2020年度測位航法学会 全国大会 セミナー②

RTK および PPP 技術の応用と実習

Theory and Practice of RTK/PPP Technology

Tokyo Univ. of Marine Science and Technology (TUMSAT)

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2020-07-16 @Zoom meeting (online)

Timetable

		July 16, 2020
1	Introduction of RTK/PPP	9:30-10:20
2	Theory of PTK/PPK	10:30-11:20
3	RTK/PPK Practice	11:30-12:30
	Lunch Break	12:30-13:30
4	Theory of PPP	13:30-14:20
5	PPP Practice	14:30-15:20
6	Advanced Topics	15:30-16:30

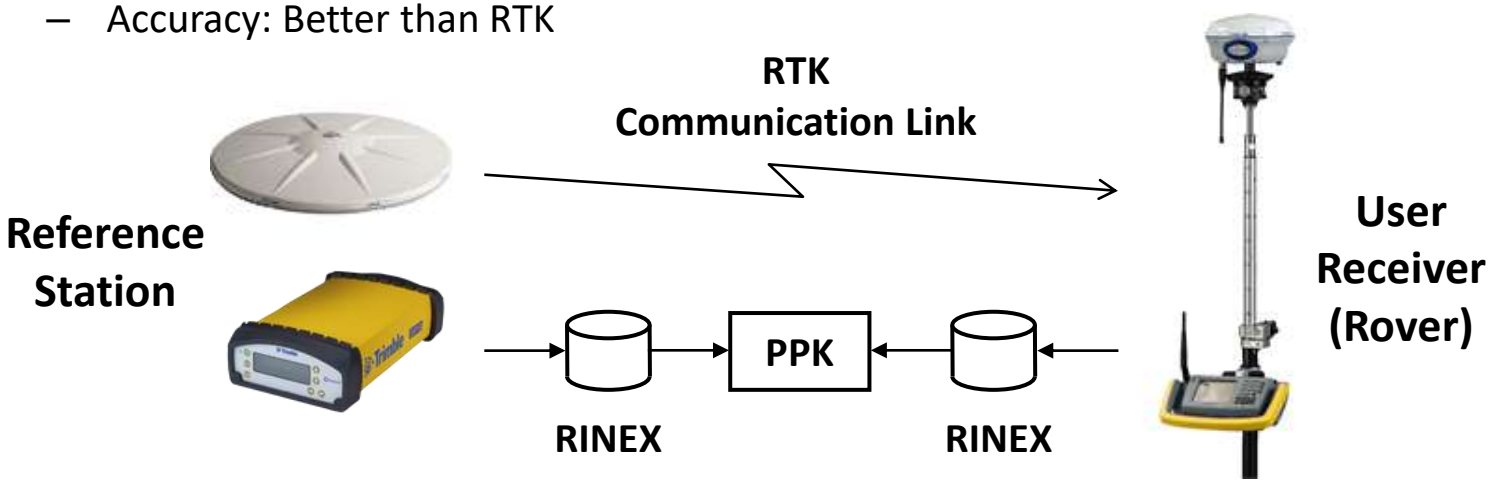
1.
Introduction of RTK/PPP

Code vs Carrier-Based Positioning

	SD (standard positioning, code-based)	**PP (precise positioning, carrier-based)**
Observables	Pseudorange (Code)	**Carrier-Phase + Pseudorange**
Receiver Noise	30 cm	3 mm
Multipath	30 cm - 30 m	1 - 3 cm
Sensitivity	High (<20dBHz)	Low (>35dBHz)
Discontinuity	No Slip	Cycle-Slip
Ambiguity	-	Estimated/Resolved
Receiver	Low-Cost (~$100)	**Expensive (~$20,000)**
Accuracy (RMS)	3 m (H), 5 m (V) (Single) 1 m (H), 2 m (V) (DGPS)	**5 mm (H), 1 cm (V) (Static)** **1 cm (H), 2 cm (V) (RTK/PPK)**
Application	Navigation, Timing, SAR,...	Survey, Mapping, ...

RTK/PPK

- **RTK (Real-time Kinematic)**
  - Real-time Position of Rover Antenna
  - Transmit Reference Station Data to Rover via Comm. Link
  - OTF (On-the-Fly) Integer Ambiguity Resolution
  - Typical Accuracy: 1 cm + 1ppm x BL RMS (Horizontal)
- **PPK (Post-processing Kinematic)**
  - Post-processed Position of Rover Antenna
  - Rover and Reference Station Data provided by Standard RINEX Formats
  - Accuracy: Better than RTK



RTK/PPK Applications

Geodetic Survey

Construction Machine Control

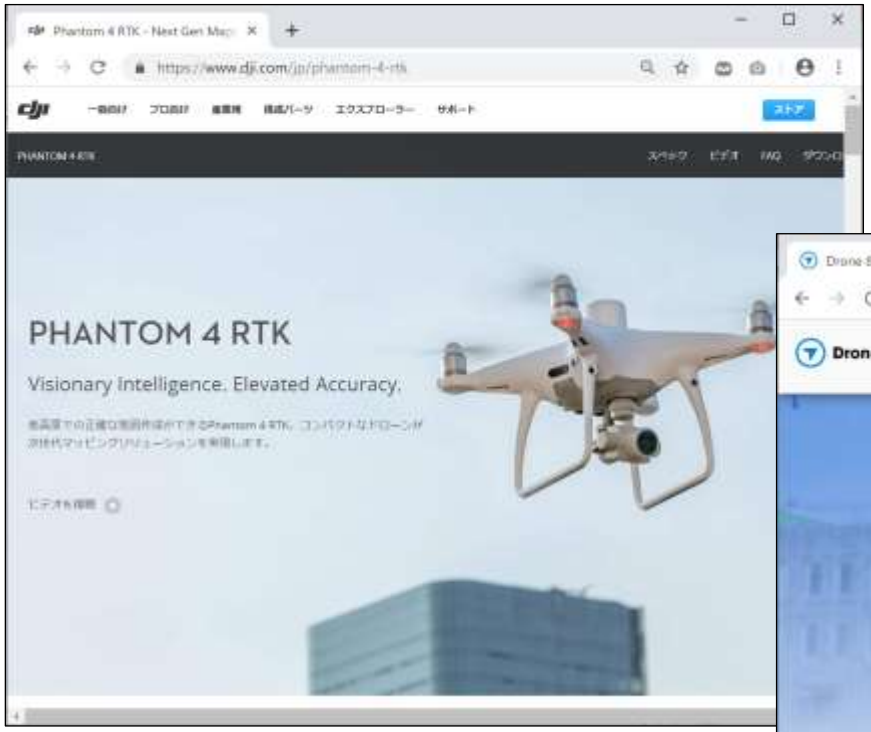
Precision Agriculture

Self-driving Car

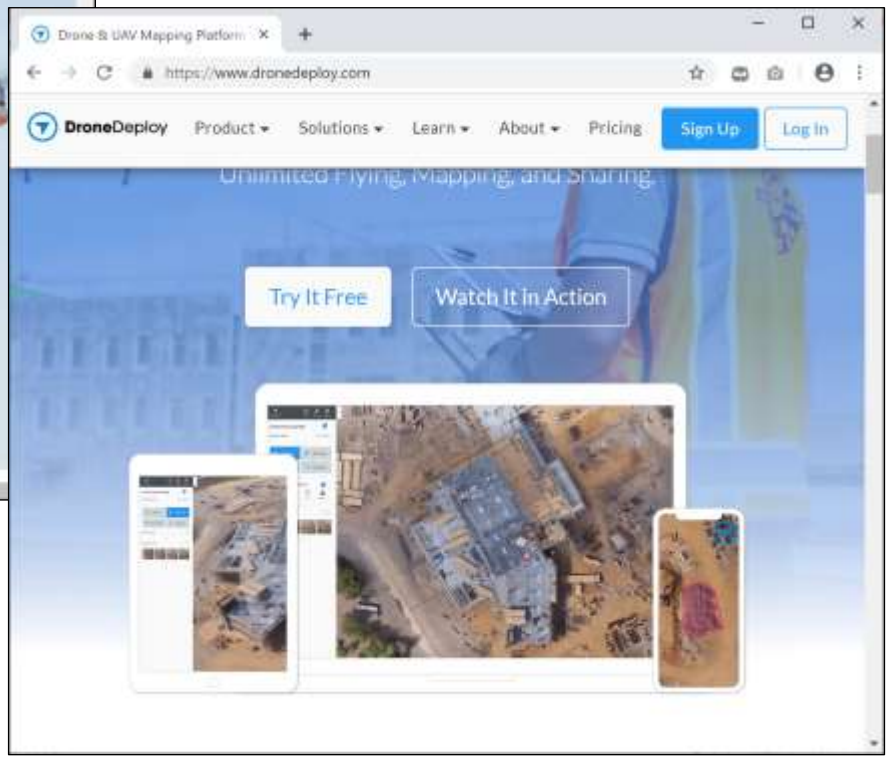
Mapping

UAV (Drone)

UAV (Drone) with RTK/PPK



https://www.dji.com/jp/phantom-4-rtk



https://www.dronedeploy.com/

PPP (Precise Point Positioning)

- **Historical View**
  - **Firstly introduced in 1990s** to analyze data of large GPS station N/W [1]
  - **Conventionally PP** with IGS precise ephemerides since 1990s
  - Several **RT-PPP services have launched via GEO satellite links** since 2000s
  - Recently improving accuracy with **multi-constellation GNSS and PPP-AR**
  - **Convergence time is still an issue** for some applications even in 2019

[1] J. F. Zumberge et al., Precise point positioning for the efficient and robust analysis of GPS data from large networks, Journal of Geophysical Research, 1997

	RTK	PPP
Base Station	Required	Not required
Area Coverage	Local (< 30 km)	Global (worldwide)
Accuracy (HRMS)	1 ~ 4 cm	2 ~ 5 cm
Convergence Time	< 60 s	15 ~ 45 min

PPP Applications

Automated Farming

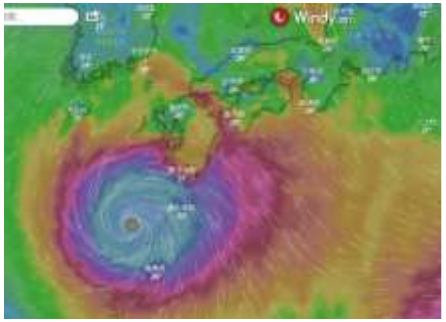
Offshore Construction/ Mining

Drone/UAV

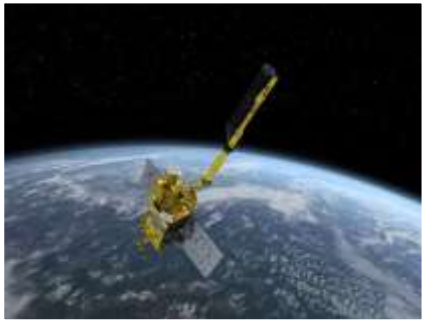
ADAS/ Autonomous Driving

Tsunami Warning

Precise Time Transfer

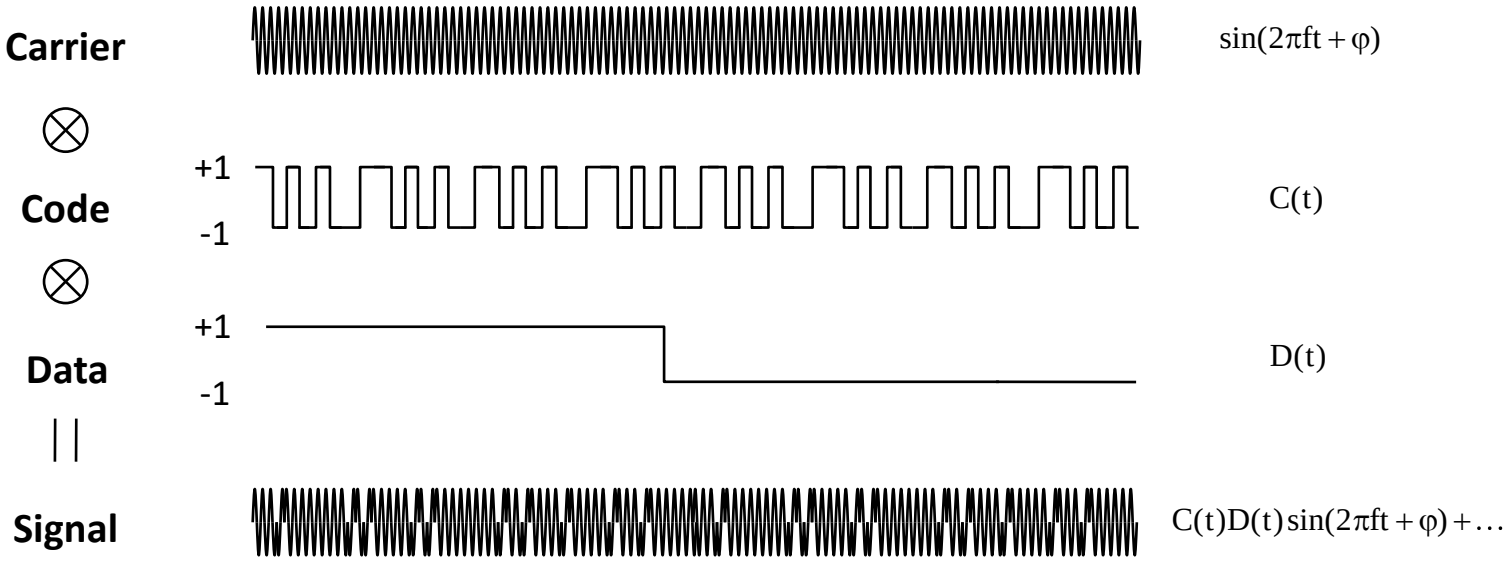


Weather Forecast (GNSS Meteorology)

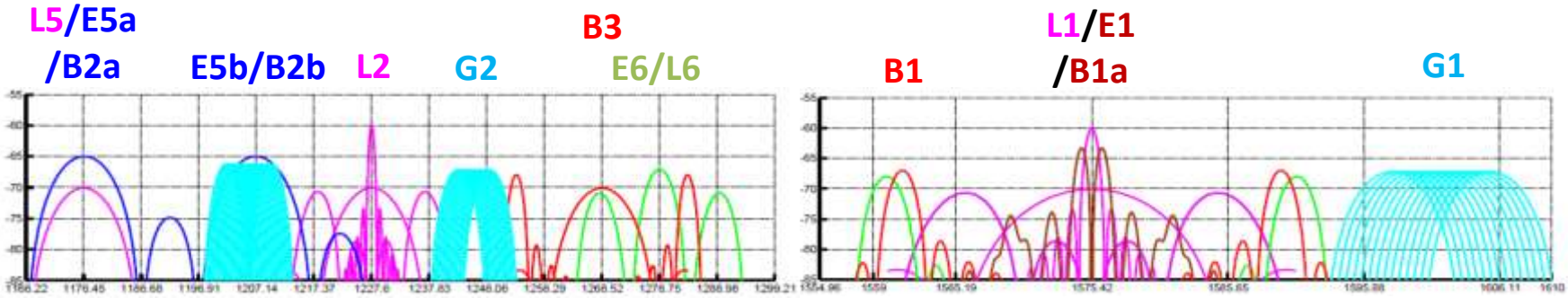


Satellite POD

GNSS Signal Structure

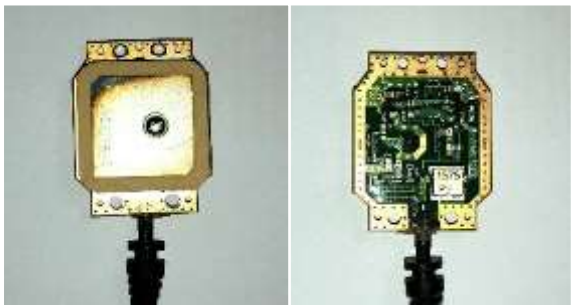


GNSS Signal Spectrum [1]



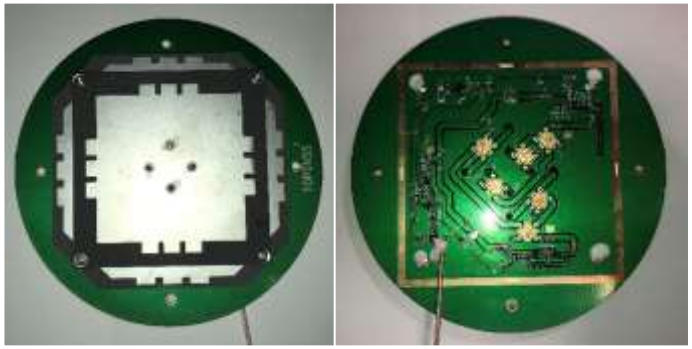
[1] Y.Yang, COMPASS: View on Compatibility and Interoperability, 2009

GNSS Antennas



Patch Antenna (L1)

Helix Antenna (L1+L2)



Patch Antenna (L1+L2)

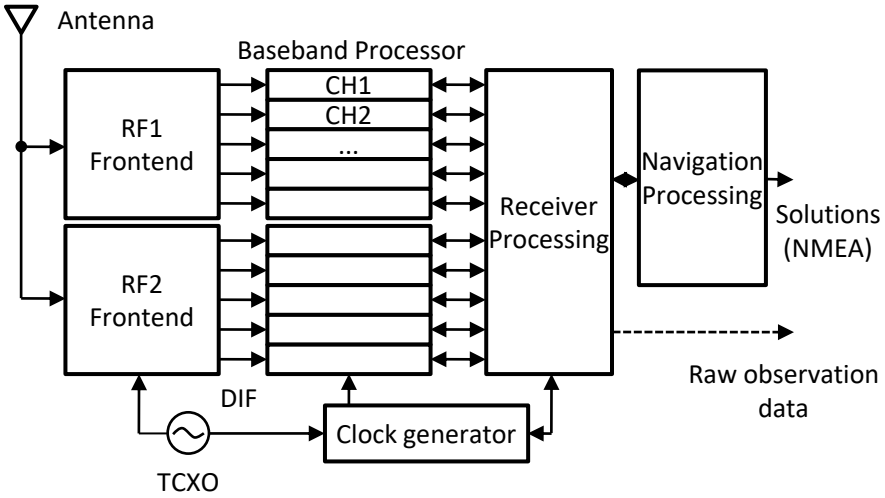
GNSS Receivers (1)

My receiver collections

NovAtel OEMV-3

Trimble BD982

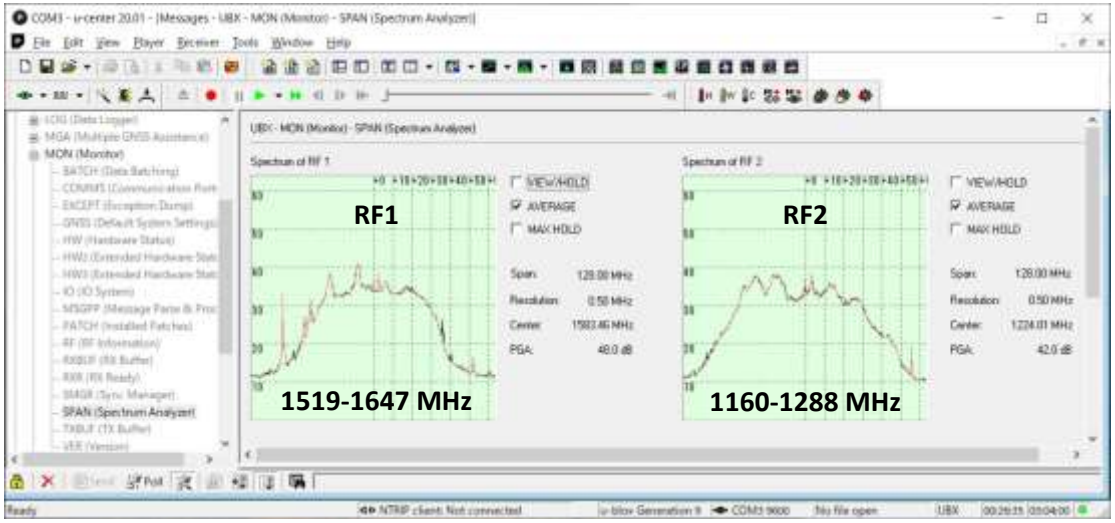
GNSS Receivers (2)



Mini-EVK of u-blox ZED-F9P [1]

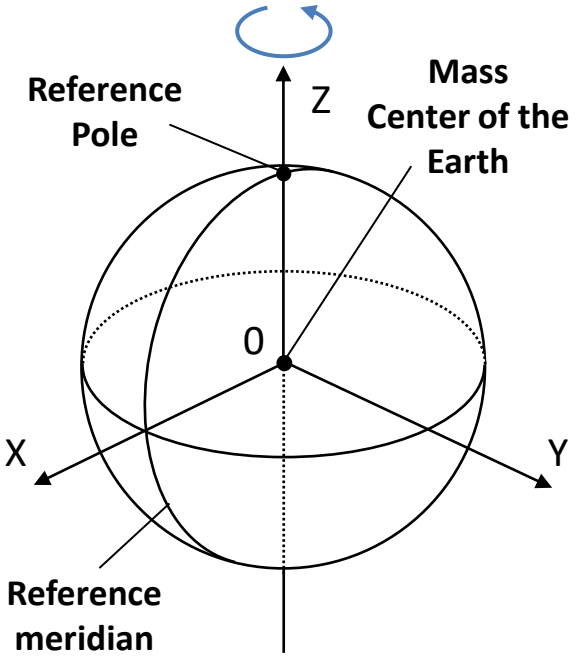
[1] https://www.u-blox.com

IF Spectrum of ZED-F9P [1]



