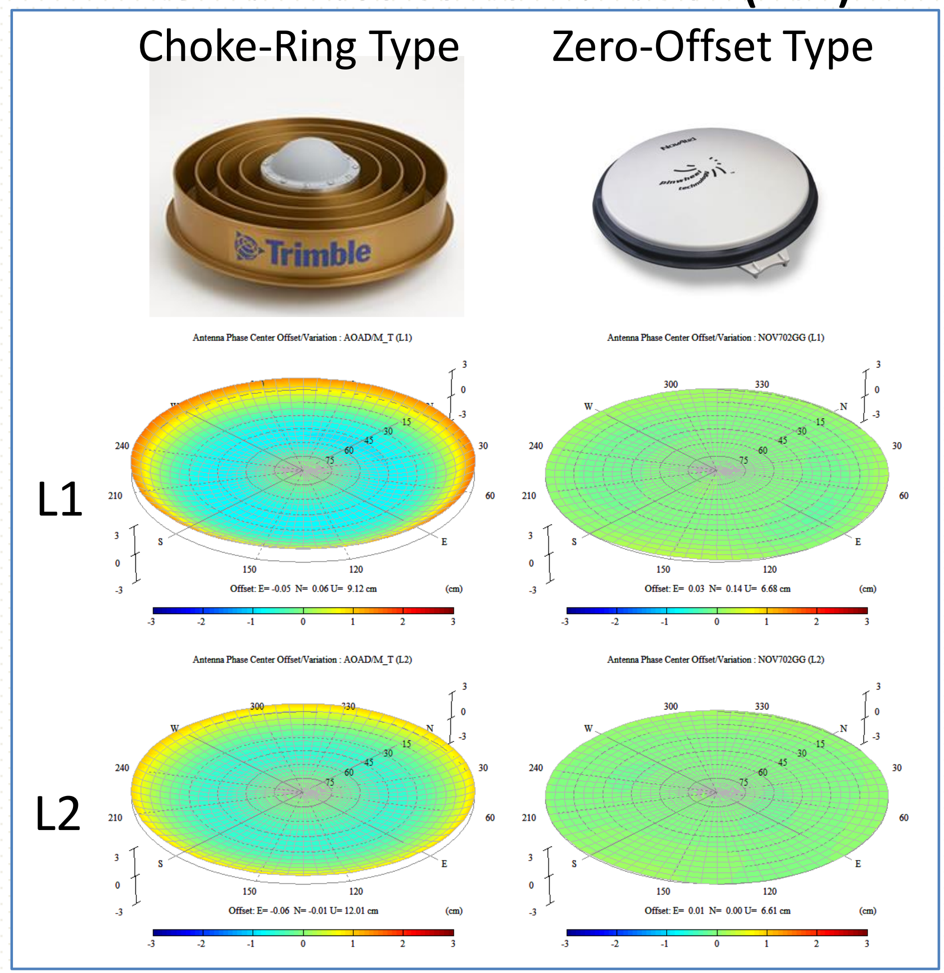
2007/1/1-12/31, 24H-Static PPP, TSKB

Antenna Phase Center 1

Receiver Antenna Phase Center:



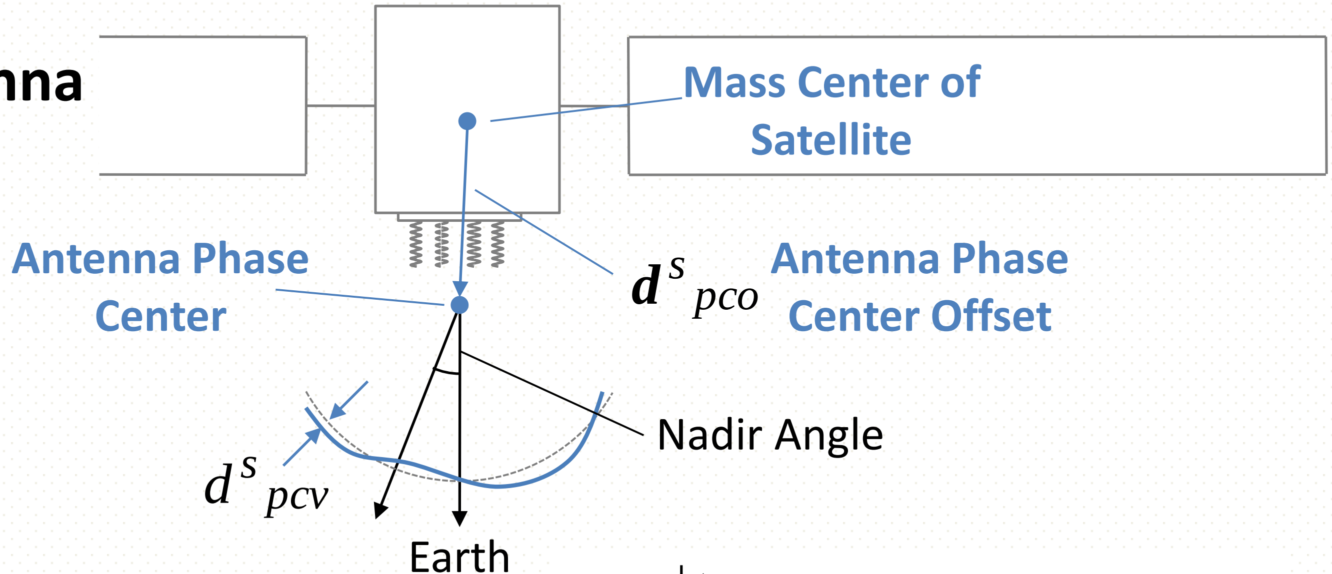
Antenna Phase Center Variation (PCV)



IGS Absolute Antenna Model (IGS05.PCV)

Antenna Phase Center 2

Satellite Antenna Phase Center:

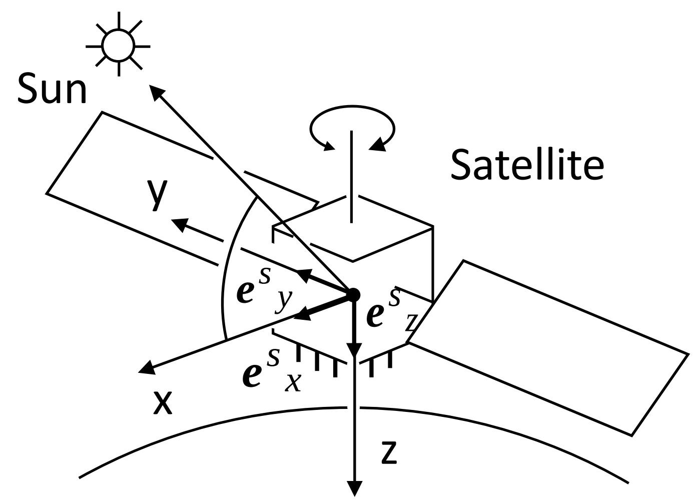


Satellite Coordinate to ECEF:

$$E_{sat \to ecef} = (e^s{}_x, e^s{}_y, e^s{}_z)$$

$$e^s{}_z = -\frac{r^s}{\|r^s\|}, e_s = \frac{r_{sun} - r^s}{\|r_{sun} - r^s\|}$$

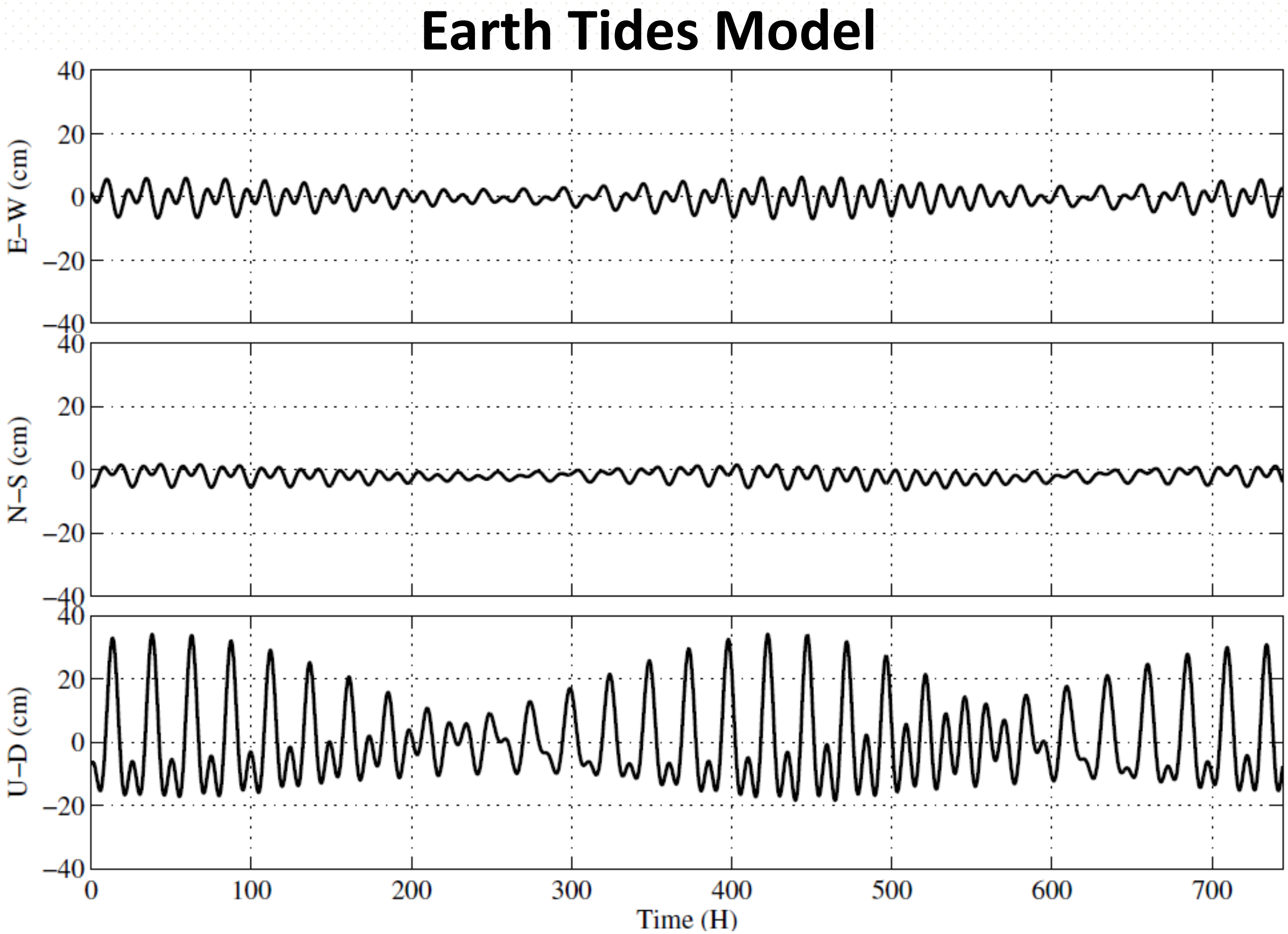
$$e^s{}_y = \frac{e^s{}_z \times e^s{}_s}{\|e^s{}_z \times e_s\|}, e^s{}_x = e^s{}_y \times e^s{}_z$$



Site Displacement

- **Displacement of Ground-Fixed Receiver**
  - Solid Earth Tide
  - Ocean Tide Loading (OTL)
  - Pole Tide
  - Atmospheric Loading
- **Tide Model**
  - IERS Conventions 1996/2003/2010
  - Ocean Loading: Schwiderski, GOT99.2/00.2, CSR 3.0/4.0, FES99/2004, NAO99.b
  - M_2,S_2,N_2,K_2,K_1,O_1,P_1,Q_1,M_1,M_m,S_{sa}

Earth Tides



IERS Conventions 1996 + NAO99.b, 2007/1/1-1/31, TSKB

Phase Wind-up Effect

- Relative rotation between satellite and receiver antennas effect to the measured phase of RHCP signal.

$$d_{pw} = \lambda \left\{ sign(\boldsymbol{e}_r^s \cdot (\boldsymbol{D}^s \times \boldsymbol{D}_r)) \arccos \frac{\boldsymbol{D}^s \cdot \boldsymbol{D}_r}{\left\| \boldsymbol{D}^s \right\| \left\| \boldsymbol{D}_r \right\|} / 2\pi + N \right\}$$

$\boldsymbol{D}^s = \boldsymbol{e}_x^s - \boldsymbol{e}_u^s(\boldsymbol{e}_u^s \cdot \boldsymbol{e}_x^s) - \boldsymbol{e}_u^s \times \boldsymbol{e}_y^s$: Dipole Vector of Satellite Antenna

$\boldsymbol{D}_r = \boldsymbol{e}_{r,x} - \boldsymbol{e}_r^s(\boldsymbol{e}_r^s \cdot \boldsymbol{e}_{r,x}) + \boldsymbol{e}_r^s \times \boldsymbol{e}_{r,y}$: Dipole Vector of Receiver Antenna

$\boldsymbol{E}_{ecef \rightarrow enu} = (\boldsymbol{e}_{r,x}{}^T, \boldsymbol{e}_{r,y}{}^T, \boldsymbol{e}_{r,z}{}^T)^T$: ECEF to ENU Transformation Matrix