Coordinate Systems

- **ECEF (earth-centered earth-fixed)**
  - ITRF
  - WGS84 (GPS)
  - PZ90.14 (GLONASS)
  - JGS (QZSS)
  - JGD (Japan, GSI) ...
- **Latitude, Longitude and Height**
  - Latitude: geodetic, geocentric
  - Height: orthometric, ellipsoidal
- **ECI (earth-centered inertial)**
  - J2000.0 (equatorial coordinate system)
  - ICRF (international celestial reference frame)



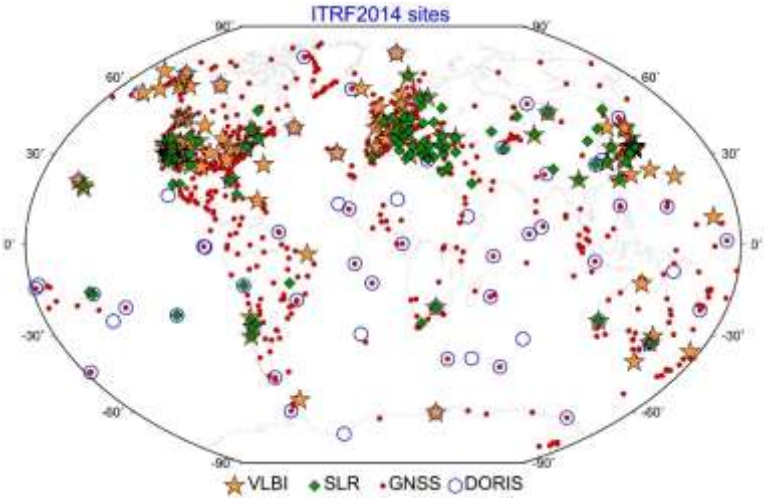
ECEF Definitions

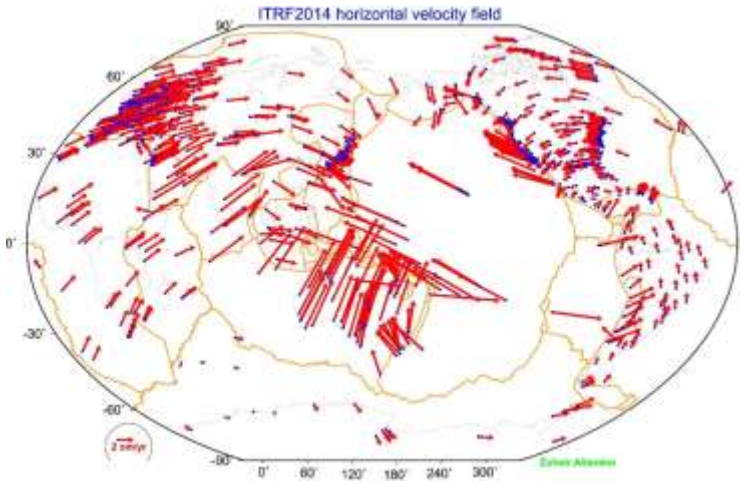
ITRF

- **International Terrestrial Reference Frame**
  - A "Realization" of ITRS developed and maintained by IERS
  - GNSS, VLBI, SLR, DORIS site position/velocity, EOP, PSD (SINEX)
  - ITRF2014, ITRF2008, ITRF2005, ITRF2000, ITRF97, ITRF96, ...

VLBI: Very Long Baseline Interferometry
SLR: Satellite Laser Ranging
DORIS: Doppler Orbit determination and Radiopositioning Integrated on Satellite

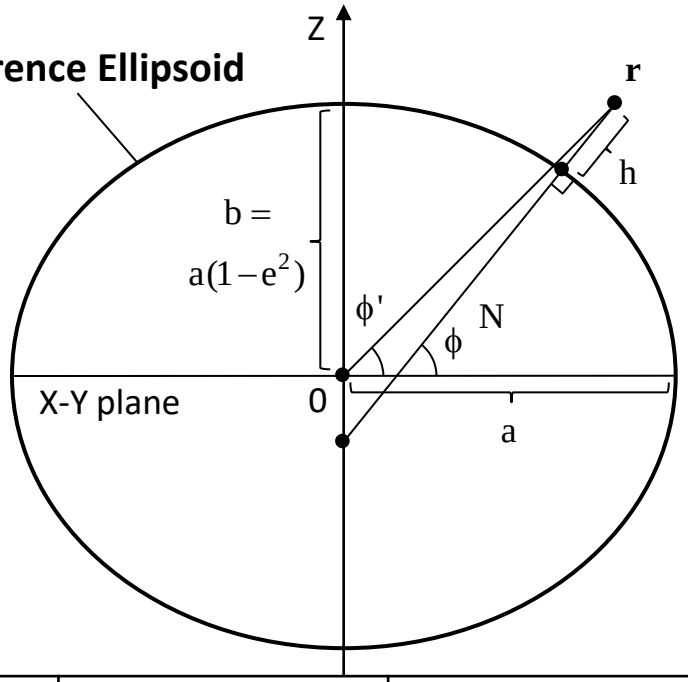
ITRS: International Terrestrial Reference System
IERS: International Earth Rotation Service





Z.Altamimi et al., ITRF2014: A new release of the International Terrestrial Reference Frame modeling nonlinear station motion, JGR Solid Earth, 2016

Latitude, Longitude and Height



	GRS80	WGS84
a	6378137 m	6378137 m
f	1/298.257222101	1/298.257223563
b	6335439.32708 m	6335439.32729 m

a : Semi-major axis length (m)
ϕ' : Geocentric latitude
ϕ : Geodetic latitude
f : Flattening
λ : Longitude
h : Ellipsoidal height

Lat/Lon/Hgt -> ECEF-XYZ

$$e^2 = f(2-f)$$

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

$$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ (N(1-e^2)+h)\sin\phi \end{pmatrix}$$

ECEF-XYZ -> Lat/Lon/Hgt

$$R = \sqrt{x^2+y^2},\ \phi_0 = 0$$

$$\phi_{i+1} = \arctan\left(\frac{z}{R} - \frac{ae^2\tan\phi_i}{R\sqrt{1+(1-e^2)\tan^2\phi_i}}\right)$$

$$\phi = \lim_{i\to\infty}\phi_i$$

$$\lambda = \mathrm{ATAN2}(y,x)$$

$$h = \frac{R}{\cos\phi} - \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

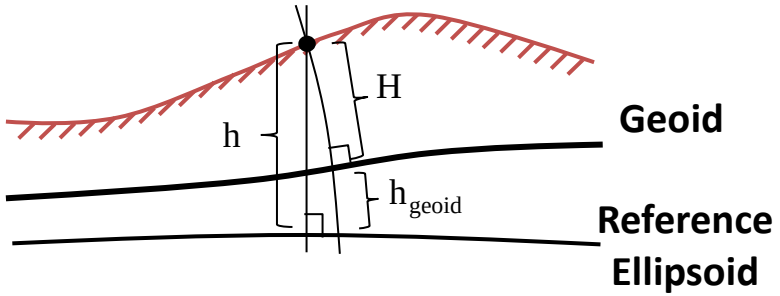
Heights

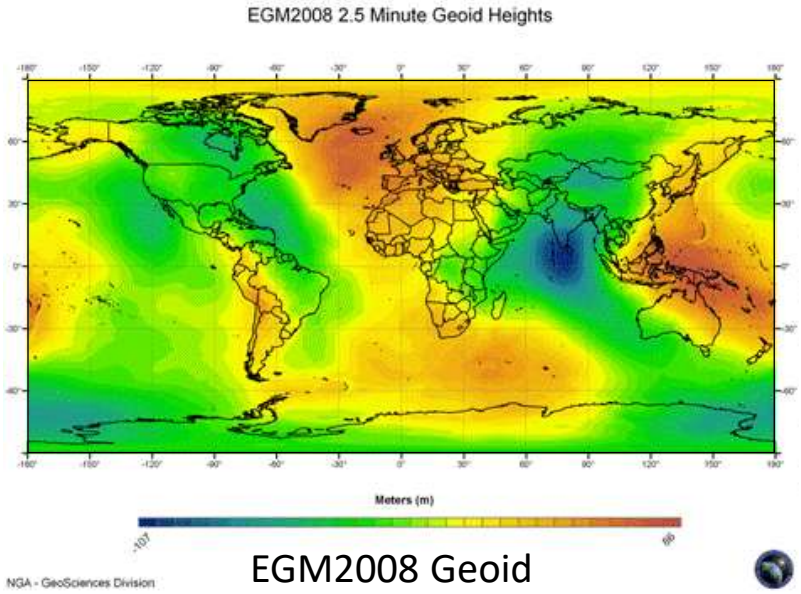
$$H \approx h - h_{geoid}(\phi, \lambda)$$

H : Orthometric height (m)

h : Ellipsoidal height (m)

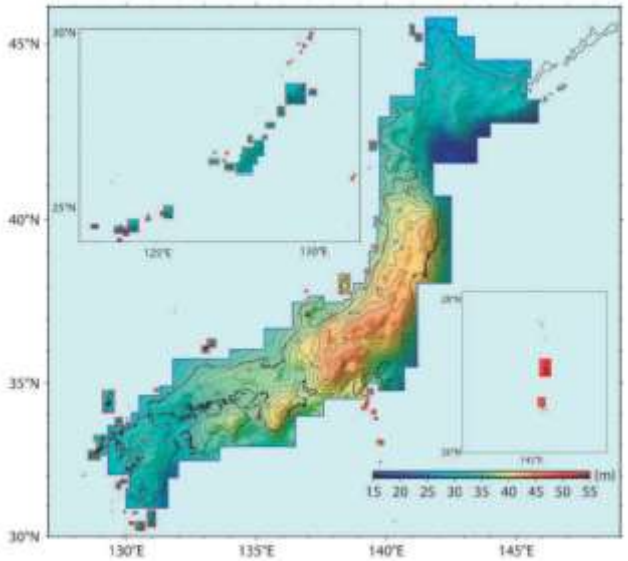
h_{geoid} : Geoid height (m)





EGM2008 Geoid

https://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html



GSIGEO2011 (Ver.2) Geoid

https://www.gsi.go.jp/buturisokuchi/grageo_geoidseika.html

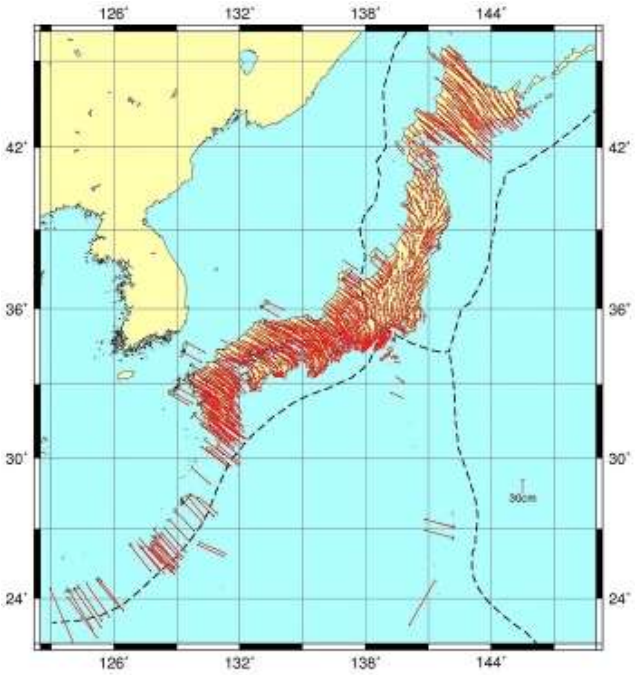
Japanese Geodetic Datum

Tokyo -> JGD2000 -> JGD2011

JGD2011 (GSI)

http://club.informatix.co.jp/?p=998

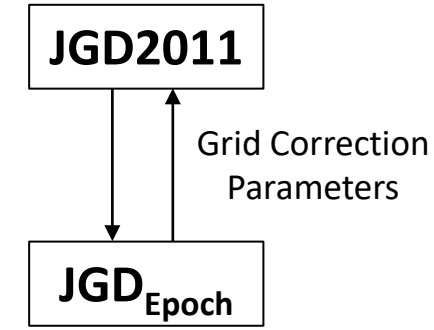
Displacement of Japanese Island



1997 -> 2009

https://www.gsi.go.jp/sokuchikijun/semidyna01.html

Semi-dynamic corrections



https://www.gsi.go.jp/sokuchikijun/semidyna03.html

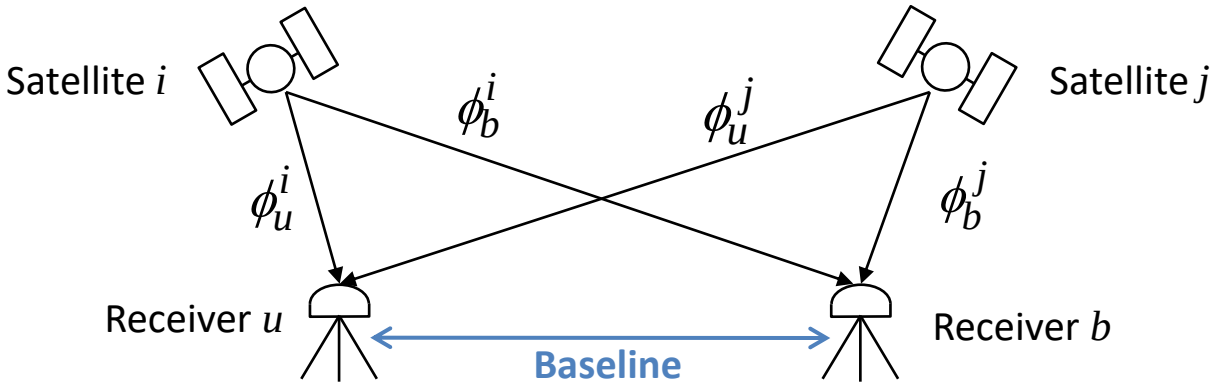
2.
Theory of RTK/PPK

DD (Double-difference)

$$\Phi_{ub}^{ij} \equiv \lambda((\phi_u^i - \phi_b^i) - (\phi_u^j - \phi_b^j))$$
$$= \rho_{ub}^{ij} + c(dt_{ub}^{ij} - dT_{ub}^{ij}) - I_{ub}^{ij} + T_{ub}^{ij} + \lambda B_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_\Phi$$
$$= \rho_{ub}^{ij} - I_{ub}^{ij} + T_{ub}^{ij} + \lambda N_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_\Phi$$
$$dt_{ub}^{ij} = dt_u^{ij} - dt_b^{ij} = 0,\ dT_{ub}^{ij} = dT_{ub}^{i} - dT_{ub}^{j} \approx 0$$
$$B_{ub}^{ij} = (\phi_{u,0} - \phi_0^i + N_u^i) - (\phi_{b,0} - \phi_0^i + N_b^i) - (\phi_{u,0} - \phi_0^j + N_u^j) + (\phi_{b,0} - \phi_0^j + N_b^j) = N_{ub}^{ij}$$

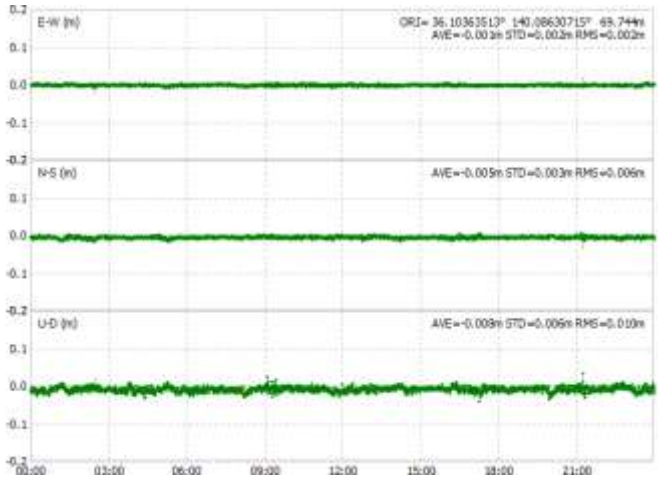
(short Baseline and same antenna type)

$$\Phi_{ub}^{ij} \approx \rho_{ub}^{ij} + \lambda N_{ub}^{ij} + \varepsilon_\Phi \quad I_{ub}^{ij} = I_{ub}^{i} - I_{ub}^{j} \approx 0, T_{ub}^{ij} = T_{ub}^{i} - T_{ub}^{j} \approx 0, d_{ub}^{ij} = d_{ub}^{i} - d_{ub}^{j} \approx 0$$



Effect of Baseline Length

BL=0.3 km

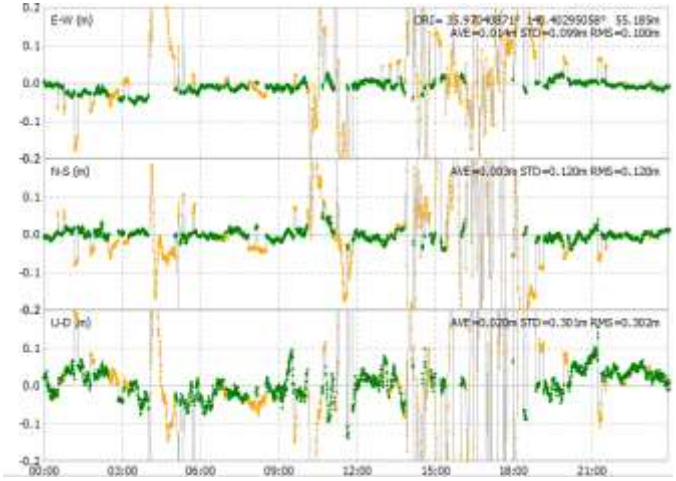


RMS Error:
E: 0.2cm
N: 0.6cm
U: 1.0cm
Fix Ratio: 99.9%

BL=13.3 km

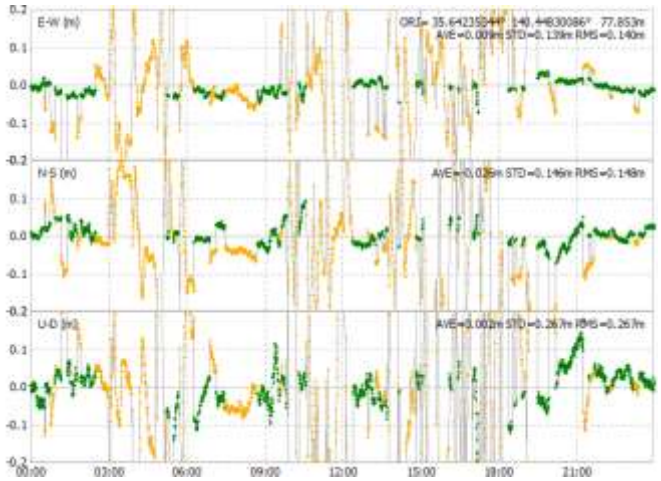
RMS Error:
E: 2.2cm
N: 2.4cm
U: 10.6cm
Fix Ratio: 94.2%

BL=32.2 km



RMS Error:
E: 10.0cm
N: 12.0cm
U: 30.2cm
Fix Ratio: 64.3%

BL=60.9 km



RMS Error:
E: 14.0cm
N: 14.8cm
U: 26.7cm
Fix Ratio: 44.4%

(24 hr Kinematic ● : Fixed Solution ● : Float Solution)

Ambiguity Resolution (AR)

Objectives of AR

More accurate solutions after proper AR (FIXED vs. FLOAT)

Faster solution convergence, ideally FIXED instantaneously

Carrier phase observables can be handled as precise pseudorange after AR

Initial phase terms should be eliminated for AR

Both of satellite and receiver initial phases do not have integer nature

DD (double difference) used for baseline processing like RTK

ZD (zero-difference) carrier-phase

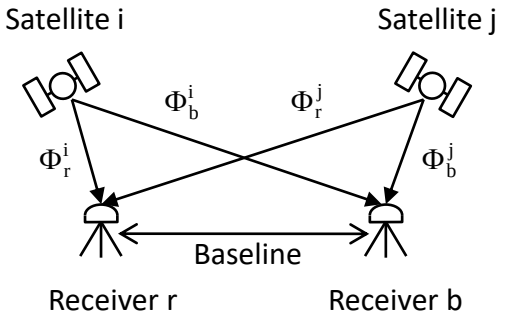
$$\Phi_r^s = \rho_r^s + c(dt_r - dT^s(t^s)) - I_r^s + T_r^s + d_r^s + \lambda d_{pw} + \lambda(\phi_{r,0} - \phi_0^s + N_r^s) + \varepsilon_\Phi$$