$\boldsymbol{e}_r^s$: LOS Vector from Receiver to Satellite Antenna

N : Integer Ambiguity

Relativistic Effects

- **Satellite/Receiver:**
  - Frequency Shift by Earth Gravity (General Rel.)
  - Frequency Shift by Sun/Moon Gravity (General Rel.)
  - Second-Order Doppler-Shift by Motion (Special Rel.)
- **Signal Propagation:**
  - Sagnac Correction (Rotating Coordinates)
  - Shapiro Time Delay Effect
  - Lense-Thirring Drag

Satellite Clock Bias/Rate Correction
+ Periodic Term:

$$d_{rel} = -\frac{2\boldsymbol{r}^s \cdot \boldsymbol{v}^s}{c^2}$$

DD (Double Difference)

$$\Phi_{ub}^{ij} \equiv \lambda((\phi_u^i - \phi_b^i) - (\phi_u^j - \phi_b^j))$$

$$= \rho_{ub}^{ij} + c(dt_{ub}^{ij} - dT_{ub}^{ij}) - I_{ub}^{ij} + T_{ub}^{ij} + \lambda B_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_\Phi$$

$$= \rho_{ub}^{ij} - I_{ub}^{ij} + T_{ub}^{ij} + \lambda N_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_\Phi$$

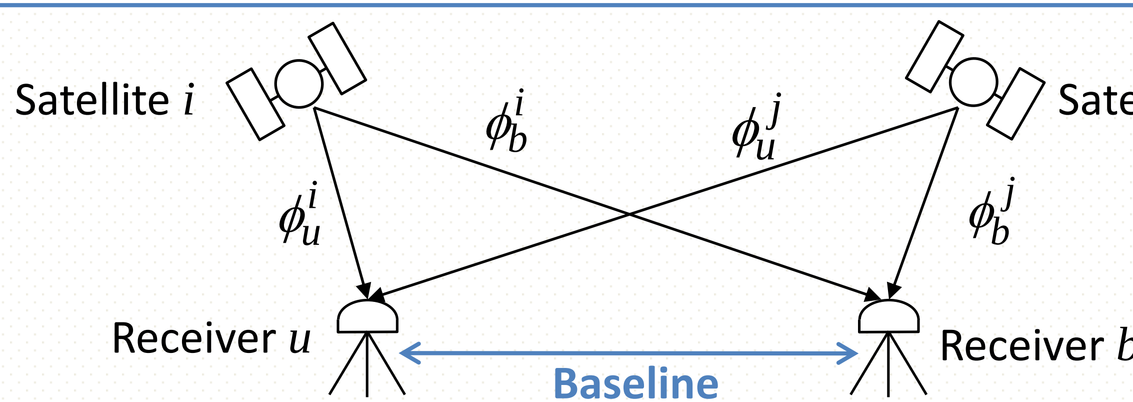
$$dt_{ub}^{ij} = dt_u^{ij} - dt_b^{ij} = 0,\ dT_{ub}^{ij} = dT_{ub}^i - dT_{ub}^j \approx 0$$

$$B_{ub}^{ij} = (\phi_{u,0} - \phi_0^i + N_u^i) - (\phi_{b,0} - \phi_0^i + N_b^i) - (\phi_{u,0} - \phi_0^j + N_u^j) + (\phi_{b,0} - \phi_0^j + N_b^j) = N_{ub}^{ij}$$

(short Baseline and same antenna type)

$$\Phi_{ub}^{ij} \approx \rho_{ub}^{ij} + \lambda N_{ub}^{ij} + \varepsilon_\Phi$$

$$I_{ub}^{ij} = I_{ub}^i - I_{ub}^j \approx 0, T_{ub}^{ij} = T_{ub}^i - T_{ub}^j \approx 0, d_{ub}^{ij} = d_{ub}^i - d_{ub}^j \approx 0$$



Memo for Misra & Enge:
http://gpspp.sakura.ne.jp/diary200608.htm

Baseline Processing

Nonlinear-LSE:

Parameter Vector:

$$\boldsymbol{x} = (\boldsymbol{r}_u{}^T, N_{ub}^{s_2 s_1}, N_{ub}^{s_3 s_1}, \ldots, N_{ub}^{s_m s_1})^T$$

Measurement Vector:

$$\boldsymbol{y} = (\boldsymbol{y}_{t_1}{}^T, \boldsymbol{y}_{t_1}{}^T, \ldots, \boldsymbol{y}_{t_n}{}^T)^T$$

Meas Model, Design Matrix:

$$\boldsymbol{h}(\boldsymbol{x}) = \left(\boldsymbol{h}_{t_1}(\boldsymbol{x})^T, \boldsymbol{h}_{t_2}(\boldsymbol{x})^T, \ldots, \boldsymbol{h}_{t_n}(\boldsymbol{x})^T\right)^T$$

$$\boldsymbol{H} = \left(\boldsymbol{H}_{t_1}{}^T, \boldsymbol{H}_{t_2}{}^T, \ldots, \boldsymbol{H}_{t_n}{}^T\right)^T$$

Meas Error Covariance:

$$\boldsymbol{R} = blkdiag\left(\boldsymbol{R}_{t_1}, \boldsymbol{R}_{t_2}, \ldots, \boldsymbol{R}_{t_n}\right)$$

Solution (Static/Float):

$$\hat{\boldsymbol{x}} = \boldsymbol{x}_0 + (\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H})^{-1} \boldsymbol{H}^T \boldsymbol{R}^{-1} (\boldsymbol{y} - \boldsymbol{h}(\boldsymbol{x}_0))$$

$$\boldsymbol{y}_{t_k} = (\Phi_{ub,t_k}^{s_2 s_1}, \Phi_{ub,t_k}^{s_3 s_1}, \ldots, \Phi_{ub,t_k}^{s_m s_1})^T$$

$$\boldsymbol{h}_{t_k}(\boldsymbol{x}) = \begin{pmatrix} \rho_{u,t_k}^{s_2 s_1} - \rho_{b,t_k}^{s_2 s_1} + \lambda N_{ub}^{s_2 s_1} \\ \rho_{u,t_k}^{s_3 s_1} - \rho_{b,t_k}^{s_3 s_1} + \lambda N_{ub}^{s_m s_1} \\ \vdots \\ \rho_{u,t_k}^{s_m s_1} - \rho_{b,t_k}^{s_m s_1} + \lambda N_{ub}^{s_m s_1} \end{pmatrix}$$

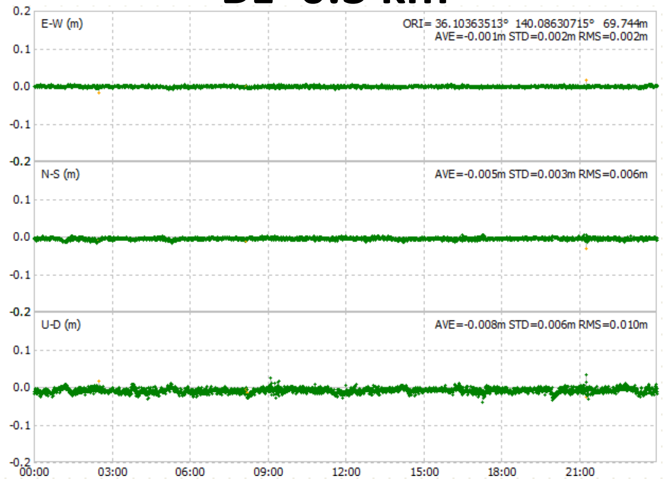
$$\boldsymbol{H}_{t_k} = \begin{pmatrix} -\boldsymbol{e}_{u,t_k}^{s_2 s_1 T} & \lambda & 0 & \cdots & 0 \\ -\boldsymbol{e}_{u,t_k}^{s_3 s_1 T} & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{e}_{u,t_k}^{s_m s_1 T} & 0 & 0 & \cdots & \lambda \end{pmatrix}$$

$$\boldsymbol{R}_{t_k} = \begin{pmatrix} 4\sigma_\Phi{}^2 & 2\sigma_\Phi{}^2 & \cdots & 2\sigma_\Phi{}^2 \\ 2\sigma_\Phi{}^2 & 4\sigma_\Phi{}^2 & \cdots & 2\sigma_\Phi{}^2 \\ \vdots & \vdots & \ddots & \vdots \\ 2\sigma_\Phi{}^2 & 2\sigma_\Phi{}^2 & \cdots & 4\sigma_\Phi{}^2 \end{pmatrix}$$

$\boldsymbol{r}_b$: Fixed Base-Station Position

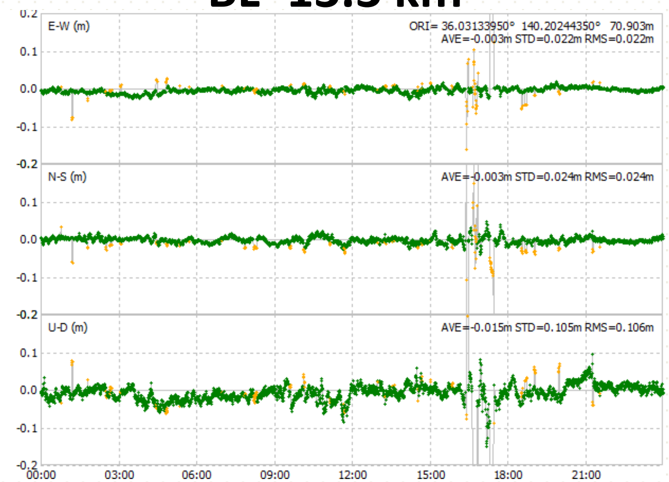
Effect of Baseline Length

BL=0.3 km



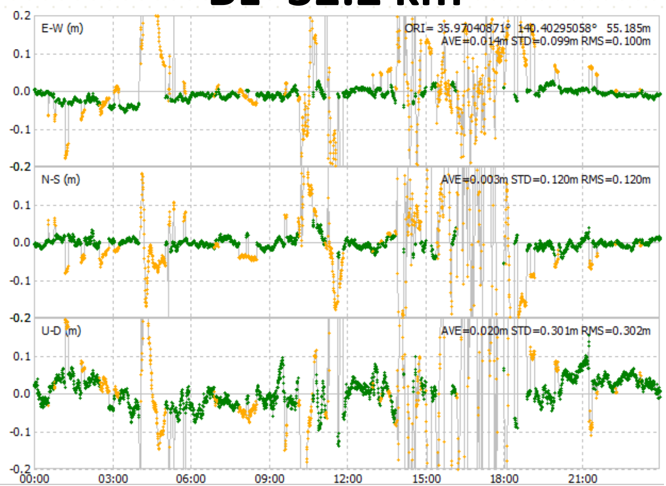
RMS Error:
E: 0.2cm
N: 0.6cm
U: 1.0cm
Fix Ratio:
99.9%

BL=13.3 km



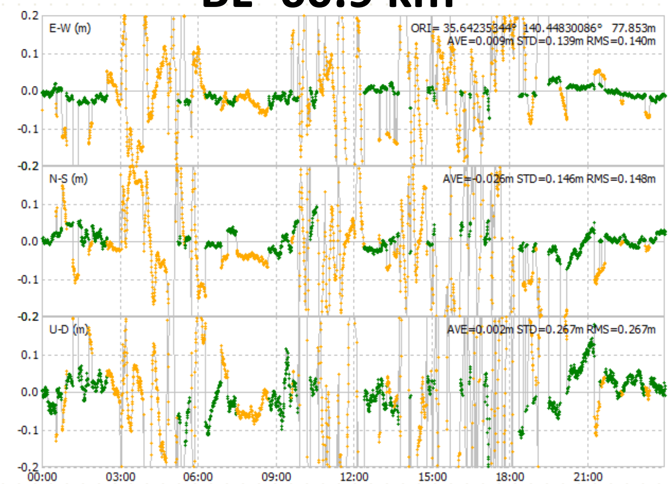
RMS Error:
E: 2.2cm
N: 2.4cm
U: 10.6cm
Fix Ratio:
94.2%

BL=32.2 km



RMS Error:
E: 10.0cm
N: 12.0cm
U: 30.2cm
Fix Ratio:
64.3%

BL=60.9 km



RMS Error:
E: 14.0cm
N: 14.8cm
U: 26.7cm
Fix Ratio:
44.4%

(24 hr Kinematic ●: Fixed Solution ●: Float Solution)

Integer Ambiguity Resolution

- **Objectives**
  - More accurate than float solutions
  - Fast converge of solutions
- **Many AR Strategies**
  - Simple Integer rounding
  - Multi-frequency wide-lane and narrow-lane generation
  - Search in coordinate domain
  - Search in ambiguity domain
  - AFM, FARA, LSAST, LAMBDA, ARCE, HB-L^3, Modified Cholesy Decomposition, Null Space, FAST, OMEGA, ...

ILS (Integer Least Square Estimation)

Problem:

$$\boldsymbol{x} = (\boldsymbol{a}^T, \boldsymbol{b}^T)^T,\ \boldsymbol{H} = (\boldsymbol{A}, \boldsymbol{B})$$

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v} = \boldsymbol{A}\boldsymbol{a} + \boldsymbol{B}\boldsymbol{b} + \boldsymbol{v}$$

$$\breve{\boldsymbol{x}} = \underset{\boldsymbol{a}\in \boldsymbol{Z}^n, \boldsymbol{b}\in \boldsymbol{R}^m}{\arg\min}\ (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^T \boldsymbol{Q}_y^{-1} (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})$$

Strategy:

(1) Conventional LSE

$$\hat{\boldsymbol{x}} = \begin{pmatrix} \hat{\boldsymbol{a}} \\ \hat{\boldsymbol{b}} \end{pmatrix} = \boldsymbol{Q}_x \boldsymbol{H}^T \boldsymbol{Q}_y^{-1} \boldsymbol{y},\ \boldsymbol{Q}_x = \begin{pmatrix} \boldsymbol{Q}_a & \boldsymbol{Q}_{ab} \\ \boldsymbol{Q}_{ba} & \boldsymbol{Q}_b \end{pmatrix} = (\boldsymbol{H}^T \boldsymbol{Q}_y \boldsymbol{H})^{-1}$$

(2) **Search Integer Vector** with Minimum Squared Residuals

$$\breve{\boldsymbol{a}} = \underset{\boldsymbol{a}\in \boldsymbol{Z}^n}{\arg\min} (\hat{\boldsymbol{a}} - \boldsymbol{a})^T \boldsymbol{Q}_a^{-1} (\hat{\boldsymbol{a}} - \boldsymbol{a})$$

(3) Improve solution

$$\breve{\boldsymbol{b}} = \hat{\boldsymbol{b}} - \boldsymbol{Q}_{ba} \boldsymbol{Q}_a^{-1} (\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}})$$

LAMBDA

Teunissen, P.J.G. (1995)
The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. *Journal of Geodesy,* Vol. 70, No. 1-2, pp. 65-82.

- **ILS Estimation with**:
  - Shrink Integer Search Space with "Decorrelation"
  - Efficient Tree Search Strategy
  - Similar to *Closest Point Search with LLL Lattice Basis Reduction* Algorithm

$$\breve{\boldsymbol{a}} = \arg\min_{\boldsymbol{a} \in \boldsymbol{Z}^n} (\hat{\boldsymbol{a}} - \boldsymbol{a})^T \boldsymbol{Q}_a{}^{-1} (\hat{\boldsymbol{a}} - \boldsymbol{a})$$

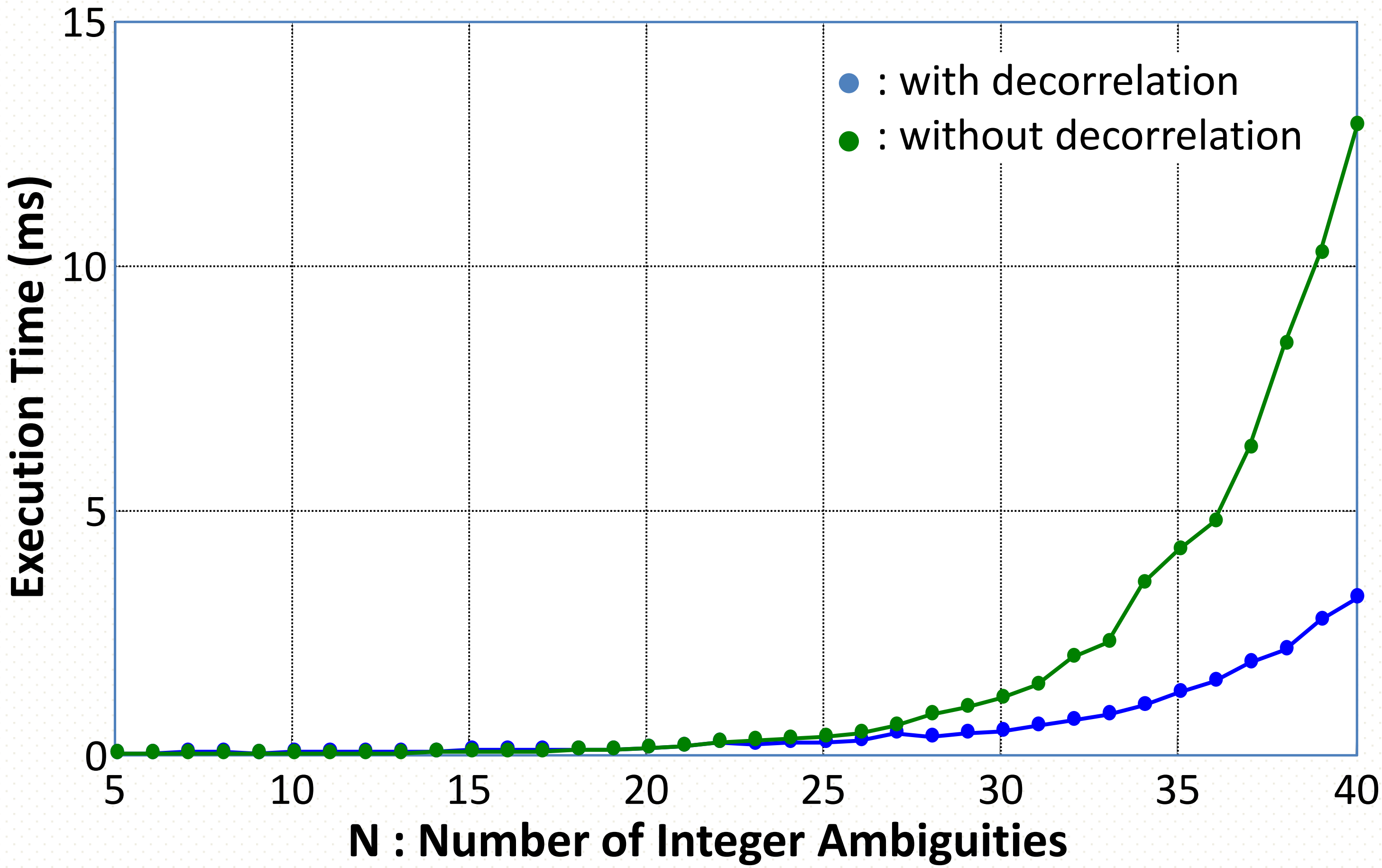
$\Leftrightarrow$

$$\hat{\boldsymbol{z}} = \boldsymbol{Z}^T \hat{\boldsymbol{a}},\ \boldsymbol{Q}_z = \boldsymbol{Z}^T \boldsymbol{Q}_a \boldsymbol{Z}$$

$$\breve{\boldsymbol{z}} = \arg\min_{\boldsymbol{z} \in \boldsymbol{Z}^n} (\hat{\boldsymbol{z}} - \boldsymbol{z})^T \boldsymbol{Q}_z{}^{-1} (\hat{\boldsymbol{z}} - \boldsymbol{z})$$

$$\breve{\boldsymbol{a}} = \boldsymbol{Z}^{-T} \breve{\boldsymbol{z}}$$

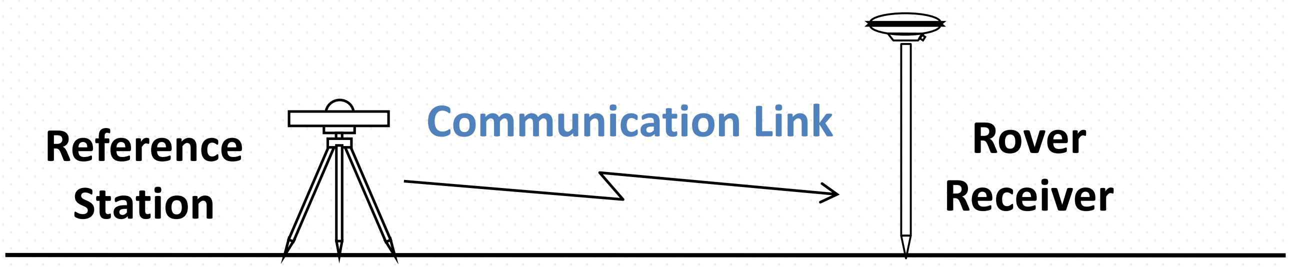
Performance of LAMBDA



(Pentium 4 3.2GHz, Intel C/C++ 8.0)

RTK (Real-Time Kinematic)

- **Technique with Baseline Processing**
  - Real-time Position of Rover Antenna
  - Transmit Reference Station Data to Rover via Comm. Link
  - OTF (On-the-Fly) Integer Ambiguity Resolution
  - Typical Accuracy: 1 cm + 1ppm x BL RMS (Horizontal)
  - Applications:
    Land Survey, Construction Machine Control, Precision Agriculture etc.



RTK Application

Geodetic Survey

Construction Machine Control

Precision Agriculture

ITS (Intelligent Transport System)

Mobile Mapping System

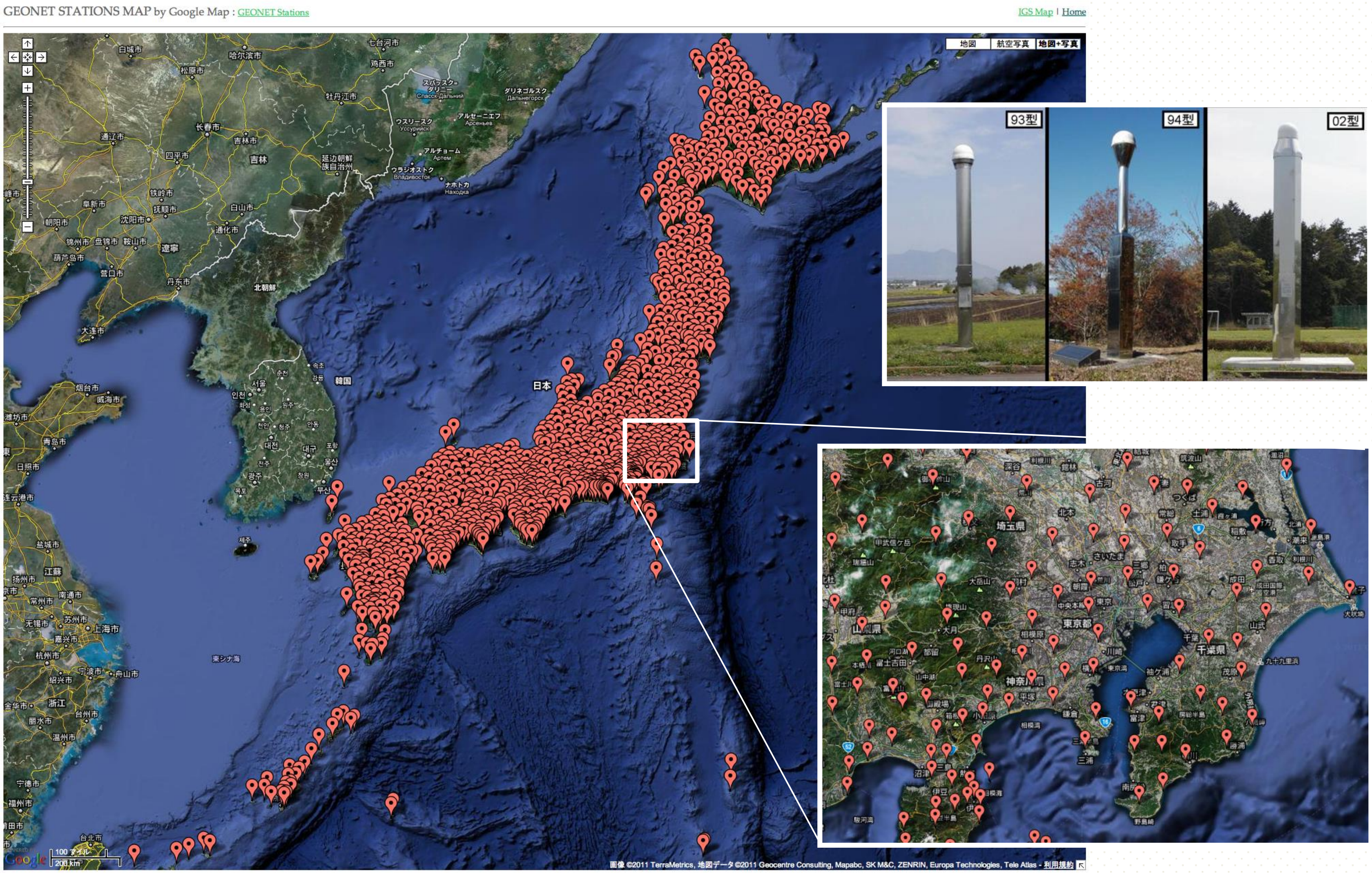
Sports

http://www.trimble.com, http://www.leica-geosystems.com, http://www.gpsworld.com

Network RTK (NRTK)

- **Extension of RTK**
  - RTK without User Reference Station
  - Sparse Networked Reference Stations
  - Correction Messages via Mobile-Phone Network
  - Format: **VRS**, **FKP**, MAC, RTCM 2.3, RTCM 3.1
  - Server S/W: Trimble GPSNet, GEO++ GNSMART, ...
  - NTRIP Networked Transport of RTCM via Internet Protocol
- **NRTK Service in Japan**
  - GEONET: ~1200 Reference Stations by GSI
  - NGDS (www.gpsdata.co.jp), JENOBA (www.jenoba.jp)

Japanese GEONET



(http://terras.gsi.go.jp/ja/index.htm)