DD (double-difference) carrier-phase for baseline processing

$$\Phi_{rb}^{ij} = \rho_{rb}^{ij} - I_{rb}^{ij} + T_{rb}^{ij} + d_{rb}^{ij} + \lambda d_{pw,rb}^{ij} + \lambda N_{rb}^{ij} + \varepsilon_\Phi$$

$$\Phi_{rb}^{ij} \approx \rho_{rb}^{ij} + \lambda N_{rb}^{ij} + \varepsilon_\Phi \qquad \text{(short-baseline, same antenna)}$$

$()^{ij}$: Difference between satellite i and j
$()_{rb}$: Difference between receiver r and b
N_r^s : Integer ambiguity (cyc)
ϕ_0^s : Satellite initial phase (cyc)
$\phi_{r,0}$: Receiver initial phase (cyc)



FIXED Solution with AR

Typical AR Steps [1]

(1) $\mathbf{y} = \mathbf{Hx} + \boldsymbol{\varepsilon} = \mathbf{Aa} + \mathbf{Bb} + \boldsymbol{\varepsilon} \quad \left(\mathbf{a} \in \mathbf{Z}^n, \mathbf{b} \in \mathbf{R}^p\right)$ Mixed-integer measurement models

(2) $\hat{\mathbf{x}} = \begin{pmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{pmatrix}, \mathbf{Q}_{\hat{\mathbf{x}}} = \begin{pmatrix} \mathbf{Q}_{\hat{\mathbf{a}}} & \mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{b}}} \\ \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}} & \mathbf{Q}_{\hat{\mathbf{b}}} \end{pmatrix}$ FLOAT solution and VC matrix by LSE or KF

(3) $\hat{\mathbf{a}} \rightarrow \breve{\mathbf{a}}$ Mapping FLOAT to FIXED

(4) $\breve{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}} \mathbf{Q}_{\hat{\mathbf{a}}}^{-1} (\hat{\mathbf{a}} - \breve{\mathbf{a}})$ FIXED solution

Mapping FLOAT to FIXED ($\hat{\mathbf{a}} \rightarrow \breve{\mathbf{a}}$)

Integer Rounding (IR)

$$\breve{\mathbf{a}} = [\hat{\mathbf{a}}] = \left([\hat{a}_1], [\hat{a}_2], \ldots, [\hat{a}_n]\right)^T$$

Integer Bootstrapping (conditional sequential rounding) (IB)

$$\breve{\mathbf{a}} = \left([\hat{a}_1], \left[\hat{a}_2 - \sigma_{21}\sigma_1^{-2}(\hat{a}_1 - \breve{a}_1)\right], \ldots, \left[\hat{a}_n - \sum_{i=1}^{n-1} \sigma_{n,i|I}\sigma_{i|I}^{-2}(\hat{a}_{i|I} - \breve{a}_i)\right]\right)^T$$

Integer Least Squares (ILS)

$$\breve{\mathbf{a}} = \underset{\mathbf{a} \in \mathbf{Z}}{\arg\min} \left(\hat{\mathbf{a}} - \mathbf{a}\right)^T \mathbf{Q}_{\hat{\mathbf{a}}}^{-1} \left(\hat{\mathbf{a}} - \mathbf{a}\right)$$

$\mathbf{a}$: Integer ambiguity parameters
$\mathbf{b}$: Other parameters
$\hat{\mathbf{a}}, \hat{\mathbf{b}}$: FLOAT solutions
$\breve{\mathbf{a}}, \breve{\mathbf{b}}$: FIXED solutions
$[\mathrm{x}]$: Rounding to the nearest integer

$$\mathbf{Q}_{\hat{\mathbf{a}}} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{pmatrix}$$

[1] P. J. G. Teunissen and O. Montenbruck (eds.), Springer Handbook of Global Navigation Satellite Systems, 2017, Springer, Section 23

LAMBDA

Least-squares AMBiguity Decorrelation Adjustment [1]

A GNSS AR strategy by ILS estimator

Shrinking integer search space with "decorrelation"

Skillful and efficient tree search strategy

Similar to "Closest Point Search with LLL Lattice Basis Reduction" Algorithm [2]

$$\breve{\mathbf{a}} = \underset{\mathbf{a}\in\mathbf{Z}^n}{\arg\min}(\hat{\mathbf{a}}-\mathbf{a})^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{a}}}{}^{-1}(\hat{\mathbf{a}}-\mathbf{a})$$

$$\breve{\mathbf{a}} = \mathbf{Z}^{-\mathrm{T}}\breve{\mathbf{z}}$$

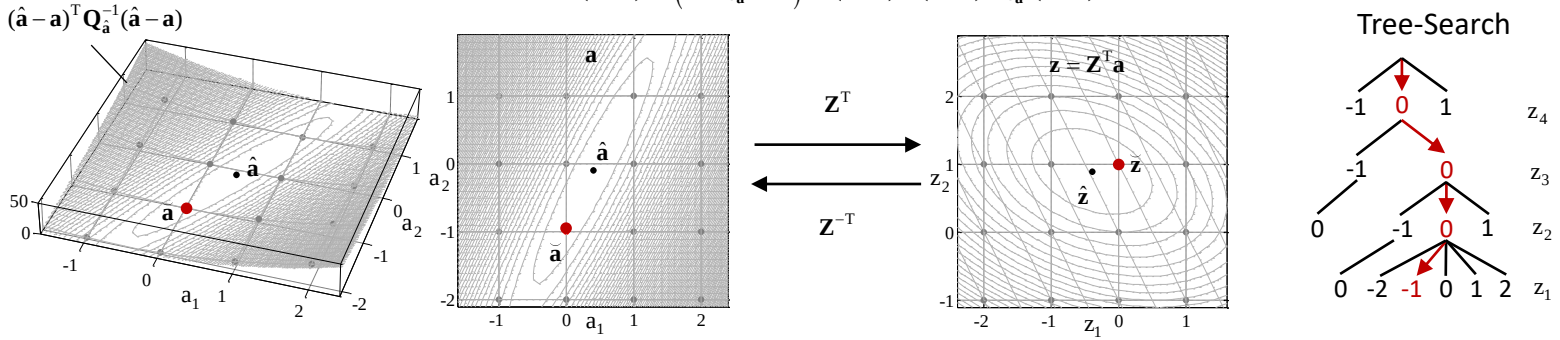
Z-transformation

$$\hat{\mathbf{z}} = \mathbf{Z}^{\mathrm{T}}\hat{\mathbf{a}},\ \mathbf{Q}_{\hat{\mathbf{z}}} = \mathbf{Z}^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{a}}}\mathbf{Z}$$

$$\breve{\mathbf{z}} = \underset{\mathbf{z}\in\mathbf{Z}^n}{\arg\min}(\hat{\mathbf{z}}-\mathbf{z})^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{z}}}{}^{-1}(\hat{\mathbf{z}}-\mathbf{z})$$

$$(\hat{\mathbf{z}}-\mathbf{z})^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{z}}}^{-1}(\hat{\mathbf{z}}-\mathbf{z}) = (\mathbf{Z}^{\mathrm{T}}\hat{\mathbf{a}}-\mathbf{Z}^{\mathrm{T}}\mathbf{a})^{\mathrm{T}}\left(\mathbf{Z}^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{a}}}\mathbf{Z}\right)^{-1}(\mathbf{Z}^{\mathrm{T}}\hat{\mathbf{a}}-\mathbf{Z}^{\mathrm{T}}\mathbf{a})$$

$$= (\hat{\mathbf{a}}-\mathbf{a})^{\mathrm{T}}\mathbf{Z}\left(\mathbf{Z}^{-1}\mathbf{Q}_{\hat{\mathbf{a}}}^{-1}\mathbf{Z}^{-\mathrm{T}}\right)\mathbf{Z}^{\mathrm{T}}(\hat{\mathbf{a}}-\mathbf{a}) = (\hat{\mathbf{a}}-\mathbf{a})^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{a}}}^{-1}(\hat{\mathbf{a}}-\mathbf{a})$$



[1] P. J. G. Teunissen, The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. Journal of Geodesy, 1994

[2] E. Agrell, Closest point search in lattices, IEEE Transaction on Information Theory, 2002

Validation of AR

Acceptance Test after AR

Wrong AR much degrades the quality of the final (FIXED) solution

Need to maximize AR success probability and to minimize AR failure rate

Remain ambiguities FLOAT if the acceptance test failed and output FLOAT solution instead

Several acceptance tests are proposed: [1]

Ratio Test

$$\text{accept } \breve{\mathbf{a}} \text{ if : } \frac{\left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}'\right\|^2_{Q_{\hat{a}}}}{\left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}\right\|^2_{Q_{\hat{a}}}} \geq c$$

Difference Test

$$\text{accept } \breve{\mathbf{a}} \text{ if : } \left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}'\right\|^2_{Q_{\hat{a}}} - \left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}\right\|^2_{Q_{\hat{a}}} \geq c$$

F-Ratio Test

$$\text{accept } \breve{\mathbf{a}} \text{ if : } \frac{\left\|\hat{\mathbf{e}}\right\|^2_{Q_y} + \left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}'\right\|^2_{Q_{\hat{a}}}}{\left\|\hat{\mathbf{e}}\right\|^2_{Q_y} + \left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}\right\|^2_{Q_{\hat{a}}}} \geq c$$

Projector Test

$$\text{accept } \breve{\mathbf{a}} \text{ if : } \left| \frac{(\breve{\mathbf{a}}'-\breve{\mathbf{a}})^T Q_{\hat{a}}^{-1}(\hat{\mathbf{a}}-\breve{\mathbf{a}})}{\left\|\breve{\mathbf{a}}'-\breve{\mathbf{a}}\right\|^2_{Q_{\hat{a}}}} \right| \leq \mu$$

$\breve{\mathbf{a}}'$: Second-best candidate of AR, c, μ : Test thresholds $\quad \|\mathbf{x}\|^2_Q = \mathbf{x}^T Q^{-1} \mathbf{x} \quad \hat{\mathbf{e}} = \mathbf{y} - \mathbf{A}\hat{\mathbf{a}} - \mathbf{B}\hat{\mathbf{b}}$

Note: The test thresholds are usually selected empirically as a fixed value (for example Ratio Test with c = 3). The threshold value, however, is often optimistic or conservative without theoretical basis.
In some literatures, variable threshold values are proposed base on fixed-failure rate approach (FF-RT) [2].

[1] S. Verhagen and P. J. G. Teunissen, New global navigation satellite system ambiguity resolution method compared to exiting approaches, Journal of Guidance, Control and Dynamics, 2006

[2] S. Verhagen and P. J. G. Teunissen, The ratio test for future GNSS ambiguity resolution, GPS Solution, 2013

WL/NL AR with Iono-free LC

FLOAT Solution and LC Ambiguity *

$$\mathbf{x} = (\mathbf{r}_r^T, T, \mathbf{N}_{LC}^T)^T, \mathbf{y} = (\mathbf{P}_{LC}^T, \mathbf{\Phi}_{LC}^T)^T, \mathbf{Q}_y$$

$$\begin{cases} P_{LC} = C_1 P_{L1} + C_2 P_{L2} = \rho + mT + \varepsilon_P \\ \Phi_{LC} = C_1 \Phi_{L1} + C_2 \Phi_{L2} = \rho + mT + N_{LC} + \varepsilon_\Phi \end{cases}$$

$$\rightarrow \hat{\mathbf{x}} = (\hat{\mathbf{r}}_r^T, \hat{T}, \hat{\mathbf{N}}_{LC}^T)^T, \mathbf{Q}_{\hat{\mathbf{x}}}$$

FLOAT WL Ambiguity *

$$\hat{N}_{WL} = \frac{1}{n}\sum_{i=1}^{n}(MW) = \frac{1}{n}\sum_{i=1}^{n}(\Phi_{WL} - P_{NL})$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{\Phi_{L1}}{\lambda_1} - \frac{\Phi_{L2}}{\lambda_2}\right) - \left(\frac{P_{L1}}{\lambda_1} + \frac{P_{L2}}{\lambda_2}\right)\right)$$

Ionosphere-free

geometry-free, Ionosphere-free

WL/NL AR by IR **

$$\breve{N}_{WL} = \left[\hat{N}_{WL}\right] \quad \text{(WL AR)}$$

$$\breve{N}_{L1} = \left[\hat{N}_{L1}\right] = \left[\frac{\hat{N}_{LC} - C_2\lambda_2\breve{N}_{WL}}{\lambda_{NL}}\right] \quad \text{(NL AR)}$$

FIXED Solution *

$$\mathbf{x} = (\mathbf{r}_r^T, T)^T, \mathbf{y} = (\mathbf{\Phi}_{LC}^T)^T$$

$$\Phi_{LC} = C_1\Phi_{L1} + C_2\Phi_{L2} = \rho + mT + \breve{N}_{LC} + \varepsilon_\Phi$$

$$\rightarrow \breve{\mathbf{x}} = (\breve{\mathbf{r}}_r^T, \breve{T})^T, \mathbf{Q}_{\breve{\mathbf{x}}}$$

N_{LC} : Iono-free LC ambiguity (m)
N_{WL} : Wide lane ambiguity (cyc)
m : Mapping function
T : ZTD or ZWD (m)

Note:

* DD applied for baseline processing to eliminate clock and initial bias terms. SD applied for PPP-AR with FCB corrections

** Alternatively, ILS or IB can be applied to NL AR considering the VC-matrix of FLOAT LC ambiguities

$$C_1 = \frac{{\lambda_2}^2}{{\lambda_2}^2 - {\lambda_1}^2}, C_2 = \frac{-{\lambda_1}^2}{{\lambda_2}^2 - {\lambda_1}^2}$$

$$C_1\lambda_1 + C_2\lambda_2 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2} = \frac{1}{1/\lambda_1 + 1/\lambda_2} = \lambda_{NL}$$

$$N_{WL} = N_{L1} - N_{L2}$$

$$N_{LC} = C_1\lambda_1 N_{L1} + C_2\lambda_2 N_{L2} = (C_1\lambda_1 + C_2\lambda_2)N_{L1} - C_2\lambda_2 N_{WL}$$

$$= \lambda_{NL} N_{L1} - C_2\lambda_2 N_{WL}$$

TCAR/CIR

TCAR (three-carrier AR) [1] /CIR (cascade integer resolution) [2]

Sequential conditional rounding EWL -> WL -> NL ambiguities

Geometry-free measurement model

Ionosphere terms can be reduced by short baseline or ionosphere corrections (for PPP)

Many modifications or enhancements including geometry-based models

$$(1)\ \breve{N}_{EWL} = \left[\frac{\Phi_{EWL} - P_X}{\lambda_{EWL}} \right]$$

$$(2)\ \breve{N}_{WL} = \left[\frac{\Phi_{WL} - (\Phi_{EWL} - \lambda_{EWL}\breve{N}_{EWL})}{\lambda_{WL}} \right]$$

$$(3)\ \breve{N}_{NL} = \left[\frac{\Phi_{NL} - (\Phi_{WL} - \hat{N}_{WL})}{\lambda_{NL}} \right]$$

$$\begin{cases} P_X = \rho + I_X + T + \varepsilon_P \\ \Phi_{EWL} = \rho - I_{EWL} + T + \lambda_{EWL} N_{EWL} + \varepsilon_{\Phi_{EWL}} \\ \Phi_{WL} = \rho - I_{WL} + T + \lambda_{WL} N_{WL} + \varepsilon_{\Phi_{WL}} \\ \Phi_{NL} = \rho - I_{NL} + T + \lambda_{NL} N_{NL} + \varepsilon_{\Phi_{NL}} \end{cases}$$

$$|I_X + I_{EWL}| \ll \lambda_{EWL}, |I_{WL}| \ll \lambda_{WL}, |I_{NL}| \ll \lambda_{NL}$$

Typical Selection of EWL, WL and NL for GPS

$$P_X = \frac{P_{L2}/\lambda_2 + P_{L5}/\lambda_5}{1/\lambda_2 + 1/\lambda_5}, \Phi_{EWL} = \frac{\Phi_{L2}/\lambda_2 - \Phi_{L5}/\lambda_5}{1/\lambda_2 - 1/\lambda_5}, \Phi_{WL} = \frac{\Phi_{L1}/\lambda_1 - \Phi_{L2}/\lambda_2}{1/\lambda_1 - 1/\lambda_2}, \Phi_{NL} = \Phi_{L1}$$

[1] U. Vollath et al., Analysis of three-carrier ambiguity resolution (TCAR) technique for precise relative positioning in GNSS-2, ION GPS-98

[2] J. Jung and P. Enge, Optimization of cascade integer resolution with three civil GPS frequency, ION GPS 2000

Triple Frequency LC

$$\Phi_{(i,j,k)} = \frac{i/\lambda_1\Phi_{L1} + j/\lambda_2\Phi_{L2} + k/\lambda_5\Phi_{L5}}{i/\lambda_1 + j/\lambda_2 + k/\lambda_5} \qquad P_{(l,m,n)} = \frac{l/\lambda_1 P_{L1} + m/\lambda_2 P_{L2} + n/\lambda_5 P_{L5}}{l/\lambda_1 + m/\lambda_2 + n/\lambda_5}$$

Type	LC	Coefficients						λ_{LC} (cm)	I_{LC} (wrt P1)	σ_{LC}		Note
		i	j	k	l	m	n			(cm)	(cyc)	
Geometry Based	$\Phi_{(1,0,0)}$	1	-	-	-	-	-	19.0	-1	0.3	0.016	NL
	$\Phi_{(0,1,0)}$	-	1	-	-	-	-	24.4	-1.647	0.3	0.012	NL
	$\Phi_{(0,0,1)}$	-	-	1	-	-	-	25.5	-1.793	0.3	0.012	NL
	$\Phi_{(1,-1,0)}$	1	-1	-	-	-	-	86.1	1.283	1.7	0.020	WL
	$\Phi_{(1,0,-1)}$	1	-	-1	-	-	-	75.1	1.339	1.5	0.020	WL
	$\Phi_{(0,1,-1)}$	-	1	-1	-	-	-	586.1	1.719	10.0	0.017	EWL
	$\Phi_{(1,-6,5)}$	1	-6	5	-	-	-	325.6	0.074	31.1	0.096	EWL
	$\Phi_{(1,-5,4)}$	1	-5	4	-	-	-	209.3	0.662	16.5	0.079	EWL
	$\Phi_{(4,-3,0)}$	4	-3	-	-	-	-	11.4	-0.090	0.8	0.073	NL
	$\Phi_{(4,0,-3)}$	4	-	-3	-	-	-	10.8	0.010	0.8	0.072	NL
Geometry Free	$\Phi_{(1,-1,0)} - P_{(1,1,0)}$	1	-1	-	1	1	-	86.1	0	21.4	0.249	WL
	$\Phi_{(1,0,-1)} - P_{(1,0,1)}$	1	-	-1	1	-	1	75.1	0	21.5	0.286	WL
	$\Phi_{(0,1,-1)} - P_{(0,1,1)}$	-	1	-1	-	1	1	586.1	0	23.4	0.040	EWL
	$\Phi_{(1,-6,5)} - P_{(1,1,1)}$	1	-6	5	1	1	1	325.6	-1.360	35.7	0.110	EWL
	$\Phi_{(1,-6,5)} - P_{(1,1,0)}$	1	-6	5	1	1	-	325.6	-1.209	37.8	0.116	EWL
	$\Phi_{(1,-5,4)} - P_{(1,1,1)}$	1	-5	4	1	1	1	209.3	-0.773	24.1	0.115	EWL
	$\Phi_{(1,-5,4)} - P_{(1,1,0)}$	1	-5	4	1	1	-	209.3	-0.622	27.0	0.129	EWL

$$\lambda_{LC} = \frac{1}{i/\lambda_1 + j/\lambda_2 + k/\lambda_5} \qquad N_{LC} = i \cdot N_{L1} + j \cdot N_{L2} + k \cdot N_{L5} \qquad I_{LC} = -\frac{i/\lambda_1 + j\cdot\lambda_2/\lambda_1^2 + k\cdot\lambda_5/\lambda_1^2}{i/\lambda_1 + j/\lambda_2 + k/\lambda_5} - \frac{l/\lambda_1 + m\cdot\lambda_2/\lambda_1^2 + n\cdot\lambda_5/\lambda_1^2}{l/\lambda_1 + m/\lambda_2 + n/\lambda_5}$$

$$\sigma_{LC} = \sqrt{\frac{(i/\lambda_1)^2 + (j/\lambda_2)^2 + (k/\lambda_5)^2}{(i/\lambda_1 + j/\lambda_2 + k/\lambda_5)^2}\sigma_\Phi^2 + \frac{(l/\lambda_1)^2 + (m/\lambda_2)^2 + (n/\lambda_5)^2}{(l/\lambda_1 + m/\lambda_2 + n/\lambda_5)^2}\sigma_P^2} \qquad (\sigma_\Phi = 0.3\text{cm},\ \sigma_P = 30\text{cm})$$