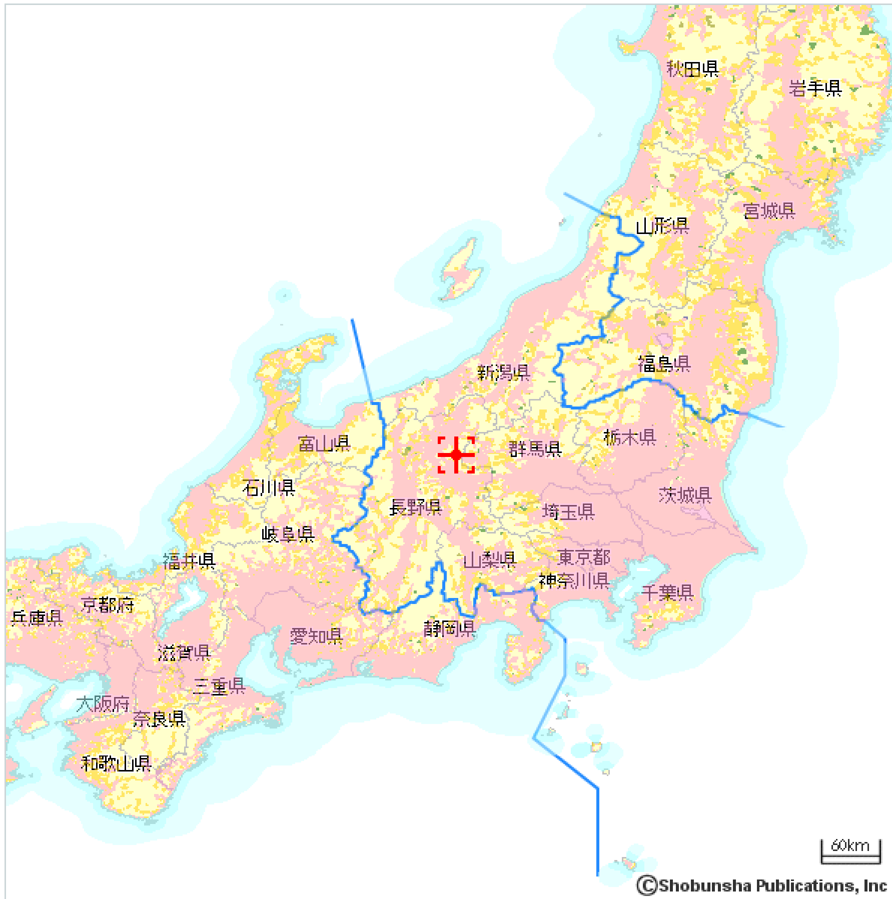
Considerations for RTK System

- **Rover**
  - Single vs. Dual-freq, Update Rate, GNSS, Receiver-cost
  - CPU Power for external processing
  - INS-integration for obstacles
- **Reference Station**
  - Baseline-Length vs. Performance
  - Self-provided vs. NRTK Service
  - Coverage, Receiver-cost, Operational-cost, Service-fee
- **Communication Link**
  - Coverage, Band-width, Latency, Link-cost

Communication Link for RTK

- **Local (<300 m)**
  - Serial, USB, LAN, ... (wired)
  - Radio Modem, WiFi, ZigBee, DSRC, ... (wireless)
- **Regional (<1,000 km)**
  - Analog-phone, ISDN, Dedicated Link, ... (wired)
  - Mobile-phone (Analog, 2G, 3G, ...), ... (wireless)
- **Global (<10,000 km)**
  - Internet
  - GEO Satellite Link (Inmarsat, WideStar II, ...)
  - LEO Satellite Link (Iridum, Orbicom, ...)

Coverage by Mobile-phone N/W



NTT docomo FOMA (2008/9)

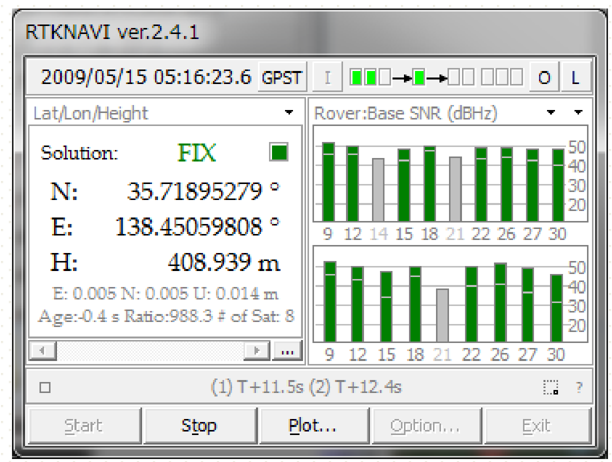
(http://servicearea.nttdocomo.co.jp)

B-5
RTKLIB Practice (3)

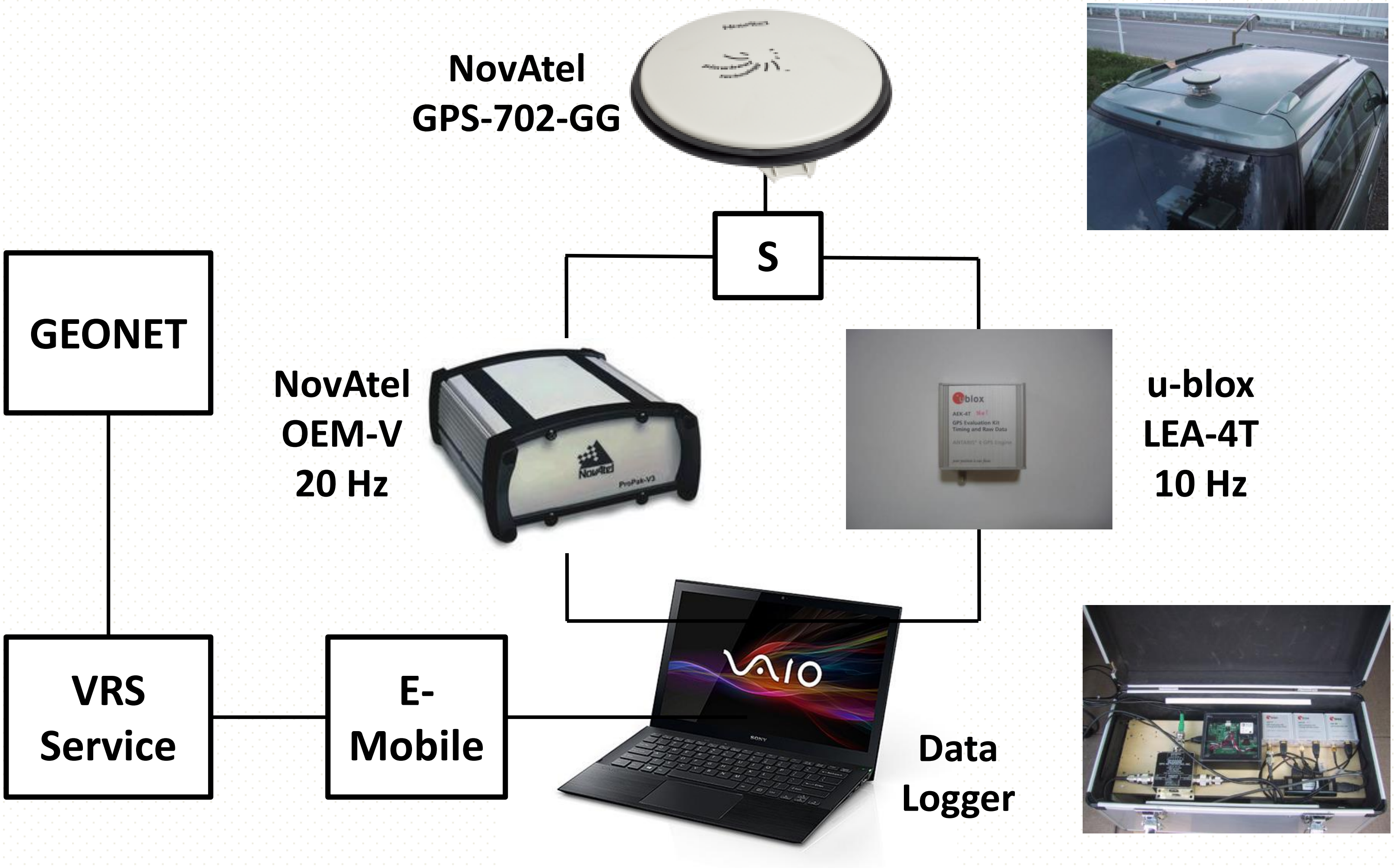
RTK by Playback Data

- **Objective**
  RTK of by Playback Data
- **Program**
  rtklib_2.4.2p9¥bin¥rtknavi.exe
- **Data**
  sample2¥
  oemv_2009515c.gps (NovAtel)
  ubx_20090515c.ubx (u-blox)
  0263_20090515c.rtcm3 (VRS)