Partial AR (PAR)

Partial AR (PAR)

Full AR (FAR) indicates low fixing ratio or long TTFF under ill conditions like long BL RTK

Newly rising satellites or cycle-slips often disturbs FAR with validation

Degraded accuracy by PAR solutions acceptable in many applications

Many criteria how to select subset of ambiguities:

Elevation angle, continuous tracking epochs, SNR, ADOP (ambiguity dilution of precision), EWL/WL/NL, success or failure probability of AR

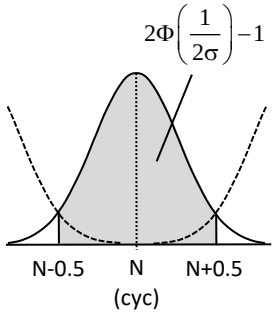
BSR (bootstrapping success rate) Criterion [1]

$$\prod_{i=k}^{n}\left(2\Phi\left(\frac{1}{2\sigma_{\hat{z}_{i|I}}}\right)-1\right)\geq P_0$$

P_0 : Minimum success probability of bootstrapping AR

$\sigma_{\hat{z}_{i|I}}$: Std-dev of Z-transformed i-th ambiguity of bootstrapping AR

$\Phi(x)$: CDF (cumulative distribution function) of normal distribution



$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{t^2}{2}\right)dt$$

$$=\frac{1}{2}\left(1-\mathrm{erf}\left(\frac{1}{\sqrt{2}}\right)\right)$$

FF-RT (fixed-failure-rate ratio-test) Criterion [1]

$$\frac{\left\|\hat{\mathbf{z}}_p-\breve{\mathbf{z}}'_p\right\|^2_{\mathbf{Q}_{\hat{z}_p}}}{\left\|\hat{\mathbf{z}}_p-\breve{\mathbf{z}}_p\right\|^2_{\mathbf{Q}_{\hat{z}_p}}}\geq c(n,P_f)$$

$p=(k,k+1,\ldots,n)$: Partial set of Z-transformed ambiguities

$\hat{\mathbf{z}}_p$: Float Z-transformed ambiguities

$\breve{\mathbf{z}}_p,\breve{\mathbf{z}}'_p$: Best-fixed, secondary-best-fixed Z-transformed ambiguities

$c(n,P_f)$: Threshold of FF-RT (fixed-failure-rate ratio-test)

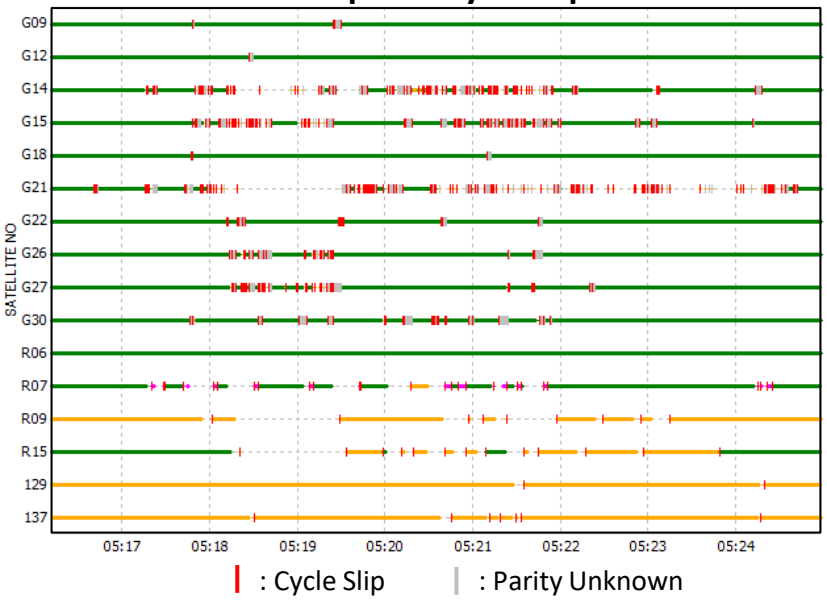
[1] S. Verhagen et al., GNSS ambiguity resolution: which subset to fix ?, International GNSS Symposium, 2011

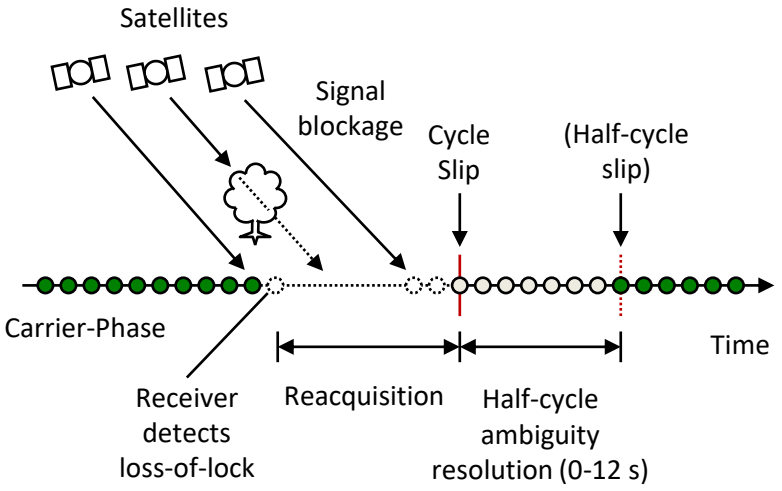
Cycle Slip

Definition

A sudden jump in carrier-phase measurements when the phase tracking loop (PLL) in receivers detects a temporary loss of lock due to signal blockage or some other disturbing factor. The carrier-cycle counter in the receiver restarts, causing a jump in the accumulated phase by an integer number of cycles.

Example of Cycle Slips





Receiver Biases and Phase Shift

DCB (Differential Code Bias)

Biases in pseudorange between different codes tracked (e.g. GPS L1C/A - L1P(Y))

Satellite DCB should be corrected by external DCB info (T_{GB}, P1-C1 DCB, P2-C2 DCB ...)

Receiver DCB usually implicitly estimated as in the receiver clock biases as LC

ISB (Inter System Bias)

Biases in pseudorange and phase between different systems (e.g. GPS L1C/A - GAL E1B)

Hard to incorporated ISB in estimation due to unstable behavior (varied by receiver F/W updates)

IFB (Inter Frequency Bias)

Biases in pseudorange and phase between different FCN GLONASS satellites (e.g. R01 - R02)

IFB in pseudorange should be estimated explicitly or corrected by external calibration data

IFB in phase is hard to be corrected due to instability including antennas and cables

ISTB (Inter Satellite Type Bias) [1]

Biases in pseudorange and phase between different BDS satellite types (e.g. BDS GEO - IGSO)

Quarter Cycle Phase Shift

Quarter cycle phase shift in phase between different codes tracked (e.g. GPS L2P(Y) - L2C)

All phases on the same frequency should be aligned by the latest RINEX or RTCM 3, but needs careful handling of OBS data in vendor proprietary formats

[1] N. Nadarajah et al., The mixed-receiver BeiDou inter-satellite-type-bias and its impact on RTK positioning, GPS Solution, 2014

GNSS Data Formats

RINEX (Receiver Independent Exchange Format)

Developed and maintained by IGS since 1990s for PP and data archive

For observation data (OBS), navigation data (NAV), meteorological data (MET) and extensions

RTCM (Radio Technical Commission for Maritime Services)

Developed and maintained by RTCM SC-104 (special-committee 104) for DGNSS, RTK and PPP

RTCM 2 (version 2) and RTCM 3 (version 3). RTCM 3 is not compatible to RTCM 2.

Compact and portable binary format

BINEX (Binary Exchange Format)

Developed and maintained by UNAVCO (university NAVSTAR consortium) for RT or PP and archive

Compact and portable binary format

Proprietary Formats by GNSS Receiver Vendors

RT17/RT27, CMR/CMR+ (Trimble), GREIS (JAVAD), NovAtel, SBF (Septentrio), UBX (u-blox), ...

Example of Data Sizes[*1]

Format	Data Size (Bytes)					
	RTCM 3 *1	**BINEX** *1	**CMR+** *1	**RINEX 2.12** *2	**RINEX 3.02** *2	**Compressed RINEX** *3
OBS Data	2,387,289	5,140,927	1,137,059	83,386,990	29,225,908	6,709,900
NAV Data	-			33,022	32,794	-

*1 GNSS receiver: Trimble BD982, Data sampling: 1 Hz x 1 H, System: GPS/GLO/GAL/BDS/QZS/SBS, *2 Converted from BINEX by RTKCONV 2.4.3 b32, *3 Converted from RINEX 3.02 by RNX2CNX 4.08

RINEX

RINEX Version/ RINEX Type

```
     3.02           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
NetR9 5.37          Receiver Operator   20190710 000000 UTC PGM / RUN BY / DATE
...
G   12 C1C L1C S1C C2W L2W S2W C2X L2X S2X C5X L5X S5X      SYS / # / OBS TYPES
R   12 C1C L1C S1C C1P L1P S1P C2C L2C S2C C3X L3X S3X      SYS / # / OBS TYPES
E   12 C1X L1X S1X C5X L5X S5X C7X L7X S7X C8X L8X S8X      SYS / # / OBS TYPES
...
                                                           END OF HEADER
> 2019  7 10  0  0  0.0000000  0 35
R10  21997839.273 7 117260873.451 7       47.800    21997838.297 7 ...
...
...
```

Header Section

Data Records

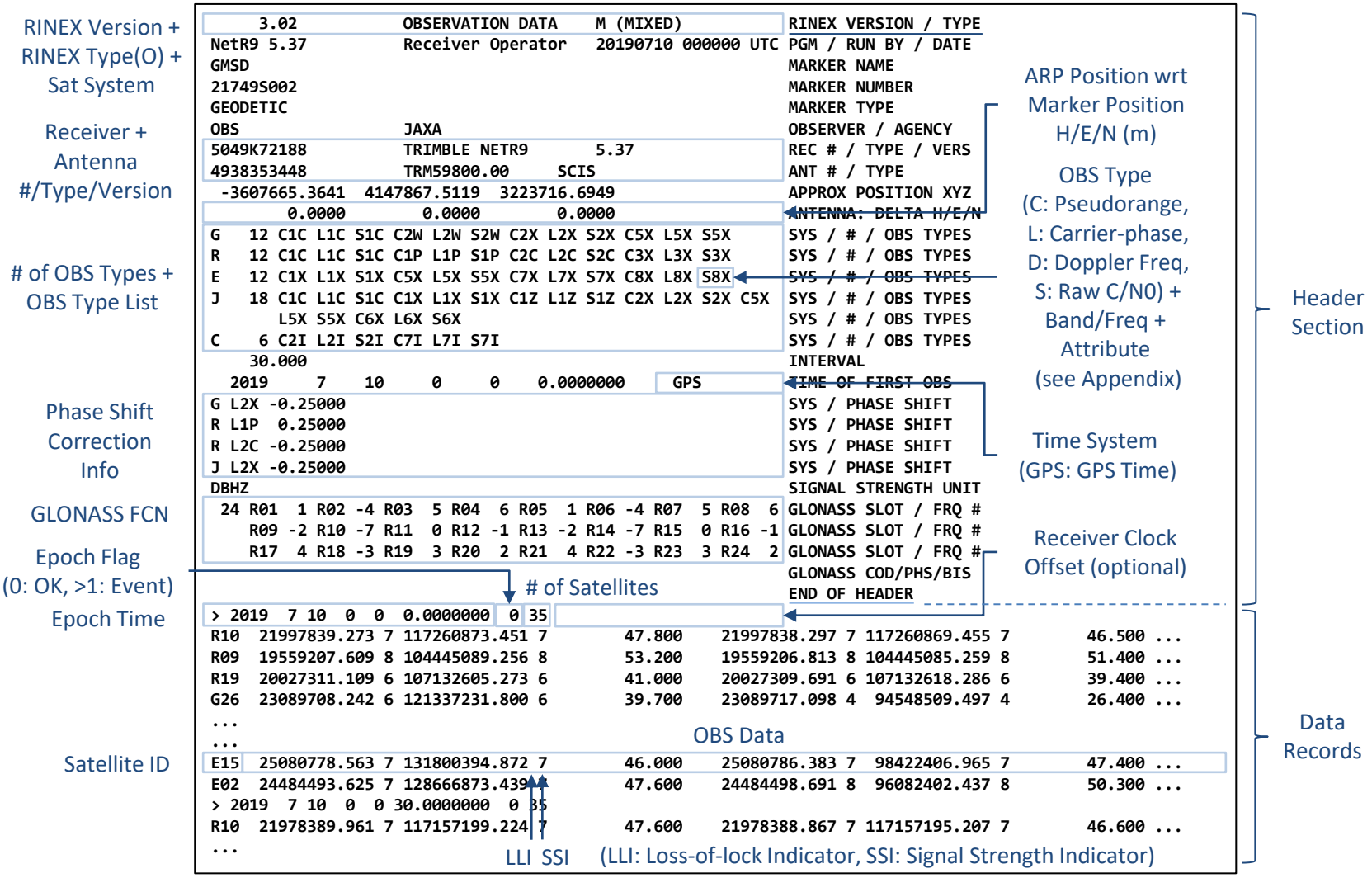
Sat System (G: GPS, R: GLONASS, E: Galileo, J: QZSS, C: BeiDou, I: IRNSS, S: SBAS, M: Mixed)

RINEX Version	RINEX Type																
	Observation Data [1]							Navigation Data [1]							MET [1]	CLK [2]	SBAS [3]
	G	R	E	J	C	I	S	G	R	E	J	C	I	S			
2.10	O	O	-	-	-	-	O	N	G	-	-	-	-	H	M	-	B
2.11	O	O	O	-	-	-	O	N	G	-	-	-	-	H	M	-	-
2.12	O	O	O	-	O	-	O	N	G	N	-	-	-	H	M	-	-
3.00	O	O	O	-	-	-	O	N	N	N	-	-	-	N	M	C	-
3.01	O	O	O	-	O	-	O	N	N	N	-	-	-	N	M	C	-
3.02	O	O	O	O	O	-	O	N	N	N	N	N	-	N	M	C	-
3.03	O	O	O	O	O	O	O	N	N	N	N	N	N	N	M	-	-
3.04	O	O	O	O	O	O	O	N	N	N	N	N	N	N	M	-	-

MET: Meteorological data, CLK: RINEX clock extension, SBAS: GEO SBAS broadcast data extension

[1] RINEX - The receiver independent exchange format version 3.04, Nov 2018, [2] RINEX extension to handle clock information, version 3.02, Sep 2010, [3] Proposal for a new RINEX-type exchange file for GEO SBAS broadcast data, Sep 2004

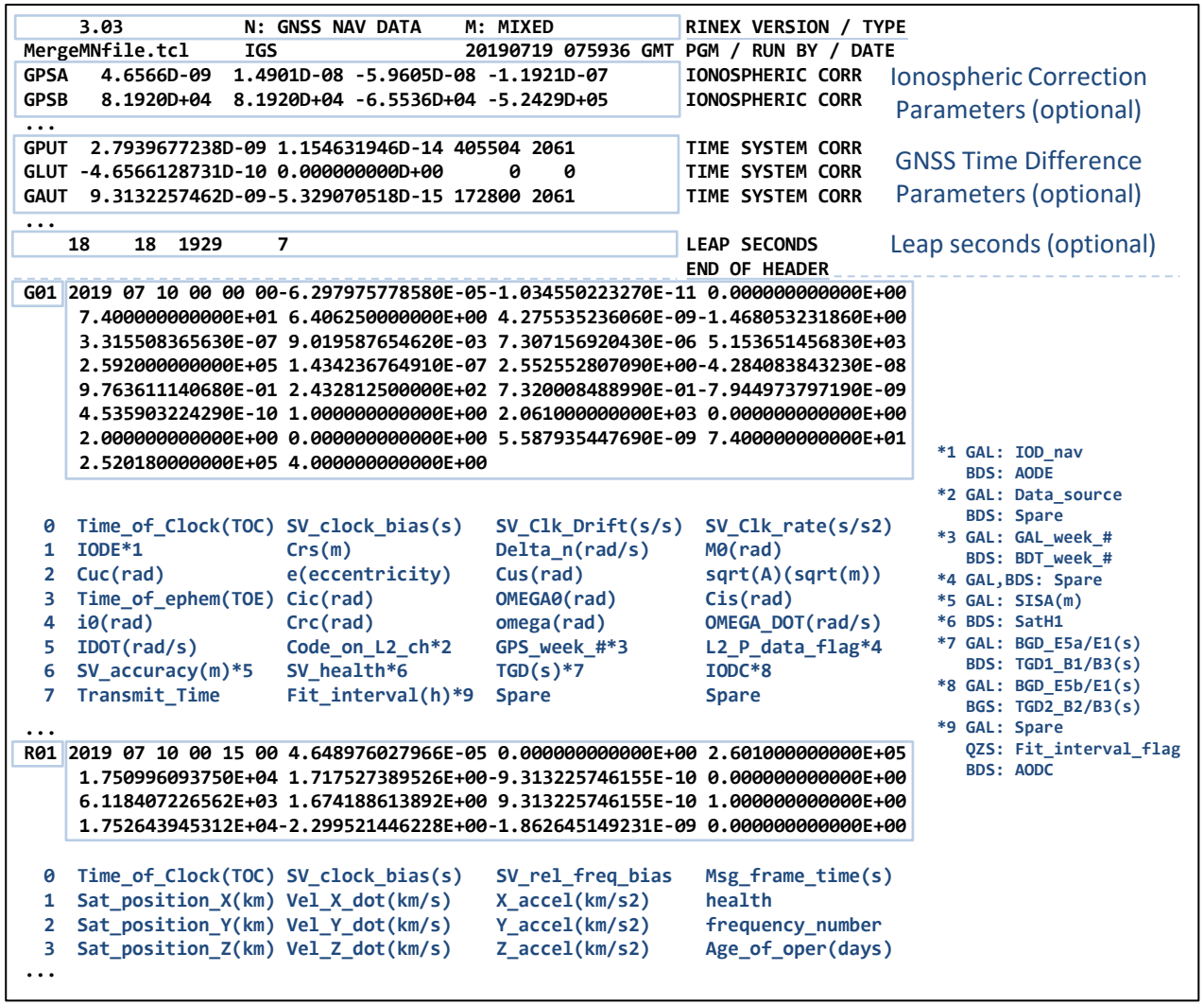
RINEX 3 OBS




```
     3.02           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
NetR9 5.37          Receiver Operator   20190710 000000 UTC PGM / RUN BY / DATE
GMSD                                                        MARKER NAME
21749S002                                                   MARKER NUMBER
GEODETIC                                                    MARKER TYPE
OBS                 JAXA                                    OBSERVER / AGENCY
5049K72188          TRIMBLE NETR9       5.37                REC # / TYPE / VERS
4938353448          TRM59800.00     SCIS                    ANT # / TYPE
 -3607665.3641  4147867.5119  3223716.6949                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
G   12 C1C L1C S1C C2W L2W S2W C2X L2X S2X C5X L5X S5X      SYS / # / OBS TYPES
R   12 C1C L1C S1C C1P L1P S1P C2C L2C S2C C3X L3X S3X      SYS / # / OBS TYPES
E   12 C1X L1X S1X C5X L5X S5X C7X L7X S7X C8X L8X S8X      SYS / # / OBS TYPES
J   18 C1C L1C S1C C1X L1X S1X C1Z L1Z S1Z C2X L2X S2X C5X  SYS / # / OBS TYPES
       L5X S5X C6X L6X S6X                                  SYS / # / OBS TYPES
C    6 C2I L2I S2I C7I L7I S7I                              SYS / # / OBS TYPES
    30.000                                                  INTERVAL
  2019     7    10     0     0    0.0000000     GPS         TIME OF FIRST OBS
G L2X -0.25000                                              SYS / PHASE SHIFT
R L1P  0.25000                                              SYS / PHASE SHIFT
R L2C -0.25000                                              SYS / PHASE SHIFT
J L2X -0.25000                                              SYS / PHASE SHIFT
DBHZ                                                        SIGNAL STRENGTH UNIT
 24 R01  1 R02 -4 R03  5 R04  6 R05  1 R06 -4 R07  5 R08  6 GLONASS SLOT / FRQ #
    R09 -2 R10 -7 R11  0 R12 -1 R13 -2 R14 -7 R15  0 R16 -1 GLONASS SLOT / FRQ #
    R17  4 R18 -3 R19  3 R20  2 R21  4 R22 -3 R23  3 R24  2 GLONASS SLOT / FRQ #
                                                            GLONASS COD/PHS/BIS
                                                            END OF HEADER
> 2019  7 10  0  0  0.0000000  0 35
R10  21997839.273 7 117260873.451 7        47.800    21997838.297 7 117260869.455 7        46.500 ...
R09  19559207.609 8 104445089.256 8        53.200    19559206.813 8 104445085.259 8        51.400 ...
R19  20027311.109 6 107132605.273 6        41.000    20027309.691 6 107132618.286 6        39.400 ...
G26  23089708.242 6 121337231.800 6        39.700    23089717.098 4  94548509.497 4        26.400 ...
...
...
E15  25080778.563 7 131800394.872 7        46.000    25080786.383 7  98422406.965 7        47.400 ...
E02  24484493.625 7 128666873.439 7        47.600    24484498.691 8  96082402.437 8        50.300 ...
> 2019  7 10  0  0 30.0000000  0 35
R10  21978389.961 7 117157199.224 7        47.600    21978388.867 7 117157195.207 7        46.600 ...
...
```