RTKNAVI

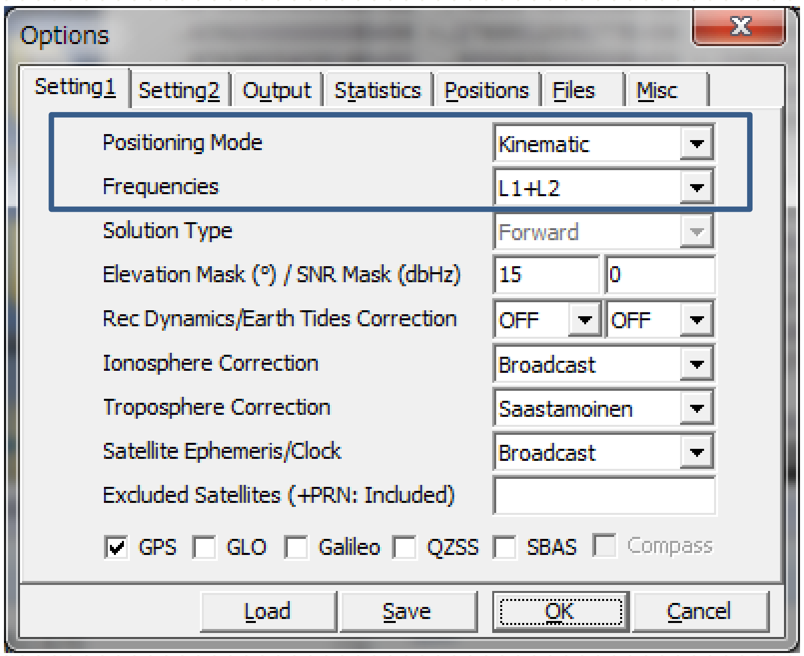


Playback Data

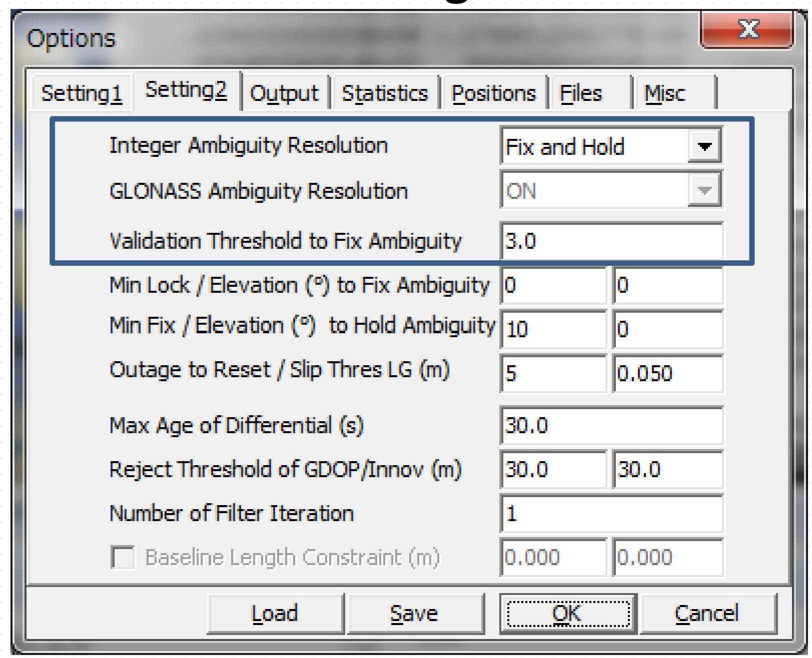


RTKNAVI - Options

Setting1



Setting2

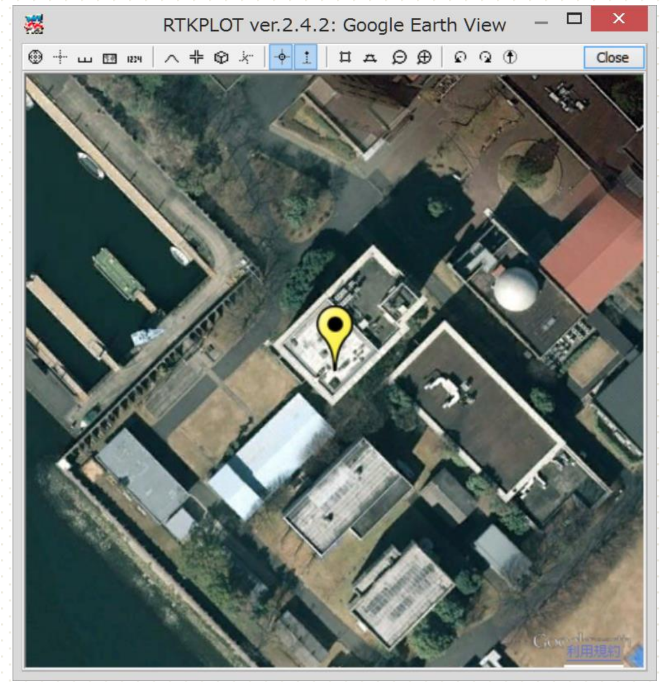


Reference Station for RTK

Receiver:
Trimble NetR9

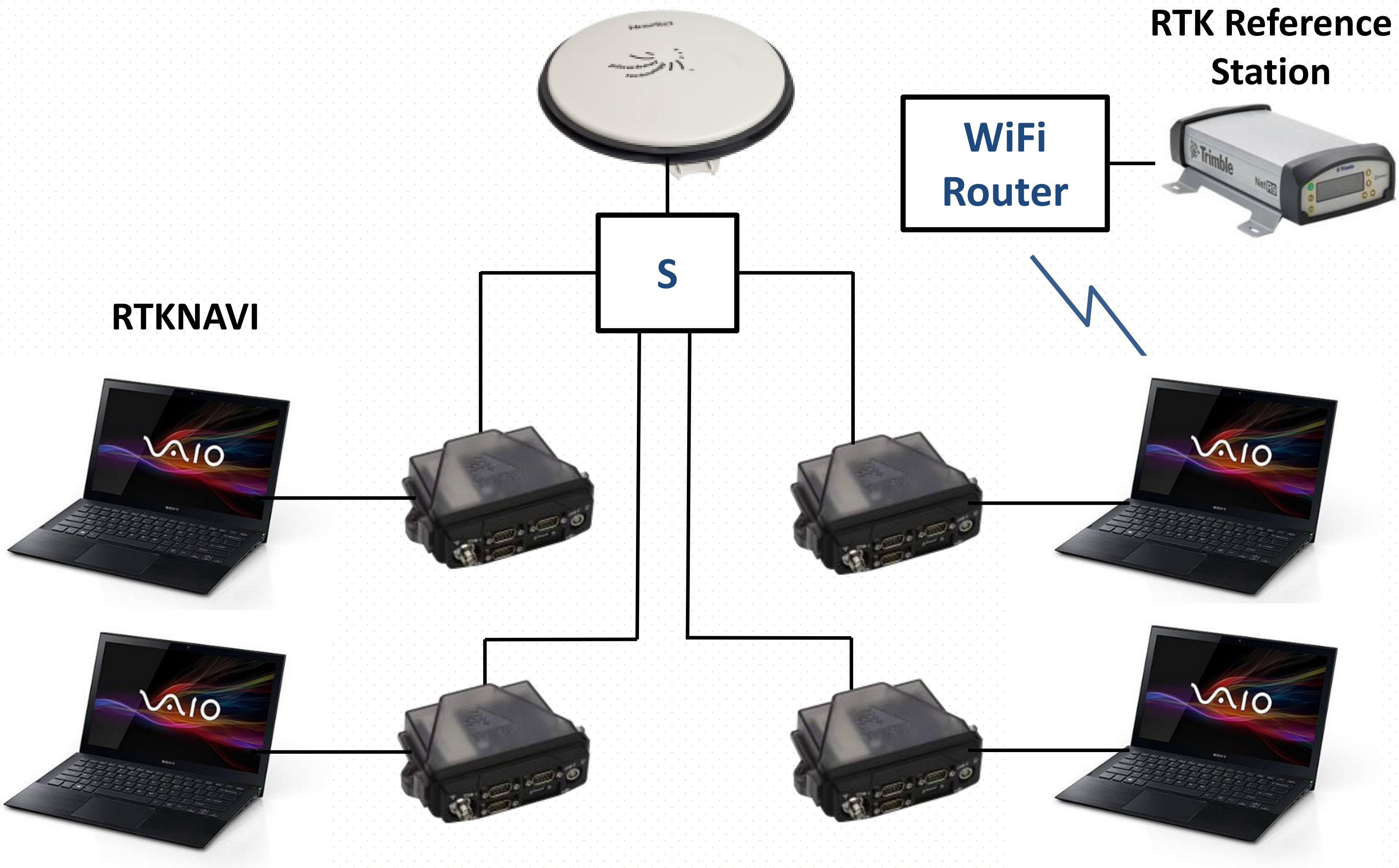
IP-Addr	: ***.***.***.***
Port	: 2101
MountP	: RTCM3
USER-ID	: ********
PW	: ****
Data	: RTCM 3, GPS+GLO

IP-Addr	: ***.***.***.***
Port	: 2102
MountP	: BINEX
USER-ID	: ********
PW	: ****
Data	: BINEX, GPS+GLO+GAL+QZS+BDS



Lat: 35.666243069
Lon: 139.792308111
Height: 59.8700

RTK Setup for Port Cruise



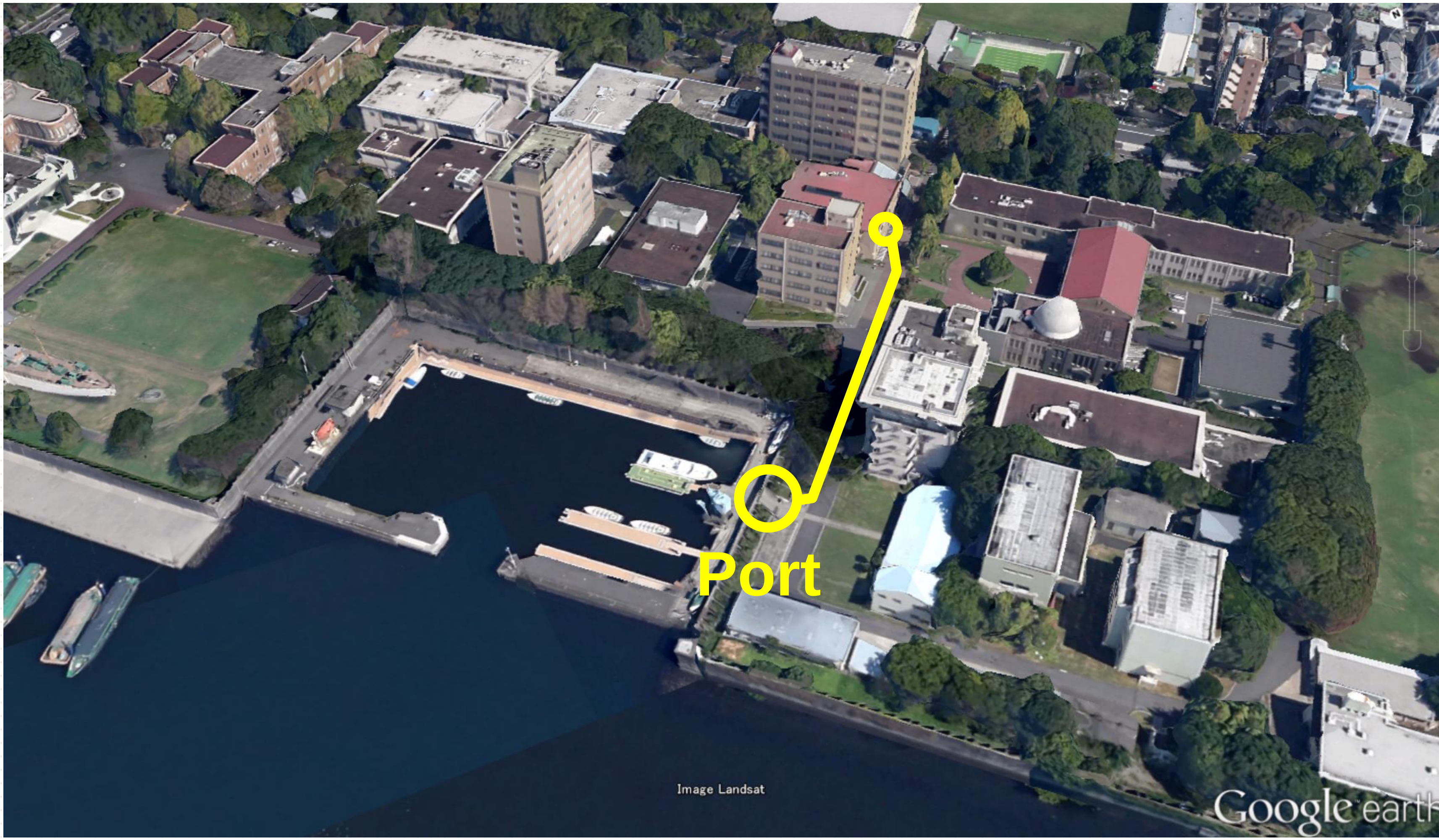
What is AIS ? by JRC

Port Cruise Demo & RTKLIB Practice (4)

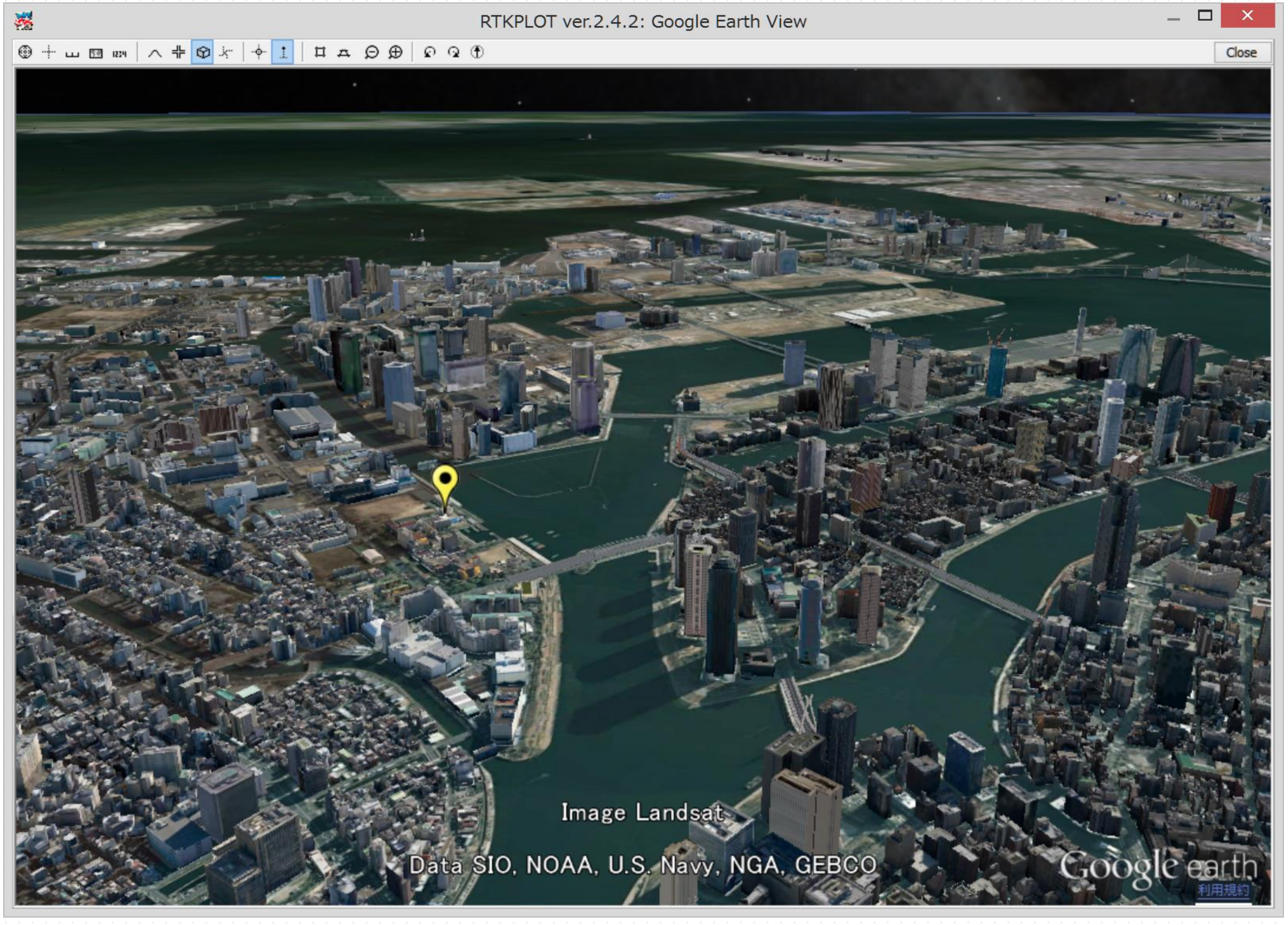
Port Cruise Schedule

- **Group 1 (A, B, C, D)**
  - **Loading : 12:55 (keep it!)**
  - **Start : 13:15**
  - **Return : 14:20**
- **Group 2 (E, F, G, H)**
  - **Loading : 14:15 (keep it!)**
  - **Start : 14:35**
  - **Return : 15:40**
- **Waiting Group**
  - Self Practice of RTKLIB on the Ground

Loading of Boat



Port Cruise with RTK



B-6
Advanced Topics

Advanced Topics

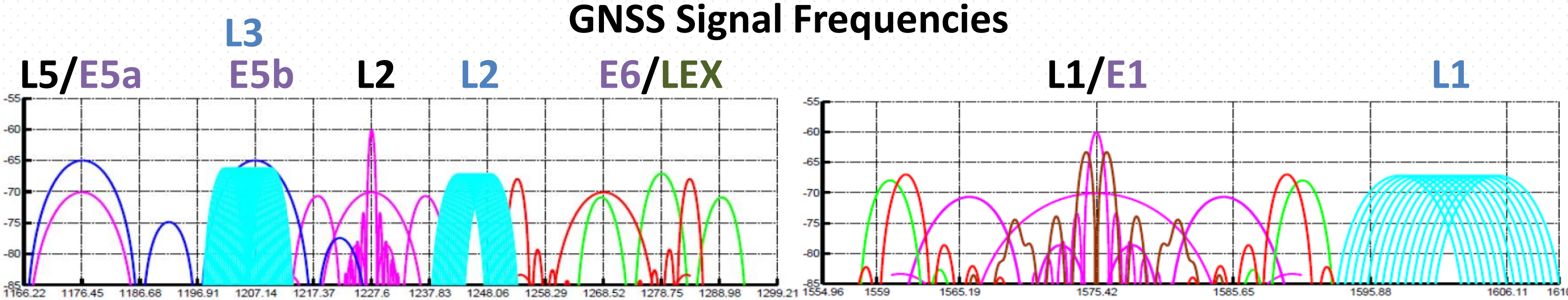
- Multi-GNSS RTK
- Long-Baseline RTK
- INS-Aided RTK
- RTK-PPP