[1] IGS RINEX WG and RTCM-SC104, RINEX - The receiver independent exchange format version 3.04, Nov 2018

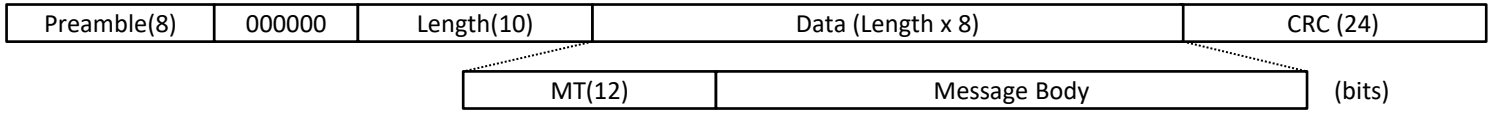
RINEX 3 NAV



[1] IGS RINEX WG and RTCM-SC104, RINEX - The receiver independent exchange format version 3.04, Nov 2018

RTCM 3

RTCM 3 Message [1]

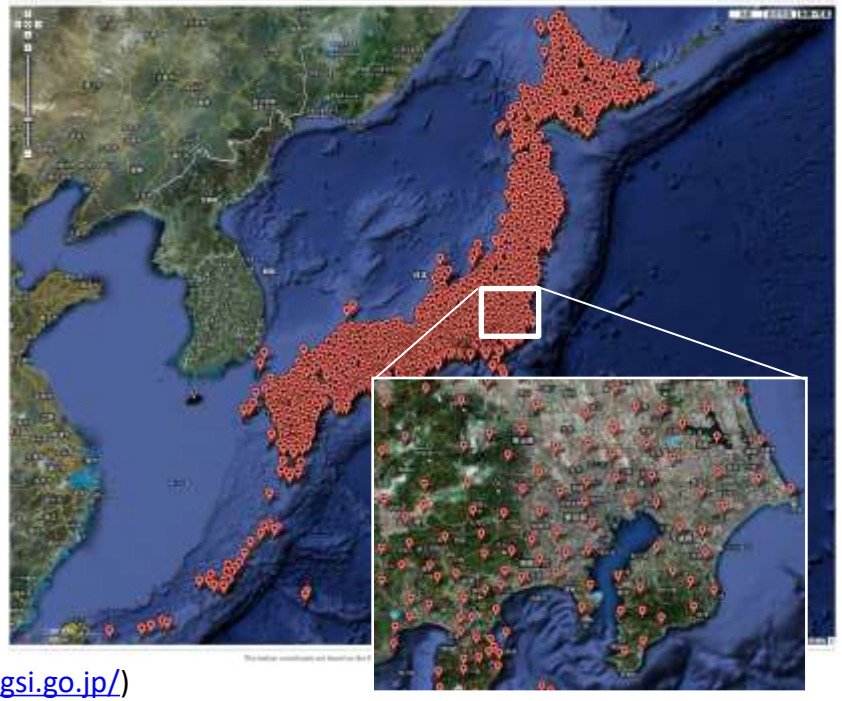


Message		MT (Message Type)					
		GPS	GLONASS	Galileo	QZSS	BeiDou	SBAS
OBS	Basic RTK L1 Only	1001	1009	-	-	-	-
	Extended RTK L1 Only	1002	1010	-	-	-	-
	Basic RTK L1 & L2	1003	1011	-	-	-	-
	Extended RTK L1 & L2	1004	1012	-	-	-	-
Satellite Ephemeris Data		1019	1020	1045, 1046*2	1044	1042	-
MSM	MSM 1 (Compact P) *3	1071	1081	1091	1111	1121	1101
	MSM 2 (Compact PR) *3	1072	1082	1092	1112	1122	1102
	MSM 3 (Compact P+PR) *3	1073	1083	1093	1113	1123	1103
	MSM 4 (Full P+PR+CN) *3	1074	1084	1094	1114	1124	1104
	MSM 5 (Full P+PR+PRR+CN) *3	1075	1085	1095	1115	1125	1105
	MSM 6 (Full P+PR+CN+H) *3	1076	1086	1096	1116	1126	1106
	MSM 7 (Full P+PR+PRR+CN+H) *3	1077	1087	1097	1117	1127	1107
SSR	Orbit Correction	1057	1063	1240*1	1246*1	1258*1	1252*1
	Clock Correction	1058	1064	1241*1	1247*1	1259*1	1253*1
	Code Bias	1059	1065	1242*1	1248*1	1260*1	1254*1
	Combined Orbit and Clock	1060	1066	1243*1	1249*1	1261*1	1255*1
	URA	1061	1067	1244*1	1250*1	1262*1	1256*1
	High-Rate Clock Correction	1062	1068	1245*1	1251*1	1263*1	1257*1
Station Coordinates/Antenna Description		1005, 1006, 1007, 1008, 1032, 1033					
Proprietary Information		4070 ~ 4095					

*1 Draft version, *2 1045: F/NAV, 1046: I/NAV, *3 P: Pseudorange, PR: PhaseRange, PRR: PhaseRangeRate, CN: CNR, H: High Resolution

[1] RTCM standard 10403.3, Differential GNSS (global navigation satellite system) services - version 3, Oct 2016

GSI GEONET



(https://terras.gsi.go.jp/)

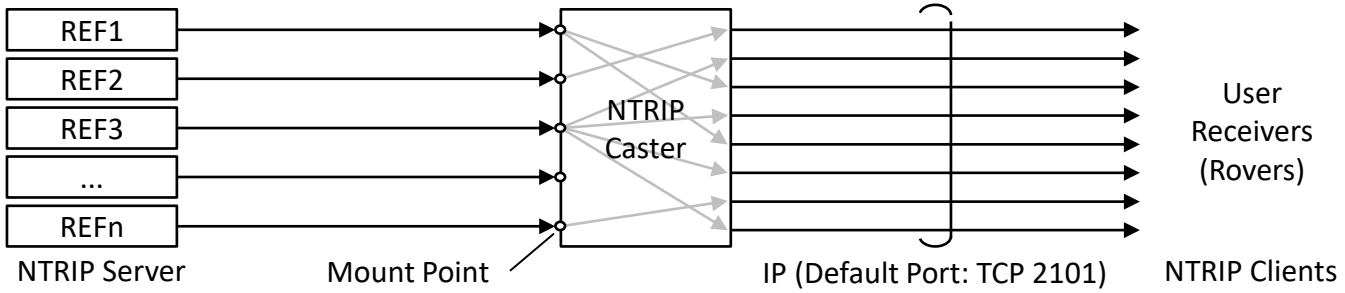
NTRIP (1)

NTRIP (Networked Transport of RTCM via Internet Protocol) [1]

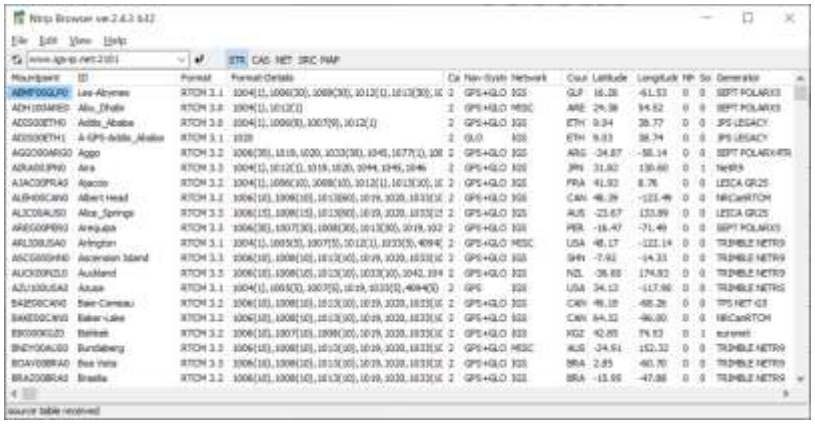
Designed based on HTTP/RTSP to disseminate DGNSS corrections and GNSS data over Internet

Ver. 1.0: Initial version only supporting TCP/IP

Ver. 2.0: Supporting RTP/UDP/IP and other feathers (chunked transfer enc., source table filtering ...)



NTRIP Mount Point List



NTRIP Source Table

[1] RTCM standard 10410.1 with amendment 1, Networked transport of RTCM via internet protocol (Ntrip) - version 2.0, June 2011

NTRIP (2)

Commercial NTRTK/RTK Services in Japan

GPSdata (https://www.gpsdata.co.jp)

Jenoba (https://www.jenoba.jp)

Nihon Terasat (https://www.terasat.co.jp)

NTT docomo (https://www.nttdocomo.co.jp/biz/service/highprecision_gnss_positioning)

Softbank (https://www.softbank.jp/biz/iot/service/ichimill)

Open RTK Reference Stations

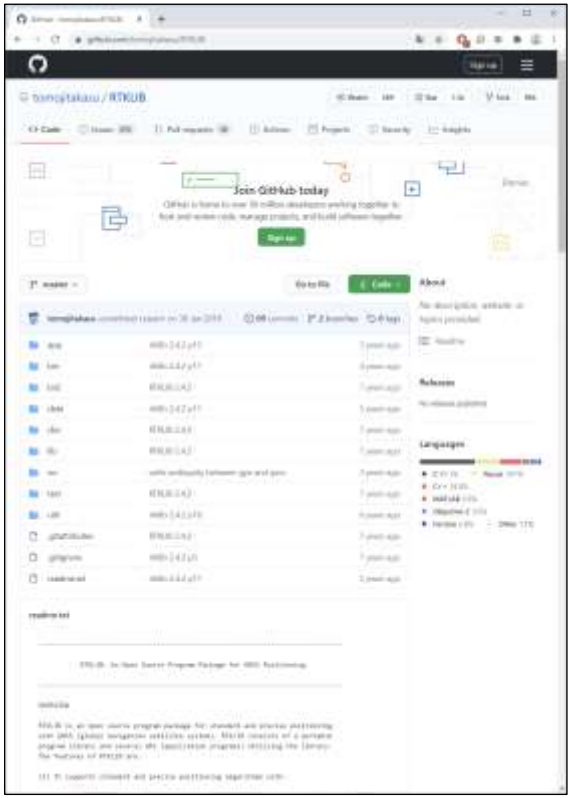
(http://rtk.silentsystem.jp)

(http://www.rtk2go.com)

(https://hamamatsu-gnss.org)

3.
RTK/PPK Practice

RTKLIB



http://www.rtklib.com/

https://github.com/tomojitakasu/RTKLIB

https://shop.cqpub.co.jp/hanbai/books/MSP/MSPZ202004.html

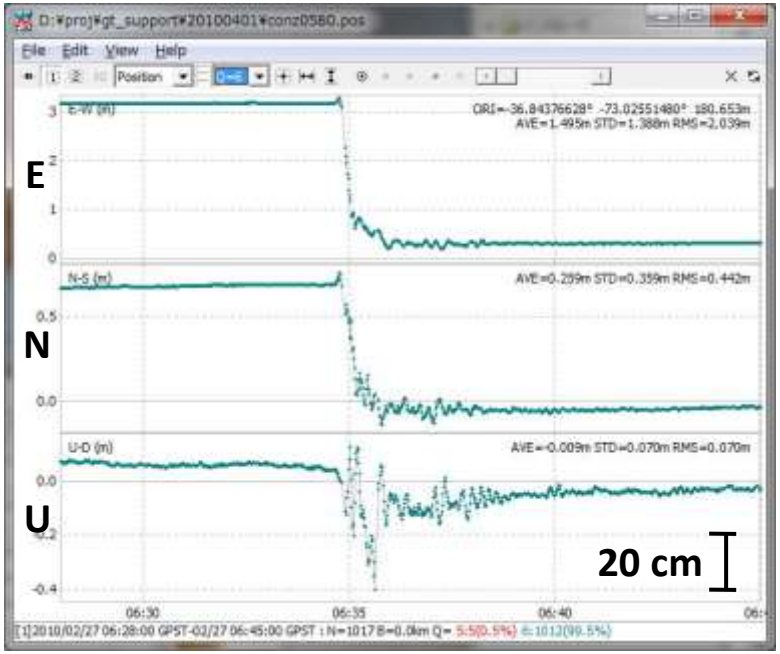
RTK/PPK Practice

(TBD)

4.
Theory of PPP

Typical Accuracy of PPP

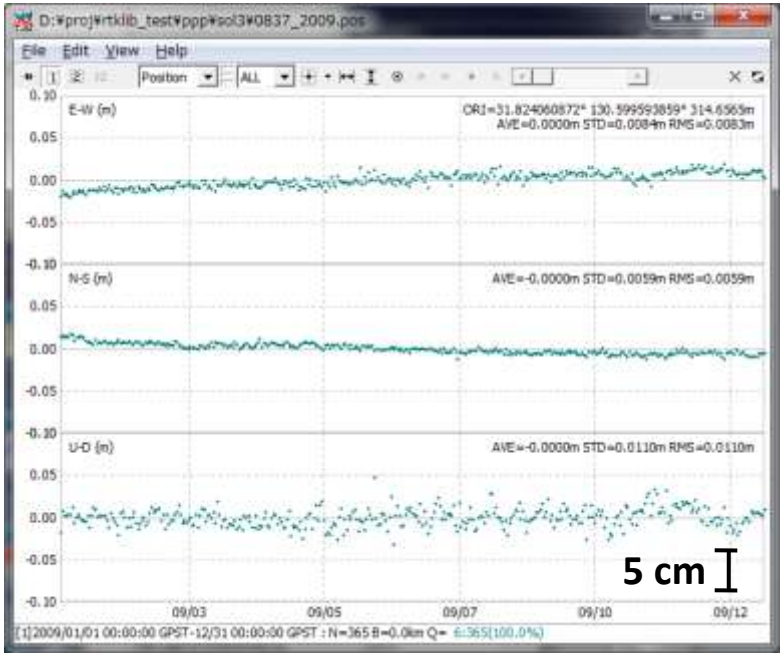
1 Hz Kinematic PPP



IGS CONZ, 2010/2/27 6:28-6:45 GPST

H-RMS: 1 cm, V-RMS: 2 cm (PP)
H-RMS: 3 cm, V-RMS: 6 cm (RT)

24 H Static PPP



GEONET 0837 (Aira), 2009/1/1-2009/12/31

H-RMS: 3 mm, V-RMS: 6 mm (PP)

PPP Model Error < 1 mm

Pseudorange/Carrier-phase Model

$$
\begin{aligned}
P_r^s &\equiv c\tau \\
&= c(\bar{t}_r - \bar{t}^s) \\
&= c((t_r + dt_r) - (t^s + dT^s(t^s))) + \varepsilon_P \\
&= c(t_r - t^s) + c(dt_r - dT^s(t^s)) + \varepsilon_P \\
&= (\rho_r^s + I_r^s + T_r^s) + c(dt_r - dT^s(t^s)) + \varepsilon_P \\
&= \rho_r^s + c(dt_r - dT^s(t^s)) + I_r^s + T_r^s + d_r^s + \varepsilon_P \\
&\quad \overline{(1)} \quad \overline{(2)} \quad \overline{(3)} \quad \overline{(4)} \quad \overline{(5)}
\end{aligned}
$$

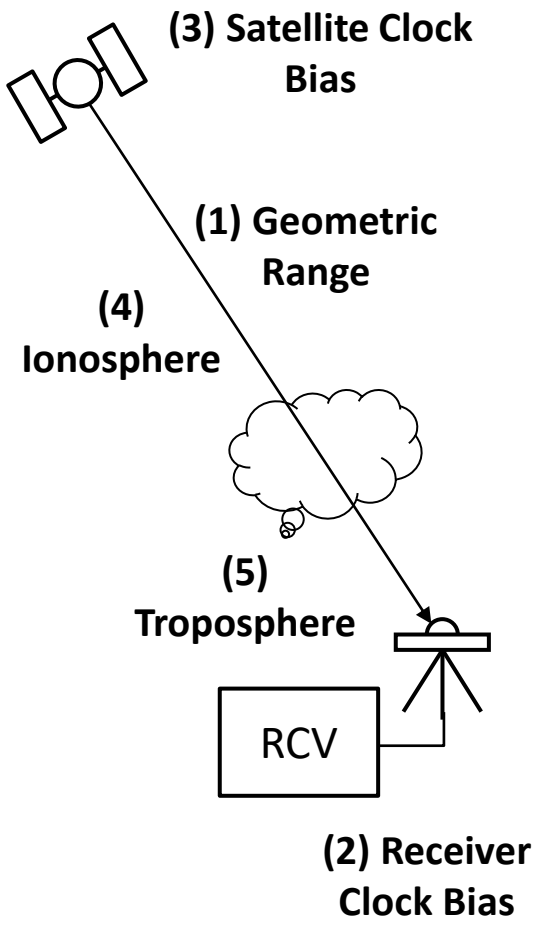
$$
\begin{aligned}
\phi_r^s &= \phi_r(t_r) - \phi^s(t^s) + N_r^s + \varepsilon_\phi \\
&= (f(t_r + dt_r - t_0) + \phi_{r,0}) - (f(t^s + dT^s(t^s) - t_0) + \phi_0^s) + N_r^s + \varepsilon_\phi \\
&= \frac{c}{\lambda}(t_r - t^s) + \frac{c}{\lambda}(dt_r - dT^s(t^s)) + (\phi_{r,0} - \phi_0^s + N_r^s) + \varepsilon_\phi \\
\Phi_r^s &\equiv \lambda\phi_r^s = c(t_r - t^s) + c(dt_r - dT^s(t^s)) + \lambda(\phi_{r,0} - \phi_0^s + N_r^s) + \lambda\varepsilon_\Phi \\
&= \rho_r^s + c(dt_r - dT^s(t^s)) - I_r^s + T_r^s + d_r^s + \lambda d_{pw} + \lambda B_r^s + \varepsilon_\Phi
\end{aligned}
$$

$$d_r^s = d_{r,ant} + d_{ant}^s - \mathbf{d}_{disp}{}^T \mathbf{e}_{r,enu}^s + d_{rel}$$

$$B_r^s = \phi_{r,0} - \phi_0^s + N_r^s$$

$\phi_{r,0}$: Receiver initial phase (cyc)

ϕ_0^s : Satellite initial phase (cyc)



LC (Linear Combination)

LC of L1, L2, L5 Carrier Phase and Pseudorange

$$LC = a\Phi_{L1} + b\Phi_{L2} + c\Phi_{L5} + dP_{L1} + eP_{L2} + fP_{L5}$$

	LC	Coefficients						Wave Length (cm)	Ionos Effect wrt L1	Typical Noise (cm)
		a	b	c	d	e	f			
L1	L1 Carrier-Phase	1	-	-	-	-	-	19.0	1.0	0.3
L2	L2 Carrier-Phase	-	1	-	-	-	-	24.4	1.6	0.3
L5	L5 Carrier-Phase	-	-	1	-	-	-	25.5	1.8	0.3
LC/L3	L1-L2 Iono-Free Phase	C_1	C_2	-	-	-	-	-	0.0	0.9
LC_{L1-L5}	L1-L5 Iono-Free Phase	C_3	-	C_4	-	-	-	-	0.0	0.8
LG/L4	L1-L2 Geometry-Free Phase	1	-1	-	-	-	-	-	-0.6	0.4
LG_{L2-L5}	L2-L5 Geometry-Free Phase	-	1	-1	-	-	-	-	-0.1	0.4
EWL	Extra-Wide-Lane Phase	-	$\lambda_{EWL}/\lambda_{L2}$	$-\lambda_{EWL}/\lambda_{L5}$	-	-	-	586.1	-0.2	0.4
WL	Wide-Lane Phase	$\lambda_{WL}/\lambda_{L1}$	$-\lambda_{WL}/\lambda_{L2}$	-	-	-	-	86.2	-1.3	1.7
NL	Narrow-Lane Phase	$\lambda_{NL}/\lambda_{L1}$	$\lambda_{NL}/\lambda_{L2}$	-	-	-	-	10.7	1.3	1.7
MW	Melbourne-Wübbena WL	$\lambda_{WL}/\lambda_{L1}$	$-\lambda_{WL}/\lambda_{L2}$	-	$-\lambda_{NL}/\lambda_{L1}$	$-\lambda_{NL}/\lambda_{L2}$	-	86.2	0.0	21
MW_{EWL}	Melbourne-Wübbena EWL	-	$\lambda_{EWL}/\lambda_{L2}$	$-\lambda_{EWL}/\lambda_{L5}$	-	$-\lambda_{ENL}/\lambda_{L2}$	$-\lambda_{ENL}/\lambda_{L5}$	586.1	0.0	21
MP1	L1-Multipath	$2C_2 - 1$	$-2C_2$	-	1	-	-	-	0.0	30
MP2	L2-Multipath	$-2C_1$	$2C_1 - 1$	-	-	1	-	-	0.0	30
MP5	L5-Multipath	$-2C_3$	-	$2C_3 - 1$	-	-	1	-	0.0	30

$$C_1 = \lambda_{L1}^2\lambda_{L2}^2/(\lambda_{L2}^2 - \lambda_{L1}^2) = 2.55, C_2 = -\lambda_{L2}^2\lambda_{L1}^2/(\lambda_{L2}^2 - \lambda_{L1}^2) = -1.55, C_3 = \lambda_{L1}^2\lambda_{L5}^2/(\lambda_{L5}^2 - \lambda_{L1}^2) = 2.26, C_4 = -\lambda_{L5}^2\lambda_{L1}^2/(\lambda_{L5}^2 - \lambda_{L1}^2) = -1.26,$$
$$\lambda_{WL} = \lambda_{L1}\lambda_{L2}/(\lambda_{L2} - \lambda_{L1}) = 86.2\,\text{cm}, \lambda_{NL} = \lambda_{L1}\lambda_{L2}/(\lambda_{L1} + \lambda_{L2}) = 10.7\,\text{cm}, \lambda_{EWL} = \lambda_{L2}\lambda_{L5}/(\lambda_{L5} - \lambda_{L2}) = 586.1\text{cm}, \lambda_{ENL} = \lambda_{L2}\lambda_{L5}/(\lambda_{L2} + \lambda_{L5}) = 12.5\text{cm}$$