GNSS Evolution

Number of Planned GNSS Satellites

System	2010	2013	2016	2019
GPS	**31**	31	32	32
GLONASS	23 (+2)	24 (+3)	24 (+3)	24 (+3)
Galileo	0	4	18	27 (+3)
Compass	6	16	30	32 (+3)
QZSS	1	1	4	7
IRNSS	0	7	7	7
SBAS	7	8	11	11
Total	**68**	**91**	**126**	**140**

GNSS Signal Frequencies



(Y.Yang, COMPASS: View on Compatibility and Interoperability, 2009)

Multi-GNSS RTK Performance

RTK Performance: Baseline 13.3 km, Instantaneous AR

GPS	Galileo	El Mask=15°				El Mask=30°			
		Fixing Ratio	RMS Error (cm)			Fixing Ratio	RMS Error (cm)		
			E-W	N-S	U-D		E-W	N-S	U-D
L1	-	49.7%	4.6	8.1	19.0	23.3%	71.4	115.0	289
L1,L2	-	99.0%	1.4	1.3	1.9	87.6%	3.4	10.5	15.5
L1,L2,L5	-	99.0%	1.4	1.3	1.9	87.3%	3.4	10.5	15.6
L1	E1	98.8%	1.3	1.2	1.9	90.1%	1.2	2.1	2.7
L1,L2	E1	98.9%	1.4	1.2	1.7	98.7%	1.2	1.0	1.6
L1,L2,L5	E1,E5a, E5b	98.9%	1.5	1.3	2.0	98.9%	1.3	1.1	1.8

GPS L1+L2

GPS+GAL L1

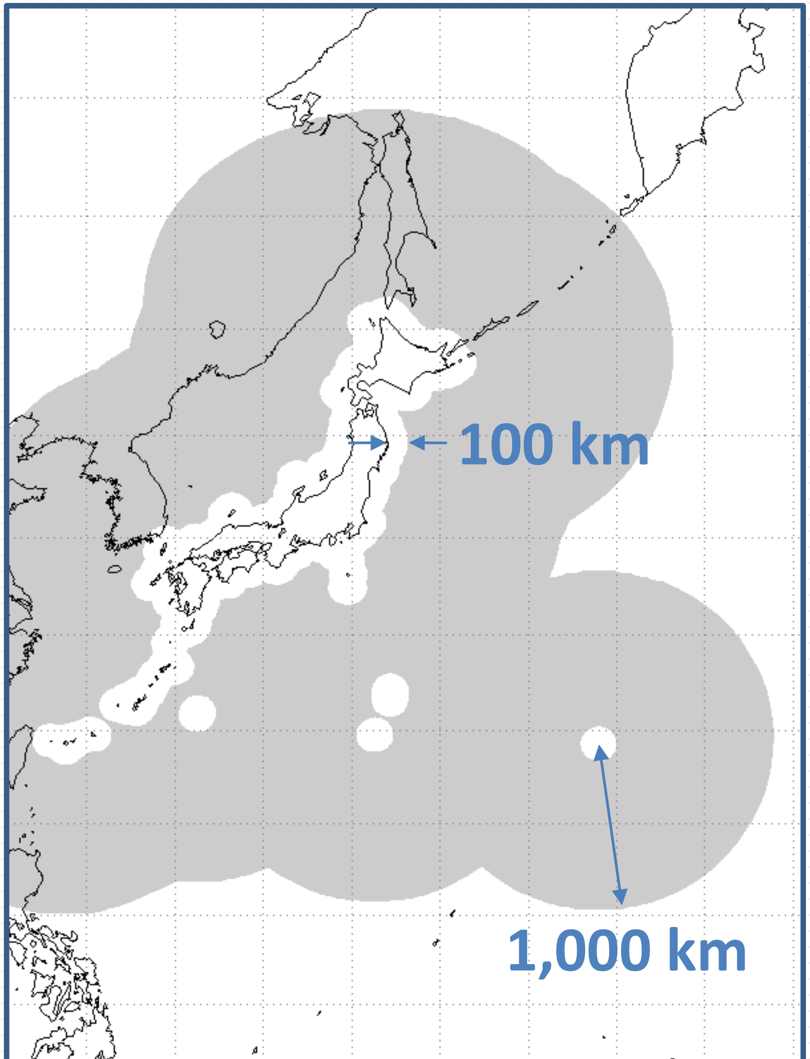
Multi-GNSS Receiver

- **Moore's Law**
  - More correlators
  - More tracking channels
  - More powerful embedded CPU
- **Consumer-grade Multi-GNSS Receiver**
  - SkyTraq: GPS + GLONASS
  - STMicro: GPS + GLONASS
  - Broadcom: GPS + GLONASS + QZSS
  - u-blox: GPS + Galileo

Issues for Multi-GNSS RTK

- **Multi-GNSS Integration Issue**
  - Time-system, Coordinate-system
  - Receiver H/W Biases
- **Multi-code System Issue**
  - L1C/A-L1P(Y)-L1Cd-L1Cp, L2P(Y)-L2C, L5I-L5Q
  - Quarter cycle phase-shift problem
- **GLONASS FDMA Issue**
  - Receiver Inter-channel biases (Receiver Interoperability)
  - Calibration Message Standard
  - Antenna Calibration

Long-Baseline RTK



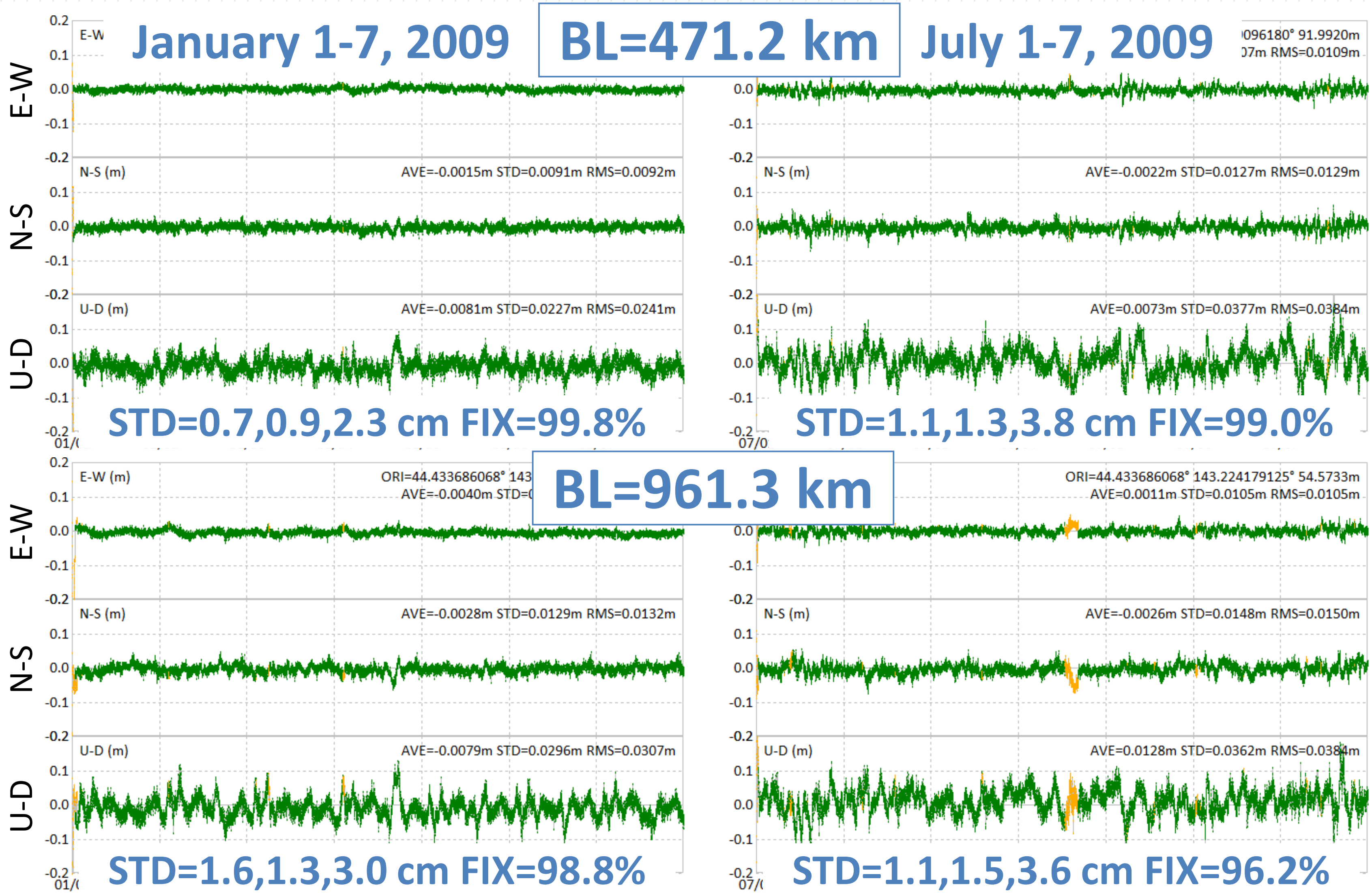
GPS Tsunami Monitoring System (Currently ~15 km off-shore)

http://www.tsunamigps.com

Long-Baseline RTK Strategy

	BL (km)	Error Elimination				Strategy
		Ephem	Ionos	Tropos	Others	
S	0 – 10	Broadcast	-	-	-	Conventional RTK
M	10 – 100	Broadcast	Dual-Freq	-	-	
			Interpolation		-	Network RTK
L	100 – 1,000	Real-time Precise (IGU)	Dual-Freq	Estimate ZTD + MF	Earth Tides	Long-Baseline RTK
VL	>1,000	Non-RT Precise (IGR, IGS)	Dual-Freq	Estimate ZTD + MF	Earth Tides, Ph-WU	Post-Processing or PPP

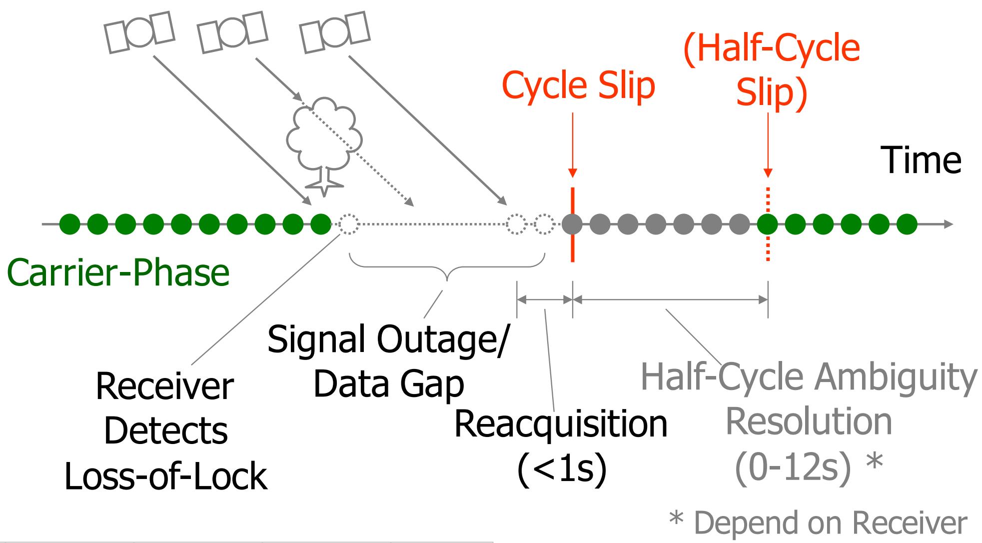
Long-Baseline RTK with RTKLIB



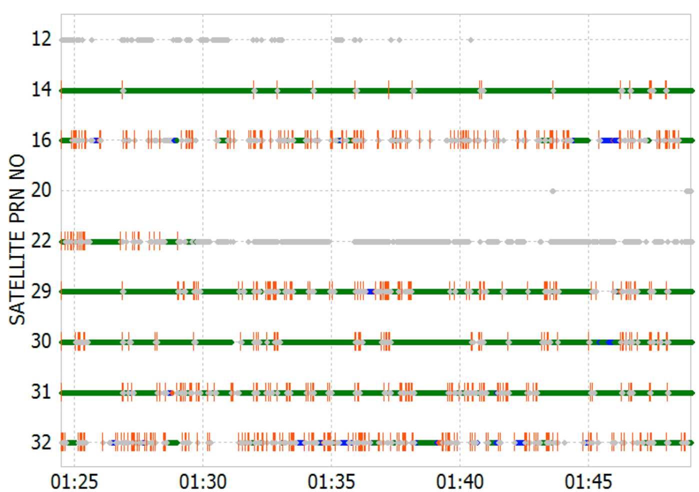
Mobile AP issues for RTK

- **Cycle-Slips**
  - Frequent cycle-slip with around obstacles
  - Miss-detection of cycle-slip
- **Low Solution Availability**
  - Long acquisition time by weak signal (Low C/N0)
  - Half-cycle ambiguity resolution with Costas-PLL
  - Low fixing ratio
- **High Noise Level**
  - High multipath level even in carrier-phase
  - Jamming by RFI

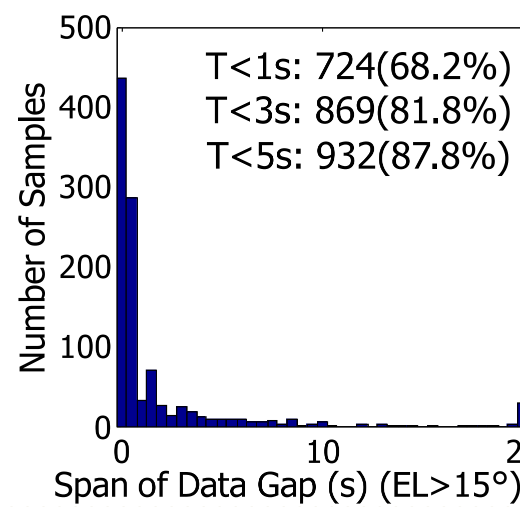
Cycle-Slips



Cycle Slip
(Half-Cycle Slip)
Time
Carrier-Phase
Signal Outage/ Data Gap
Receiver Detects Loss-of-Lock
Reacquisition (<1s)
Half-Cycle Ambiguity Resolution (0-12s) *
* Depend on Receiver

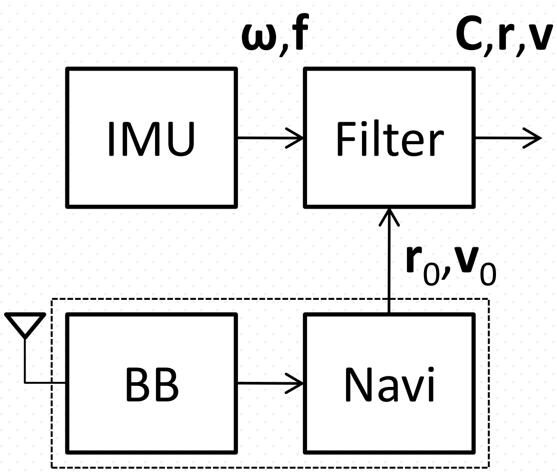


SATELLITE PRN NO
12
14
16
20
22
29
30
31
32
01:25
01:30
01:35
01:40
01:45

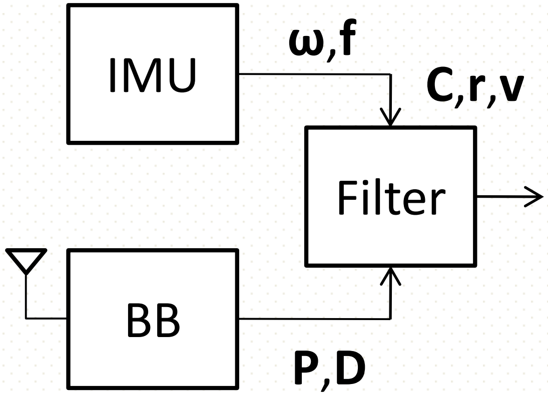


T<1s: 724(68.2%)
T<3s: 869(81.8%)
T<5s: 932(87.8%)
Number of Samples
0
100
200
300
400
500
0
10
20
Span of Data Gap (s) (EL>15°)

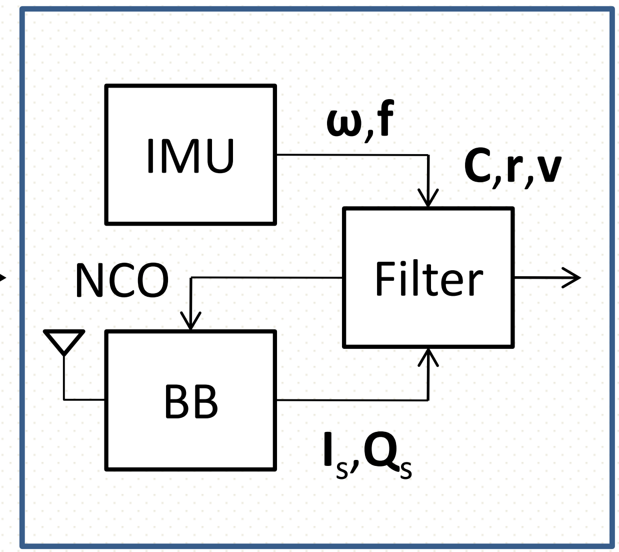
INS-Aided RTK



Loosely-Coupled Integration



Tightly-Coupled Integration



Deep Integration (Ultra-Tightly)

High sensitivity (DLL, PLL)

Slip resistance

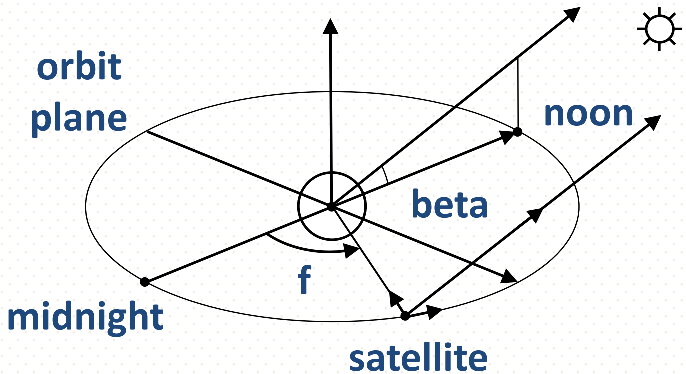
RTK-PPP

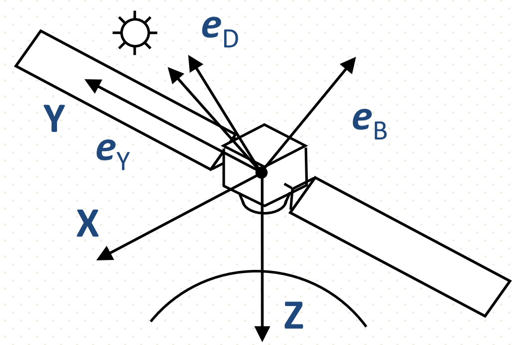
- **Combination of RTK and PPP**
  - Precise Orbits and Clocks
  - Ambiguity Resolution in PPP
  - Local Augmentation (Iono. and Tropos.)
- **RTCM SSR (RTCM 3.2, 3.5.12)**
  - Level 1: Precise Orbits, Clock and Code Biases
  - Level 2: Iono. (VTEC)
  - Level 3: Iono. (STEC), Tropos. and Phase Biases

Precise Orbits and Clock

- **Satellite Orbit Models**
  - EGM 2008+solid earth tide+FES2004
  - Sun, Moon, Venus and Jupiter with JPL DE421
  - Empirical SRP model, ...
- **Measurement Models**
  - ZD Iono-free phase+ pseudorange, 2nd-order-iono
  - ZTD+gradient estimation with GPT+GMF/VMF1
  - IERS DEHANTIDEINEL+FES2004+pole tide+CMC
- **ECI-ECEF Coordinates Transformation**
  - IAU 2000A/2006 by IAU SOFA

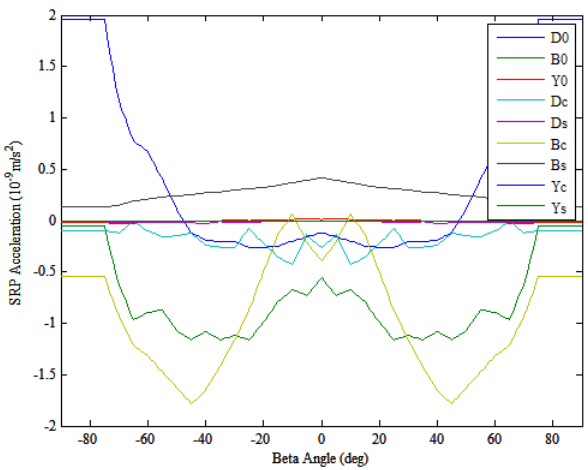
SRP Model



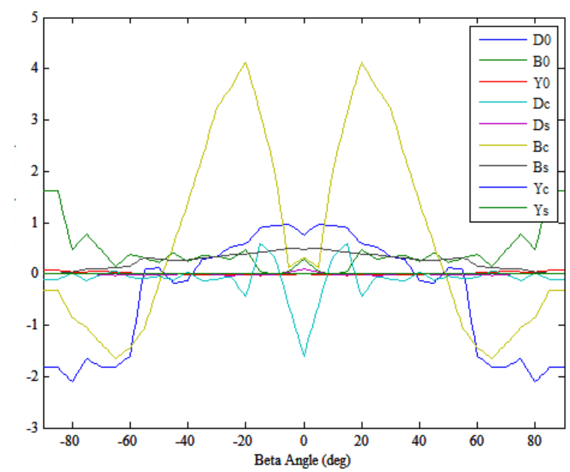


$$a_{srp} = S\,((D_0 + D_C \cos f + D_S \sin f)\, \boldsymbol{e}_D + (B_0 + B_C \cos f + B_S \sin f)\, \boldsymbol{e}_B + (Y_0 + Y_C \cos f + Y_S \sin f)\, \boldsymbol{e}_Y) \times 10^{-9}\ (m/s^2)$$

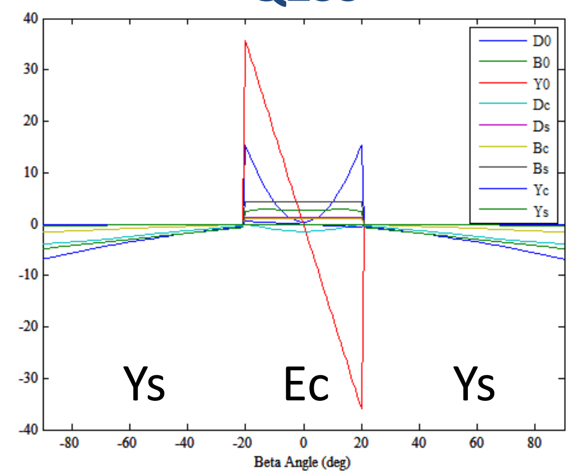
GPS Block IIR



GLONASS



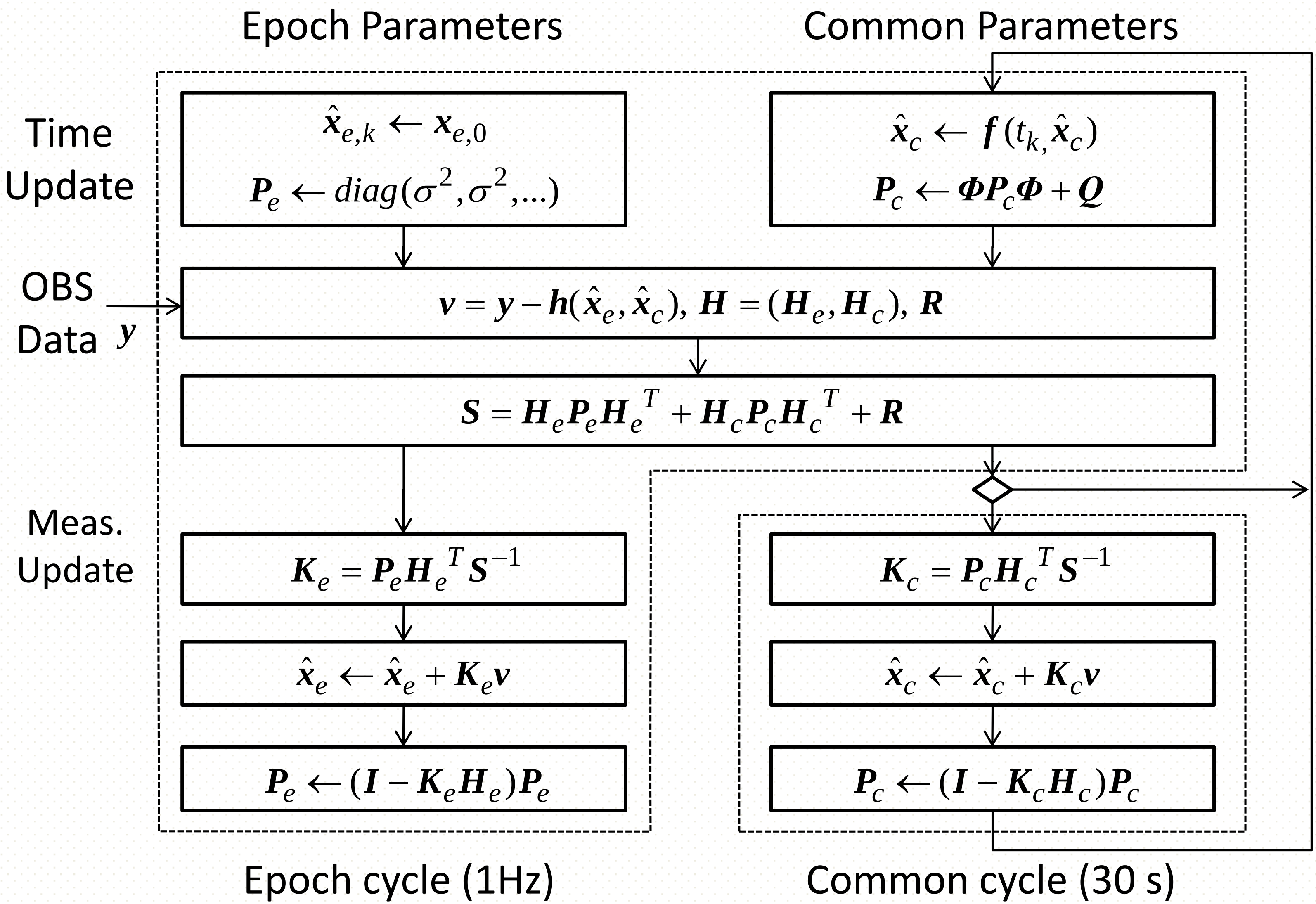
QZSS



Parameter Adjustment

	Offline	Real-Time
Algorithm	**Iterated Weighted LSQ**	**Dual-Cycle-EKF**
Estimated Parameters	Orbit, SRP/Emp-Acc, Clock, Position, ZTD/Grad, Ambiguity, Bias, EOP	
Measurements	ZD Carrier-Phase and Peudo-range	
Numerical Solver	NEQ by Cholesky Factorization	**Numerical Stable EKF**
Clock Estimation	**Parameter Elimination in NEQ**	State as White-Noise or Random-Walk
Integer Ambiguity Resolution	**Network AR** (Ge., 2005)	Real-Time Network AR