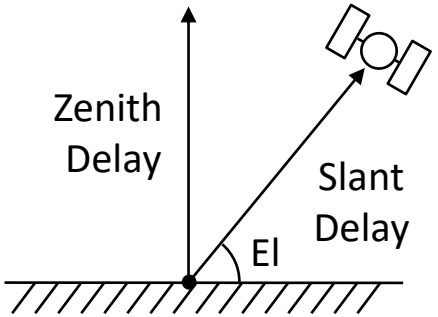
Tropospheric Delay

Tropospheric Delay

$$T_r^s = m_h(El)Z_h + m_w(El)Z_w \quad (1)$$

$$T_r^s = m_h(El)Z_h + m_w(El)(Z_t - Z_h) \quad (2)$$

(m)



Zenith hydrostatic delay

$$Z_h = \frac{0.0022768\,p}{1 - 0.00266\cos 2\phi - 2.8\times10^{-7}H}$$

p : Total atmospheric pressure (hPa)

Troposphere Model by Standard Atmosphere

$$p = 1013.25\times(1 - 2.2557\times10^{-5}h)^{5.2568}$$

$$T = 15.0 - 6.5\times10^{-3}h + 273.15$$

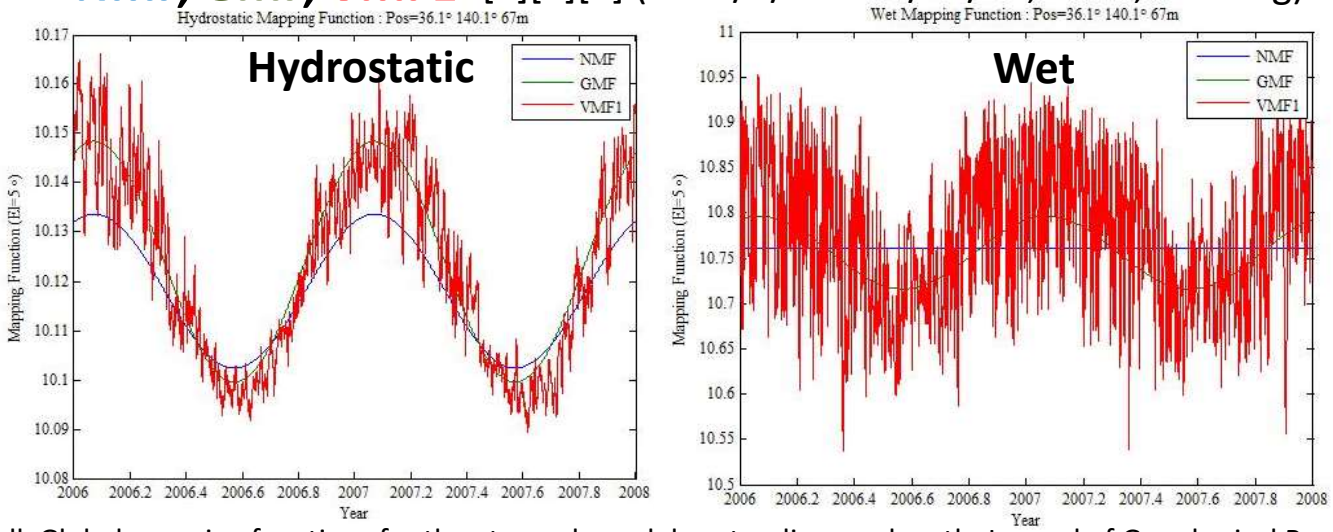
$$e = 6.108\times\exp\left\{\frac{17.15T - 4684.0}{T - 38.45}\right\}\times\frac{h_{rel}}{100}$$

Mapping Function

$$m_x(El) = \frac{1 + \dfrac{a_x}{1 + \dfrac{b_x}{1 + c_x}}}{\sin(El) + \dfrac{a_x}{\sin(El) + \dfrac{b_x}{\sin(El) + c_x}}}$$

a_x, b_x, c_x : Mapping Function Coefficients

NMF, GMF, VMF1 [1][2][3] (2006/1/1-2007/12/31, TSKB, El=5deg)



[1] A.E.Niell, Global mapping functions for the atmosphere delay at radio wavelength, Journal of Geophysical Research, 1996

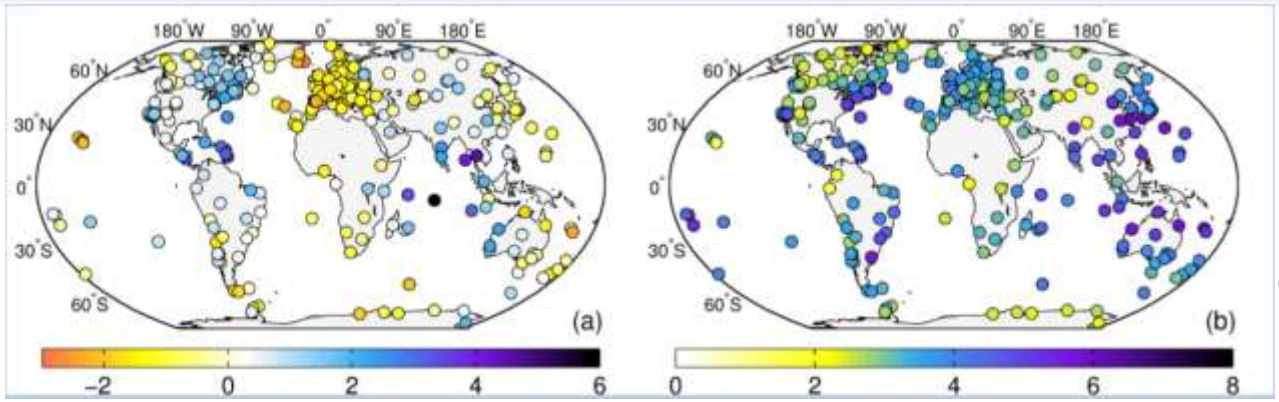
[2] J.Boehm, A.Niell, P.Tregoning and H.Shuh, Global Mapping Function (GMF): A new empirical mapping function base on numerical weather model data, G/formeophysical Res Lett, 33, L07304, 2006

[3] J.Boehm, R.Heinkelmann and H.Schuh, Short note: A global model of pressure and temperature for geodetic applications, Journal of Geodesy, 2007

Empirical Tropospheric Model

Model ZTD Bias and RMS error (cm) (2012)

	mean bias	mean standard deviation	
RTCA-MOPS (1999)	-2.50 cm	4.55 cm	[1]
ESA (Martellucci 2012)	0.83 cm	3.82 cm	[2]
GPT2 (Lagler et al. 2013)	-0.28 cm	3.79 cm	[3]
GPT2w (Böhm et al. 2014)	-0.02 cm	3.61 cm	



GPT2w ZTD Bias and RMS error (cm)

J. Bohm et al., Development of an improved empirical model for slant delays in the troposphere (GPT2w), GPS Solutions, 2015

[1] RTCA/DO-229, Minimum operational standards for global positioning system/wide area augmentation system airborne equipment. 1999

[2] A. Martellucci, Galileo reference troposphere model for the user receiver. ESA-APPNG-REF/00621-AM v2.7, 2012

[3] K. Lagler et al., GPT2: empirical slant delay model for radio space geodetic techniques. Geophys Res Lett, 2013

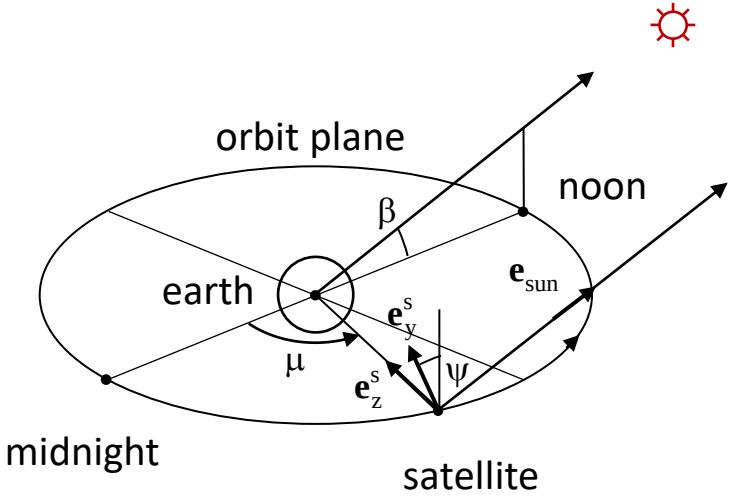
Satellite Attitude Model

Nominal Yaw Attitude Model

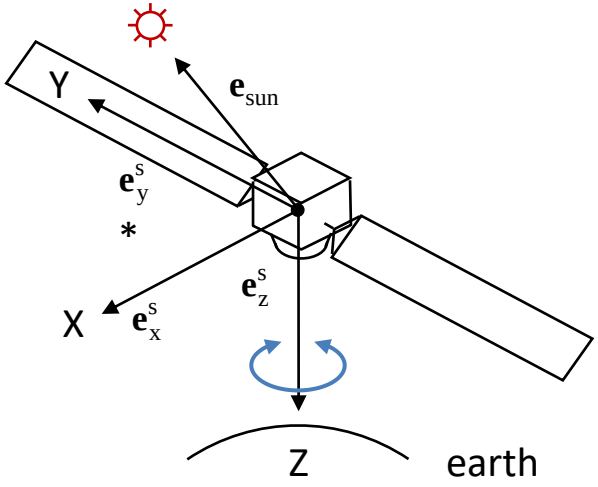
$$\mathbf{E}^s = (\mathbf{e}_x^{s\,T}, \mathbf{e}_y^{s\,T}, \mathbf{e}_z^{s\,T})^T$$

$$\mathbf{e}_z^s = -\frac{\mathbf{r}^s(t^s)}{\left\|\mathbf{r}^s(t^s)\right\|}, \mathbf{e}_{sun} = \frac{\mathbf{r}_{sun}(t^s) - \mathbf{r}^s(t^s)}{\left\|\mathbf{r}_{sun}(t^s) - \mathbf{r}^s(t^s)\right\|}$$

$$\mathbf{e}_y^s = \frac{\mathbf{e}_z^s \times \mathbf{e}_{sun}}{\left\|\mathbf{e}_z^s \times \mathbf{e}_{sun}\right\|}, \mathbf{e}_x^s = \mathbf{e}_y^s \times \mathbf{e}_z^s$$

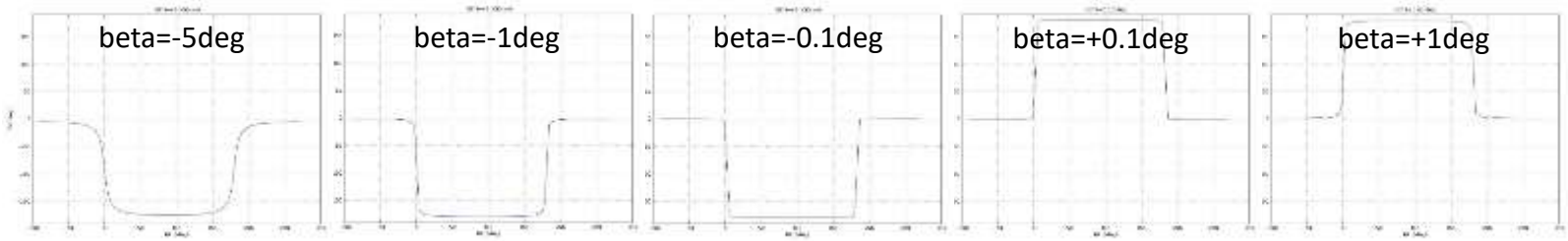


$\mathbf{r}_{sun}(t)$: Sun position in ECEF (m)
β : Beta angle of orbit plane (rad)
μ : Orbit angle from midnight (rad)
ψ : Yaw angle of satellite attitude (rad)



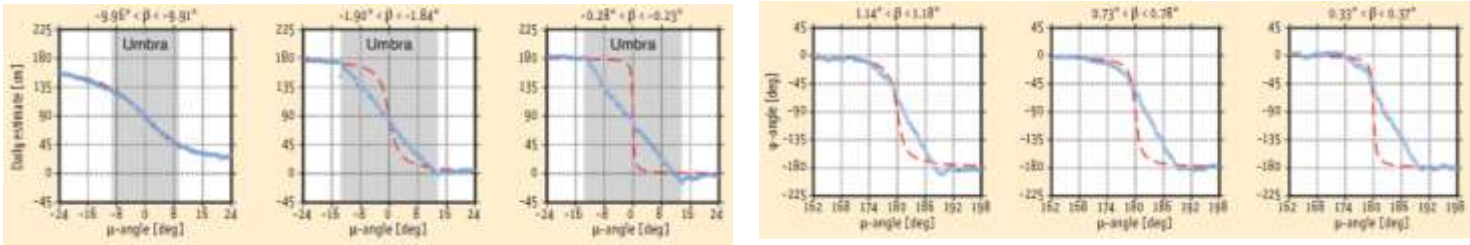
* Some satellites take opposite direction of Y-axis

Yaw Attitude of Eclipsing Satellites



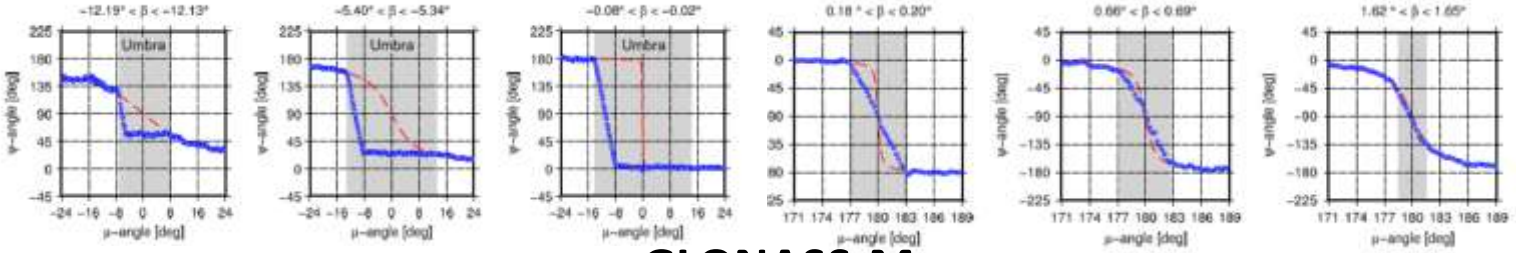
GPS Block IIR

J.Kouba, A simplified yaw-attitude model for eclipsing GPS satellites, GPS Solutions, 2009



GPS Block IIF

F. Dilssner, GPS IIF-1 satellite antenna phase center and attitude modeling, Inside GNSS, 2010



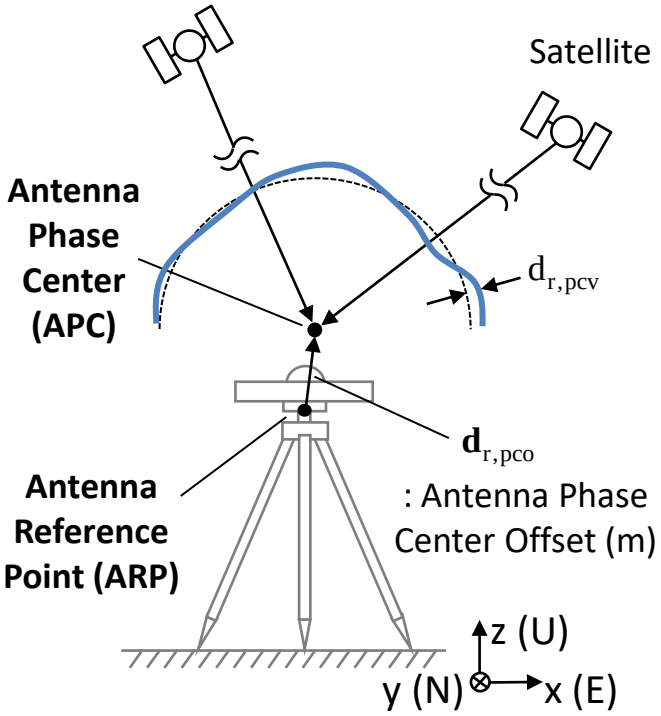
GLONASS-M

F. Dilssner, The GLONASS-M satellite yaw-attitude model, Advances in Space Research, 2010

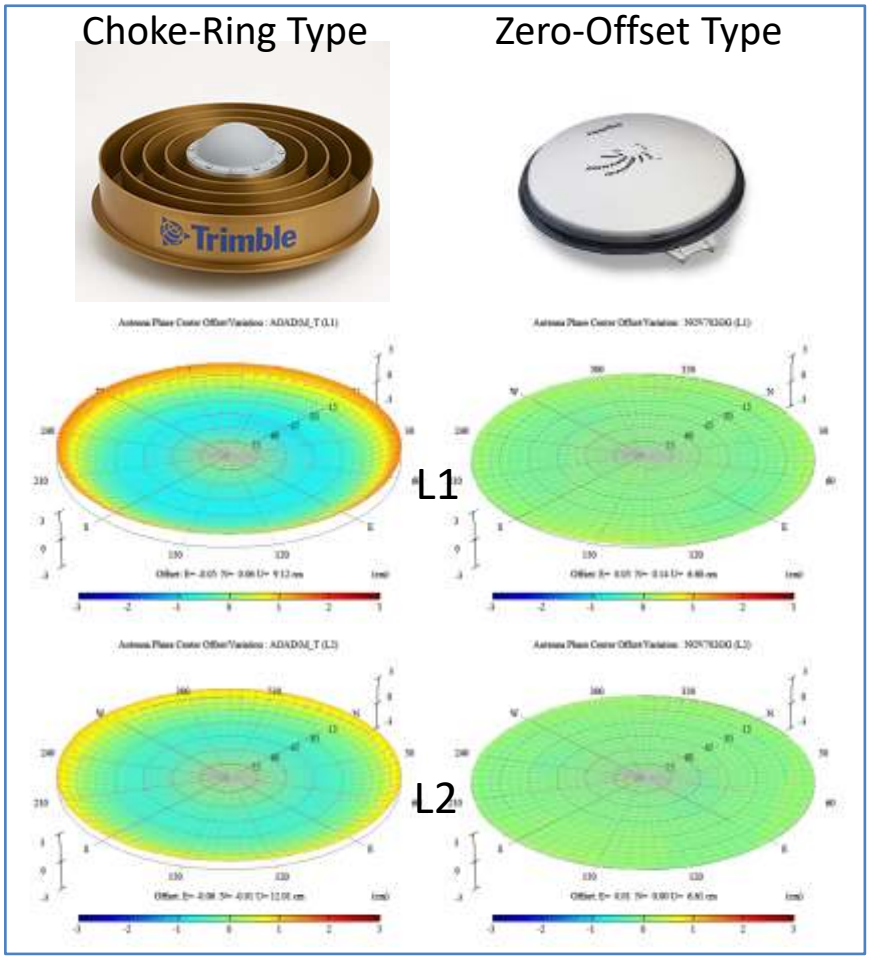
Antenna Phase Center 1

Receiver Antenna Phase Center Corrections

$$d_{r,ant} = -\mathbf{d}_{r,pco}{}^{T}\mathbf{e}^{s}_{r,enu} + d_{r,pcv}(El, Az) \qquad (m)$$



$d_{r,pcv}$: Antenna Phase Center Variation (PCV)



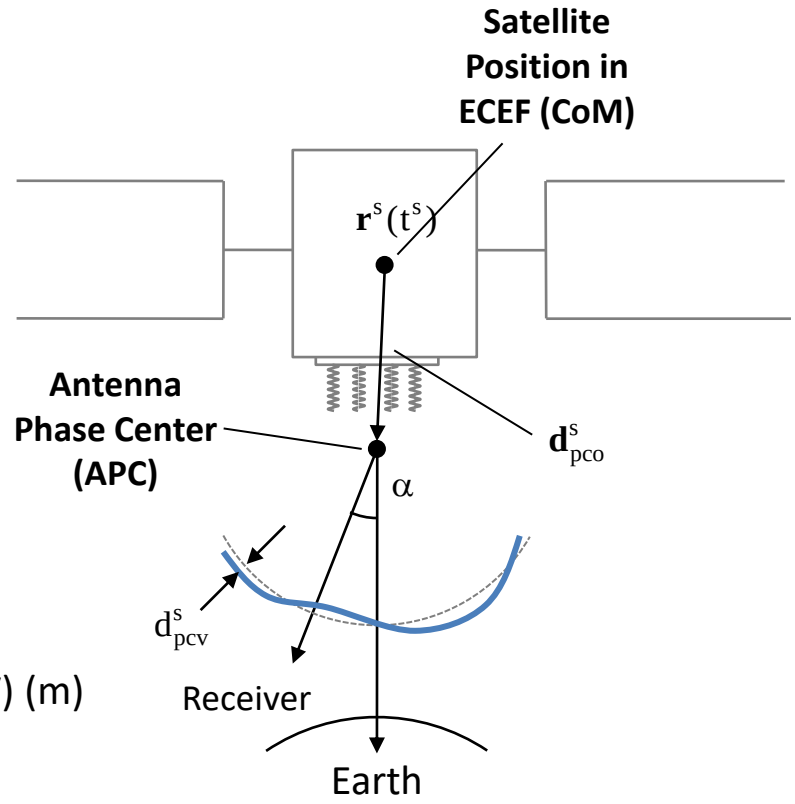
IGS Absolute Antenna Model (IGS05.PCV)

Antenna Phase Center 2

Satellite Antenna Phase Center Corrections

$$d^s_{ant} = \left(\mathbf{E}^{sT}\mathbf{d}^s_{pco}\right)^T \mathbf{e}^s_r + d^s_{pcv}(\alpha)$$

$$\alpha = \arccos\left(\frac{\mathbf{e}_r^{sT}\mathbf{r}^s(t^s)}{\left\|\mathbf{r}^s(t^s)\right\|}\right)$$



$\mathbf{d}^s_{pco}$: Antenna Phase Center Offset (m)

d^s_{pcv} : Antenna Phase Center Variation (PCV) (m)

α : Off-nadir Angle (rad)

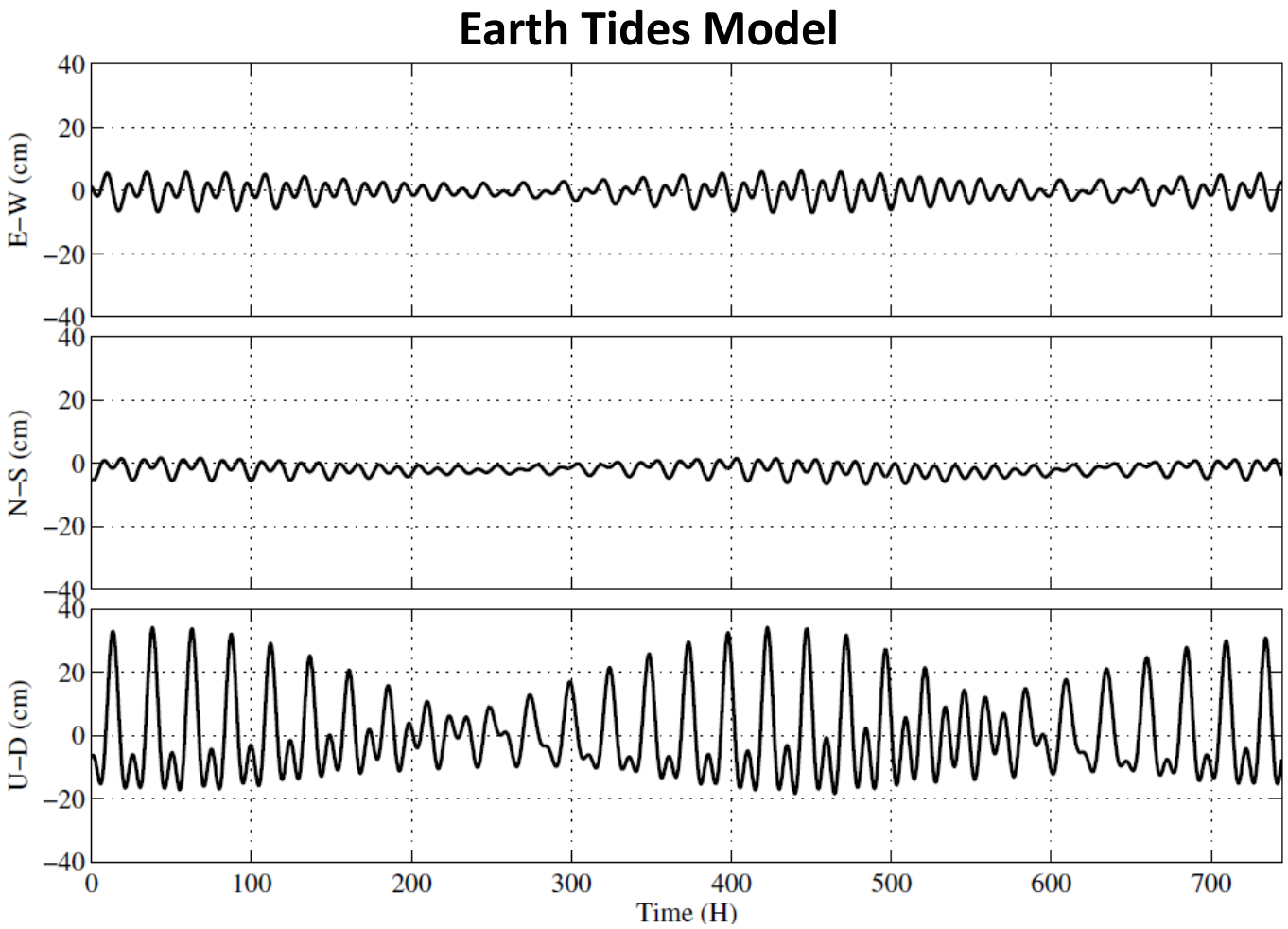
Site Displacement

$$d\mathbf{r}_{tide} = \mathbf{E}_r^{\,T}\left(d\mathbf{r}_{solid}(t_r,\mathbf{r}_r) + d\mathbf{r}_{otl}(t_r,\mathbf{r}_r) + d\mathbf{r}_{pole}(t_r,\mathbf{r}_r) + d\mathbf{r}_{atmos}(t_r,\mathbf{r}_r)\right) \qquad \text{(m)}$$

- **Displacement of Ground-Fixed Receiver**
  - Solid Earth Tide $d\mathbf{r}_{solid}(t_r,\mathbf{r}_r)$
  - Ocean Tide Loading (OTL) $d\mathbf{r}_{otl}(t_r,\mathbf{r}_r)$
  - Pole Tide $d\mathbf{r}_{pole}(t_r,\mathbf{r}_r)$
  - Atmospheric Loading $d\mathbf{r}_{atmos}(t_r,\mathbf{r}_r)$
- **Tide Models**
  - IERS Conventions 1996/2003/2010
  - Ocean Loading: Schwiderski, GOT99.2/00.2, CSR 3.0/4.0, FES99/2004, NAO99.b
  - $M_2,S_2,N_2,K_2,K_1,O_1,P_1,Q_1,M_1,M_m,S_{sa}$ (11 constituents)
- **IERS Subroutines**
  - DEHANTTIDEINEL.F

http://iers-conventions.obspm.fr/content/chapter7/software/dehanttideinel/DEHANTTIDEINEL.F

Earth Tides



IERS Conventions 1996 + NAO99.b, 2007/1/1-1/31, TSKB

Phase Wind-up

- Relative rotation between satellite and receiver antennas effect to the measured phase of RHCP signal.

$$d_{pw} = \mathrm{sign}\left(\mathbf{e}_r^{sT}(\mathbf{D}^s \times \mathbf{D}_r)\right)\arccos\left(\frac{\mathbf{D}^s \cdot \mathbf{D}_r}{\|\mathbf{D}^s\|\|\mathbf{D}_r\|}\right)/2\pi + N \qquad \text{(cyc)}$$

$$\mathbf{E}^s = (\mathbf{e}_x^{sT}, \mathbf{e}_y^{sT}, \mathbf{e}_z^{sT})^T$$

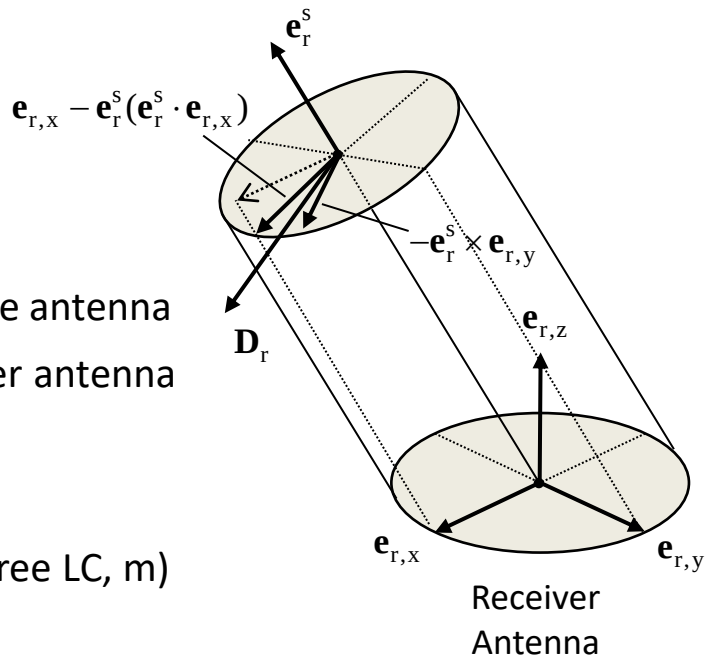
$$\mathbf{E}_r = (\mathbf{e}_{r,x}^{T}, \mathbf{e}_{r,y}^{T}, \mathbf{e}_{r,z}^{T})^T$$

$\mathbf{D}^s = \mathbf{e}_x^s - \mathbf{e}_r^s(\mathbf{e}_r^s \cdot \mathbf{e}_x^s) + \mathbf{e}_r^s \times \mathbf{e}_y^s$: Dipole vector of satellite antenna

$\mathbf{D}_r = \mathbf{e}_{r,x} - \mathbf{e}_r^s(\mathbf{e}_r^s \cdot \mathbf{e}_{r,x}) - \mathbf{e}_r^s \times \mathbf{e}_{r,y}$: Dipole vector of receiver antenna

N : Integer ambiguity

$$\delta_{pw,LC} = C_1\lambda_1 d_{pw} + C_2\lambda_2 d_{pw} = \frac{c}{f_1 + f_2} d_{pw} = \lambda_{NL} d_{pw} \qquad \text{(Iono-free LC, m)}$$



J. T. Wu et al., Effects of antenna orientation on GPS carrier phase, Manuscripta Geodaetica, 1993

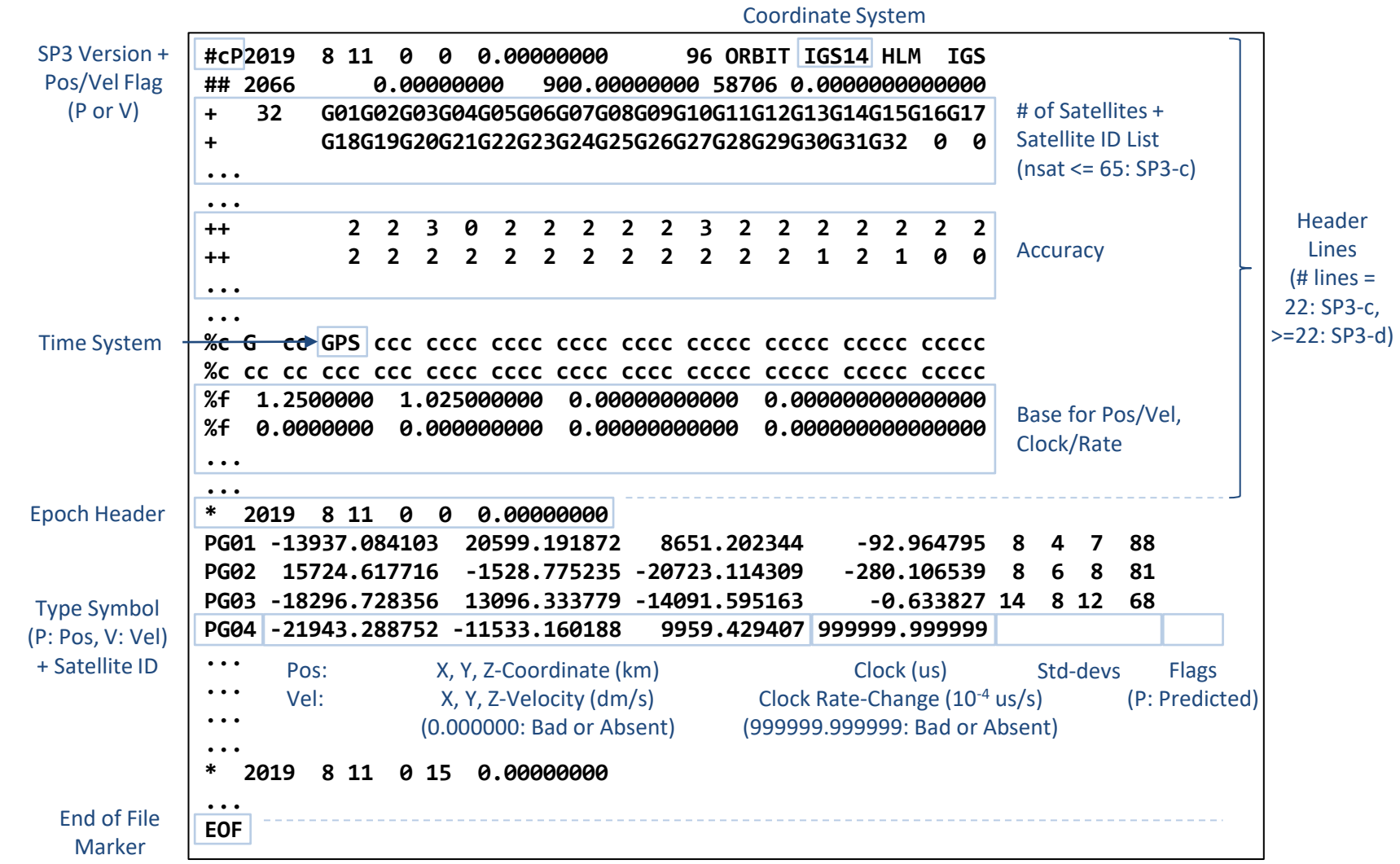
IGS Precise Ephemeris

IGS Products (Satellite Orbit, Clock and EOP) [1]

Product			Final (IGS)	Rapid (IGR)	Ultra-Rapid (IGU)		Broadcast
					Observed	Predicted	
Accuracy	Orbit	GPS	~ 2.5 cm	~ 2.5 cm	~ 3 cm	~ 5 cm	~ 100 cm
		GLONASS	~ 3 cm	-	-	-	-
	Clock (GPS)	RMS	~ 75 ps	~ 75 ps	~ 150 ps	~ 3 ns	~ 5 ns
		STD	~ 20 ps	~ 25 ps	~ 50 ps	~ 1.5 ns	~ 2.5 ns
	EOP	PM	~ 30 uas	~ 50 uas	~ 50 uas	~ 300 uas	-
		PM Rate	~ 150 uas/day	~ 250 uas/day	~ 250 uas/day	~ 300 uas/day	-
		LOD	~ 10 us	~ 10 us	~ 10 us	~ 50 us	-
Sample Interval	Orbit		15 min	15 min	15 min		-
	Clock	Satellite	30 s	5 min	15 min		-
		Station	5 min	-	-		-
	EOP		1 day	1 day	1 day		-
Update Interval			1 week (Thursday)	1 day (17:00 UTC)	6 h (3:00, 9:00, 15:00, 21:00 UTC)		-
Latency			12 ~ 18 days	17 ~ 41 h	3 ~ 9 h	Realtime	Realtime
Format			SP3-c, RINEX Clock, IGS ERP ver.2	SP3-c, RINEX Clock, IGS ERP ver.2	SP3-c, IGS ERP ver.2		RINEX 2 (NAV)
Combination of IGS ACs			COD, EMR, ESA, GFZ, GRG, JPL, MIT, NGS, SIO	COD, EMR, ESA, GFZ, JPL, NGS, SIO, USN, WHU	COD, EMR, ESA, GFZ, NGS, SIO, USN		-