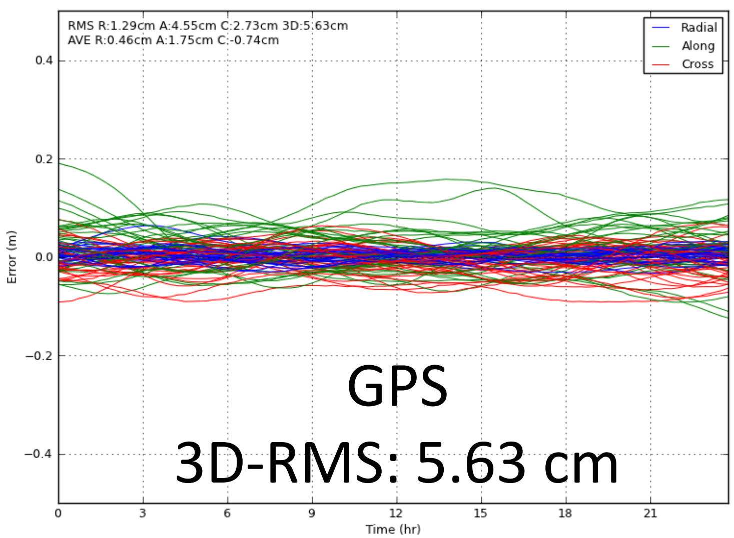
Dual-Cycle-EKF



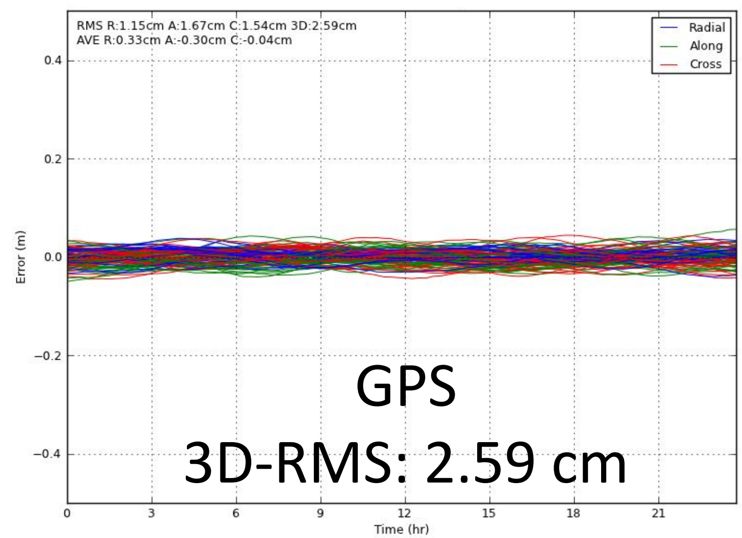
Network AR

- **Dynamic baseline selection to convert ZD to DD**
- **WL and NL DD ambiguities by rounding**
- **Validation by confidence function and FCB**
- **For GPS, QZSS and Galileo (no GLONASS)**

AR-OFF



AR-ON



Numerically Stable EKF

Measurement Update of EKF

$$K = P^- H^T (HP^- H^T + R)^{-1}$$

$$x^+ = x^- + K(y - h(x^-))$$

$$P^+ = (I - KH)P^-$$

Standard EKF

(1) $\mathbf{v} = \mathbf{y} - \mathbf{h}(\mathbf{x})$, $\mathbf{H}$, $\mathbf{R}$	
(2) $\mathbf{D} = \mathbf{P}\,\mathbf{H}^T$	(sparse)
(3) $\mathbf{S} = \mathbf{H}\,\mathbf{D} + \mathbf{R}$	(sparse)
(4) $\mathbf{U} = \mathrm{chol}(\mathbf{S})$	DPOTRF
(5) $\mathbf{K} = (\mathbf{D}\,\mathbf{U}^{-1})\,\mathbf{U}^{-T}$	DTRSM
(6) $\mathbf{x} = \mathbf{x} + \mathbf{K}\,\mathbf{v}$	DGEMV
(7) $\mathbf{P} = \mathbf{P} - \mathbf{K}\,\mathbf{D}^T$	DGEMM

Numerically Stable EKF

(1) $\mathbf{v} = \mathbf{y} - \mathbf{h}(\mathbf{x})$, $\mathbf{H}$, $\mathbf{R}$	
(2) $\mathbf{D} = \mathbf{P}\,\mathbf{H}^T$	(sparse)
(3) $\mathbf{S} = \mathbf{H}\,\mathbf{D} + \mathbf{R}$	(sparse)
(4) $\mathbf{U} = \mathrm{chol}(\mathbf{S})$	DPOTRF
(5) $\mathbf{E} = \mathbf{D}\,\mathbf{U}^{-1}$	DTRSM
(6) $\mathbf{K} = \mathbf{E}\,\mathbf{U}^{-T}$	DTRSM
(7) $\mathbf{x} = \mathbf{x} + \mathbf{K}\,\mathbf{v}$	DGEMV
(8) $\mathbf{P} = \mathbf{P} - \mathbf{E}\,\mathbf{E}^T$	DSYRK

GPS Orbit Quality by MADOCA

IGS AC	Analysis Software	# of Stas	GPS Orbit RMS (cm)				Clock (ns)	
			R	A	C	3D	STD	RMS
	MADOCA 0.3.0	**77**	**0.89**	**1.10**	**1.12**	**1.81**	**0.109**	**0.131**
ESA	NAPEOS 3.5	110	0.97	1.33	1.09	1.98	0.116	0.183
CODE	Bernese 5.1	231	1.01	1.36	1.14	2.04	0.075	0.089
NGS	arc, orb, pages, gpscom	199	0.95	1.46	1.41	2.24	-	-
GFZ	EPOS.P.V2	191	1.15	1.64	1.59	2.56	0.146	0.169
MIT	GAMIT 10.33, GLOBK 5.16	263	1.37	2.12	1.39	2.88	0.277	0.316
NRCan	GIPSY/OASIS-II 5.0	91	2.58	1.72	1.77	3.57	0.128	0.148
JPL	GIPSY/OASIS-II 5.0	142	2.62	1.67	1.98	3.68	0.168	0.226
SIO	GAMIT 10.20, GLOBK 5.08	258	2.42	2.26	1.77	3.75	-	-
GRG	GINS, DYNAMO	134	2.47	2.80	1.74	4.12	0.172	0.212

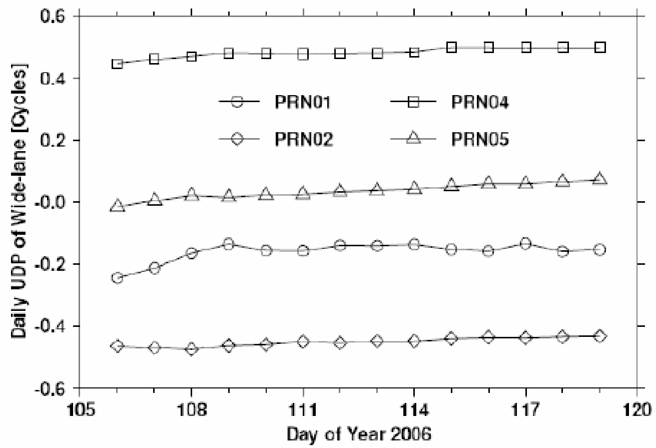
2011/01/01 -2011/12/31 (365 days), wrt IGS Final

Ambiguity Resolution in PPP

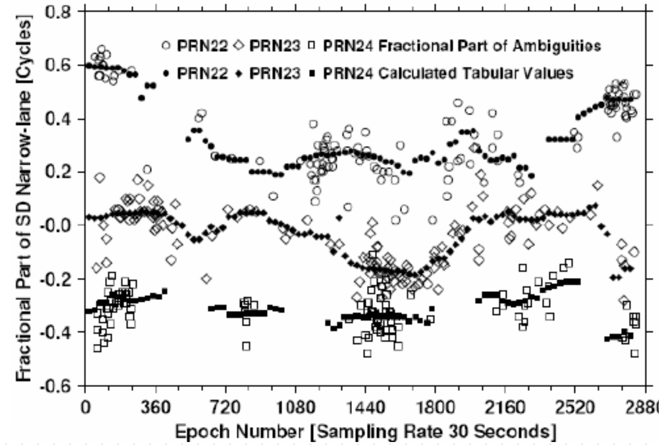
- **with AR for PPP**
  - Improve Convergence Time
  - Improve Accuracy of Static Solution (EW, UD)
  - Improve Stability of Kinematic Solution
- **Difficulties of AR for PPP**
  - Unknown Satellite Initial Phase Biases
  - Effect of Precise Orbit/Clock Error
  - Effect of Ionospheric Delay
  - Code/Phase Bias Instability
  - Multipath Effect at Reference Station Network

M.Ge et al., EGU 2007

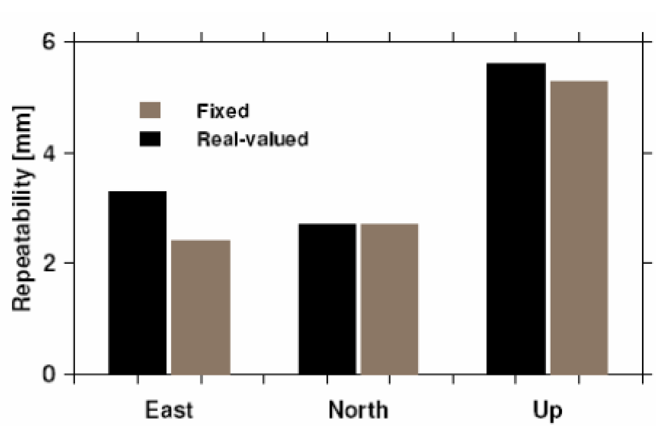
WL Phase Bias Stability



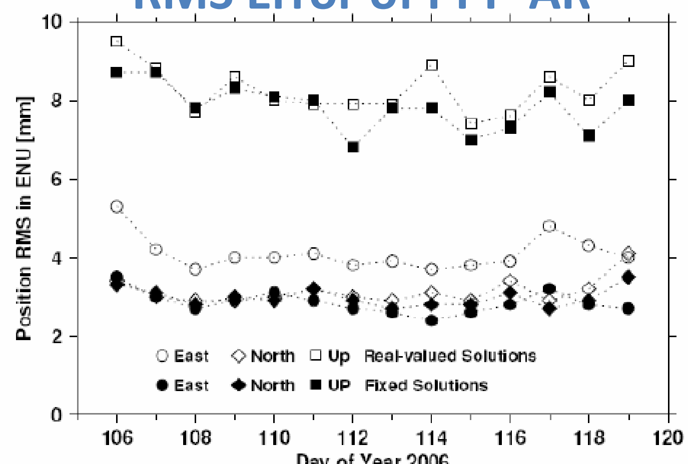
NL Phase Bias Stability



Repeatability of PPP-AR



RMS Error of PPP-AR



M.Ge et al., Resolution of GPS carrier-phase ambiguity in precise point positioning, EGU Assembly 2007

D.Laurichesse, ION 2010

- **Real-Time Implementation of PPP-AR**
  - Network WL ambiguity fixing
  - Parameter estimation by EKF with iono-free code/phase: phase-clock, code-phase-bias, ZTD, station position, orbit correction to IGU, phase ambiguity
  - Orbit construction + high-rate clock generation
- **Evaluation of Accuracy**
  - Orbit: 4cm, code-clock: 5 cm, phase-clock: 1cm
- **RT-PPP with AR ("CNES Integer PPP")**
  - 1 cm HRMS

Future Precise Positioning

	Requirement	Technologies in 2020
Coverage	Global (world-wide)	RT-Orbit/Clock with AR, Broadcast via Internet + GEO/QZSS Satellite
Dynamics	Stationary - Space	Static - Kinematic
Accuracy	1 cm (HRMS) 2 cm (VRMS)	Local iono/tropos corrections (land) Iono estimation by triple-freq (sea)
Availability	99 % (open sky)	30 sats + triple-freq
	95 % (urban)	30 sats+INS-aided PLL, slip-resistant
Latency	1 s	Real-time corrections
TTFF	10 s (land)	Local iono/tropos corrections
	1 min (sea)	Iono estimation by triple-freq
User Cost	< $100	No patent problem, need killer-AP