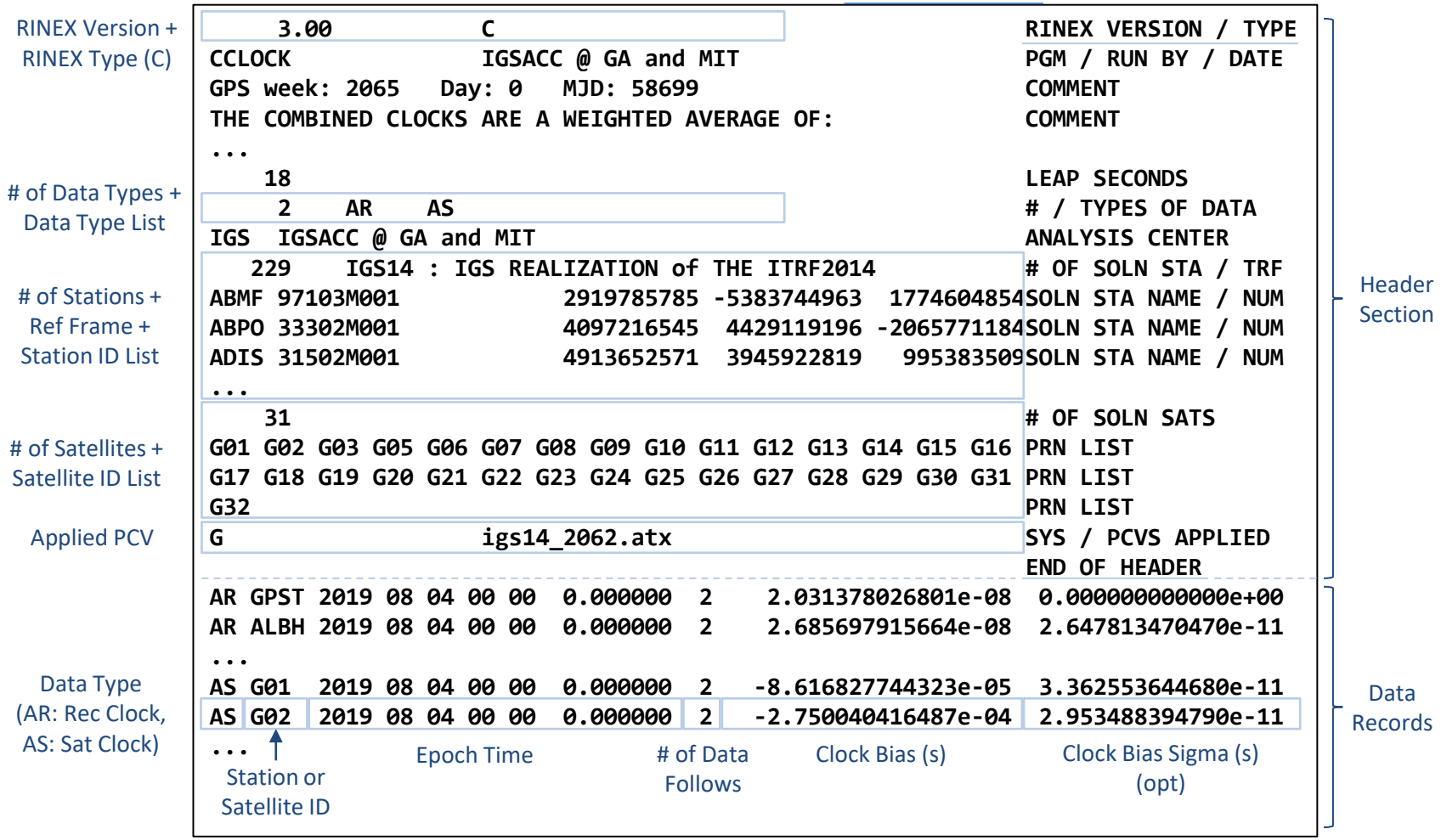
[1] http://www.igs.org/products

SP3



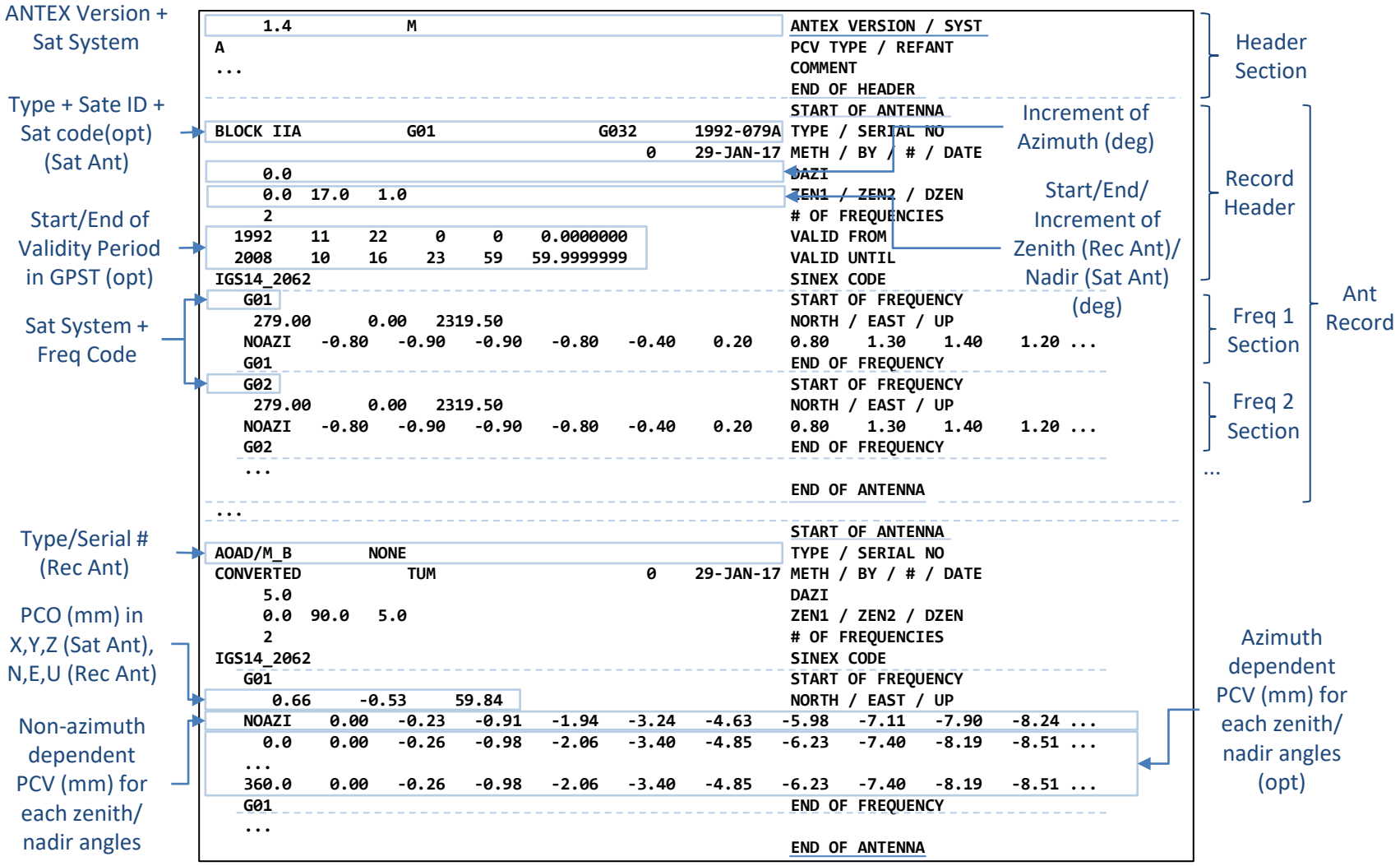
[1] S. Hilla, The extended standard product 3 orbit format (SP3-d), Feb 2016

RINEX Clock



[1] J. Ray and W. Gurtner, RINEX extensions to handle clock information, version 3.02, Sep 2010

ANTEX



[1] M. Rothacher, R. Schmid and M. Chen, ANTEX: The antenna exchange format, version 1.4, Sep 2010

SSR Orbit and Clock Corrections

SSR Orbit and Clock Corrections [1]

$$(t_0, \mathrm{IOD}, \delta O_{radial}, \delta O_{along}, \delta O_{cross}, \dot{\delta O}_{radial}, \dot{\delta O}_{along}, \dot{\delta O}_{cross}, C_0, C_1, C_2, C_{HR})$$

Satellite Position and Clock Bias with SSR Corrections [1]

$$t_O = t_{O0} + \mathrm{UDI}_O / 2$$

$$\delta \mathbf{O} = \begin{pmatrix} \delta O_{radial} \\ \delta O_{along} \\ \delta O_{cross} \end{pmatrix} + \begin{pmatrix} \dot{\delta O}_{radial} \\ \dot{\delta O}_{along} \\ \dot{\delta O}_{cross} \end{pmatrix} (t - t_O)$$

$$\mathbf{r}^s(t) = \mathbf{r}^s_{brdc}(t, \mathrm{IOD}) + \left(\mathbf{e}_{radial}, \mathbf{e}_{along}, \mathbf{e}_{cross}\right)\delta \mathbf{O}$$

$$\mathbf{v}^s_{brdc}(t) = \frac{\mathbf{r}^s_{brdc}(t + \Delta t, \mathrm{IOD}) - \mathbf{r}^s_{brdc}(t, \mathrm{IOD})}{\Delta t}$$

$$\mathbf{e}_{along} = \frac{\mathbf{v}^s_{brdc}(t)}{\left|\mathbf{v}^s_{brdc}(t)\right|}, \quad \mathbf{e}_{cross} = \frac{\mathbf{r}^s_{brdc}(t, \mathrm{IOD}) \times \mathbf{v}^s_{brdc}(t)}{\left|\mathbf{r}^s_{brdc}(t, \mathrm{IOD}) \times \mathbf{v}^s_{brdc}(t)\right|}$$

$$\mathbf{e}_{radial} = \mathbf{e}_{along} \times \mathbf{e}_{cross}$$

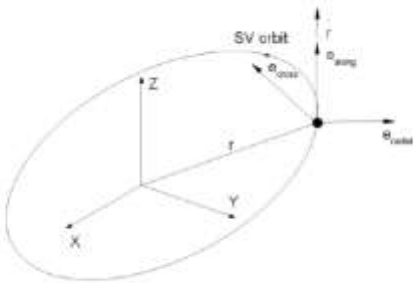
$$t_C = t_{C0} + \mathrm{UDI}_C / 2$$

$$\delta C = C_0 + C_1(t - t_C) + C_2(t - t_C)^2 + C_{HR}$$

$$dT^s(t) = dT^r_{brdc}(t, \mathrm{IOD}) + \frac{\delta C}{c}$$

t_{O0}, t_{C0} : Epoch time of orbit and clock corrections (s)
$\mathrm{UDI}_O, \mathrm{UDI}_C$: Update interval of orbit and clock corrections (s)
IOD : IOD indicating corresponding broadcast ephemeris
$\delta O_i, \dot{\delta O}_i$, : Satellite position/velocity corrections (m, m/s)
C_0, C_1, C_2 : Satellite clock corrections (m, m/s, m/s^2)
C_{HR} : Satellite high-rate clock correction (m)
$\mathbf{r}^s_{brdc}(t, \mathrm{IOD})$: Satellite position by broadcast ephemeris with IOD (m)
$\mathbf{v}^s_{brdc}(t)$: Satellite velocity by broadcast ephemeris (m/s)
$dT^s_{brdc}(t, \mathrm{IOD})$: Satellite clock bias by broadcast SV clock with IOD (s) *
$\delta \mathbf{O}$: Satellite orbit corrections (m)
δC : Satellite clock bias correction (m)
$\mathbf{r}^s(t)$: Satellite position with SSR corrections
$dT^s(t)$: Satellite clock bias with SSR corrections

* including relativistic correction



[1]

Figure 3.12-1. Radial, along-track and cross-track orbit components

[1] RTCM standard 10403.3, Differential GNSS (global navigation satellite system) services - version 3, Oct 2016 (3.5.13)

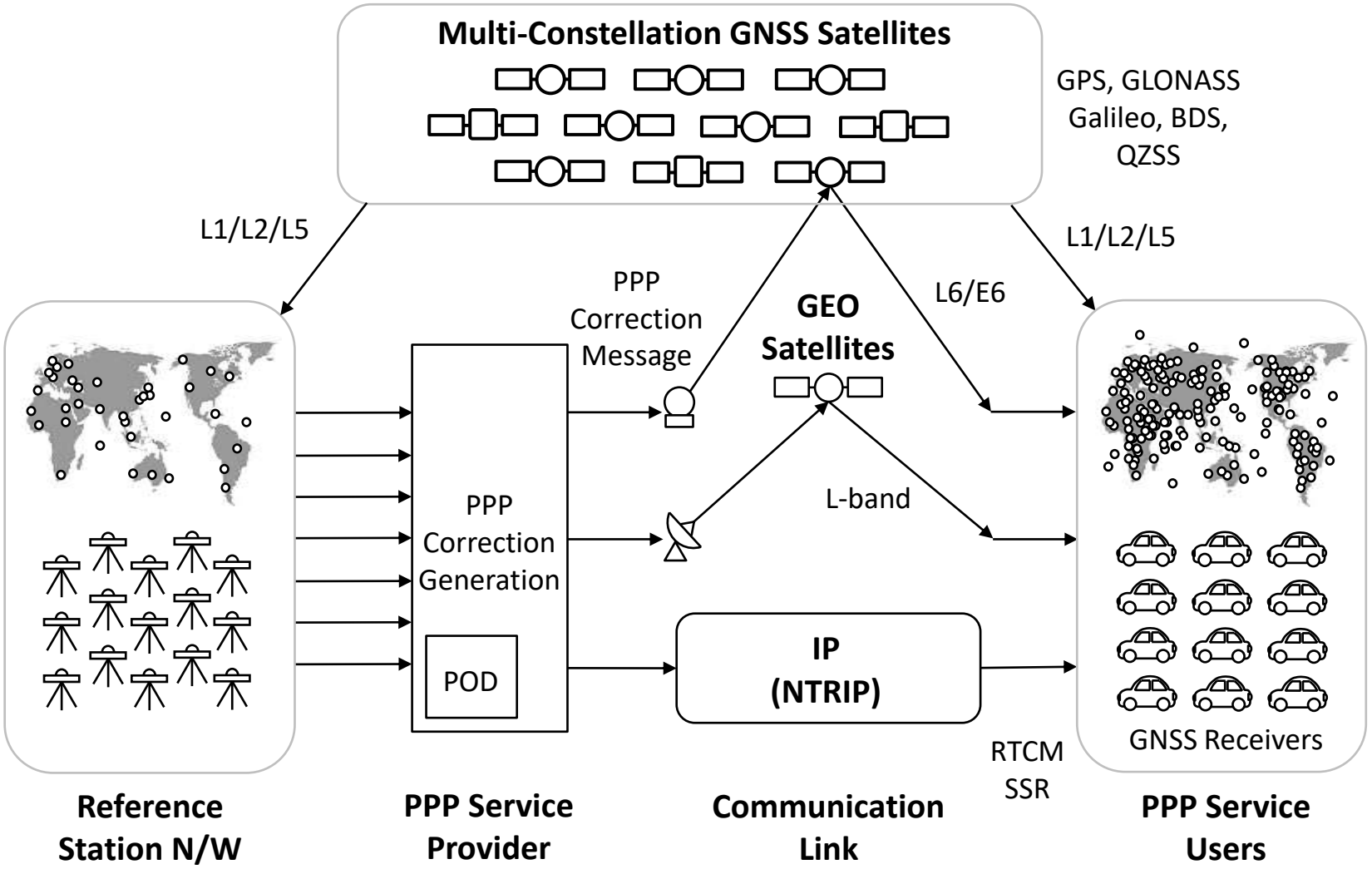
5.
PPP Practice

PPP Practice

(TBD)

6.
Advanced Topics

PPP Service Architecture

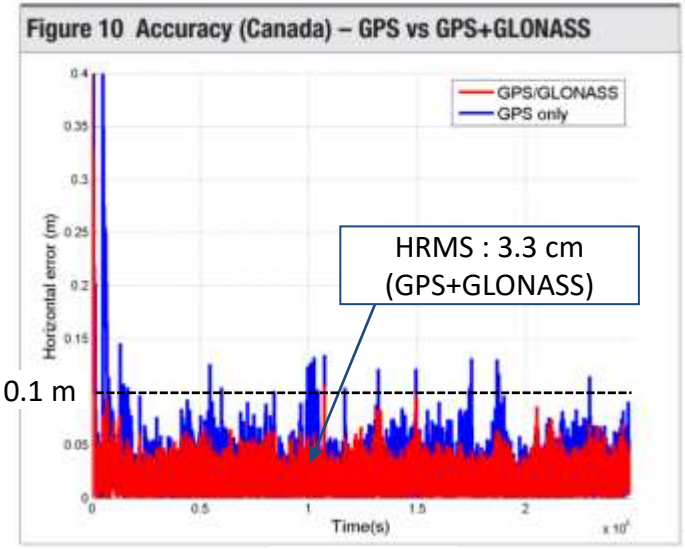


Commercial PPP Services

Service	Provider	Supported GNSS	# of Ref. Stations	Comm. Link	Receivers	Accuracy
StarFire™ [1]	NAVCOM (US)	GPS, GLO	> 40	3 GEO (L-band), IP	NavCom	< 5 cm
Seastar™ [2]	FUGRO (NED)	GPS, GLO, GAL, BDS (G4)	~ 80	6 GEO (L-band), IP (NTRIP)	Fugro	10 cm H 15 cm V (95%)
Apex/Ultra [3] **TerraStar®** [4]	veripos (UK)	GPS, GLO, GAL, BDS, QZS (Apex[5])	~ 80	7 GEO (L-band)	VERIPOS, NovAtel [7], Septentrio [8], TOPCON [9], Hemisphere [10]	< 5 cm H < 12 cm V (95%)
CenterPoint RTX [5]	Trimble (US)	GPS, GLO, GAL, BDS, QZS	~ 100	6 GEO (L-band), IP (NTRIP)	Trimble, Qualcomm (?)	2 cm H 5 cm V (RMS)
magicGNSS [6]	gmv (Spain)	GPS, GLO, GAL, BDS, QZS	~ 80	IP (NTRIP)	(RTCM SSR)	5 cm H 8 cm V (RMS)
GEOFLEX [11]	geoflex (France)	GPS, GLO, (GAL, BDS)	~ 100	GEO, IP, GPRS/UMTS	(RTCM SSR)	4 cm (2D-95%)

[1] https://www.navcomtech.com, [2] https://www.fugro.com, [3] https://veripos.com, [4] https://www.terrastar.net, [5] https://positioningservices.trimble.com, [6] https://magicgnss.gmv.com, [7] https://www.notavel.com, [8] https://www.septentrio.com, [9] https://www.topconpositioning.com, [10] https://www.hemispheregnss.com, [11] http://www.geoflex.fr

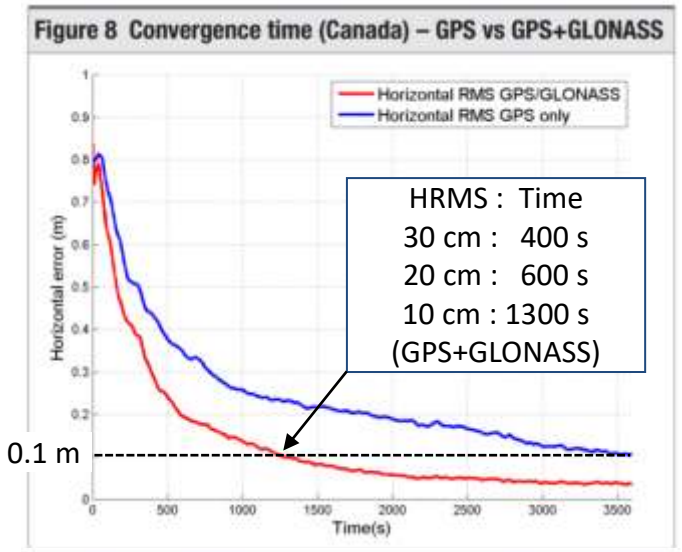
PPP Service Performance

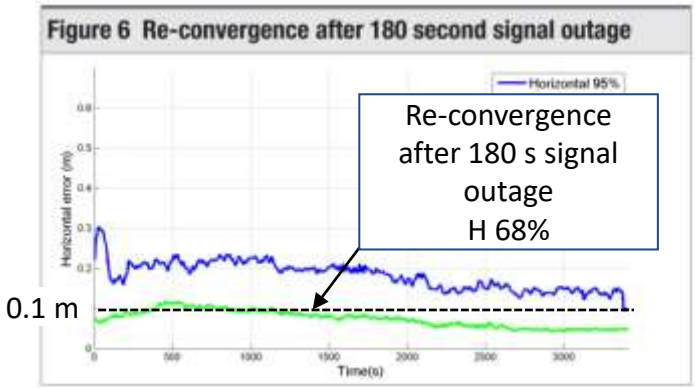


(3 days)

Table 5: Error Statistics at the Canada Station

PPP Correction Source	Horizontal RMS Error (cm)	Vertical RMS Error (cm)
TerraStar-C (GPS/GLONASS)	3.3	4.9
TerraStar-C (GPS only)	4.4	6.5





(NovAtel Correct with TerraStar-C in 2015)

[1] NovAtel White Paper, Precise Positioning with NovAtel CORRECT Including Performance Analysis, April 2015 (https://www.novatel.com/assets/Documents/Papers/NovAtel-CORRECT-PPP.pdf)

PPP for Next-Gen Vehicle Positioning

Qualcomm

Snapdragon Automotive 4G/5G Platform:
supporting
GPS, GLONASS, Galileo, BDS and QZSS (L1, L2, L5)

+

Trimble

RTX Auto:
ASIL and ASPICE compliant RTX precise positioning software library

⇩

Absolute In-lane positioning for ADAS/autonomous-driving

GPS World, Trimble, Qualcomm partner on connected vehicle positioning, September 26, 2019 (https://www.gpsworld.com)

Faster Convergence for PPP-AR

PPP-RTK

- PPP-AR with local STEC and troposphere corrections
- High bandwidth required to support broad coverage larger than nation-wide
- Dense CORS N/W required for local corrections
- Supported by CLAS and some commercial PPP services

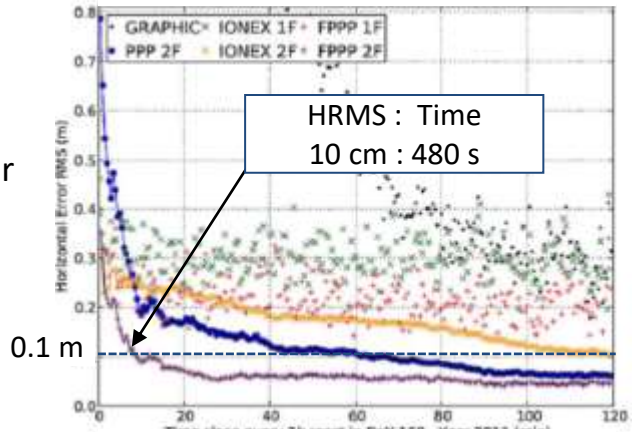
Fast-PPP [1]

- PPP-AR with global VTEC corrections
- Multi-layer model for global VTEC model
- Low bandwidth to transmit corrections for global coverage

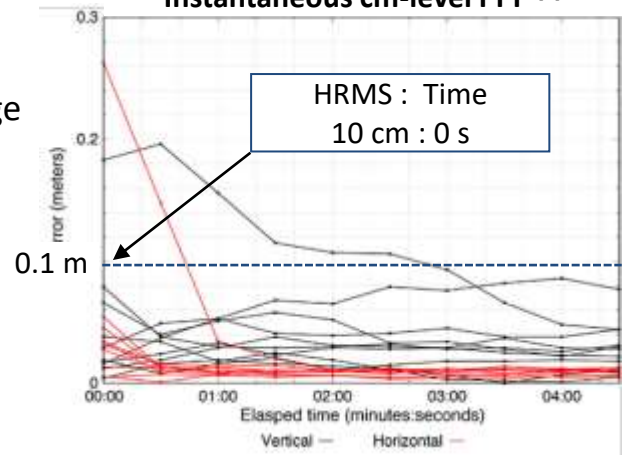
Instantaneous cm-level PPP [2]

- No local or global TEC corrections
- Cascading PPP-AR with triple or quad frequencies (L1-L2-L5, E1-E5a-E5b-E6)

Fast-PPP [1]



Instantaneous cm-level PPP [2]



[1] A. R. Garcia et al., A worldwide ionospheric model for fast precise point positioning, IEEE Transaction on Geoscience and Remote Sensing, 2015

[2] D. Laurichesse and S. Banville, Innovation: Instantaneous centimeter-level multi-frequency precise point positioning, GPS World, 2018

QZSS L6 (1)

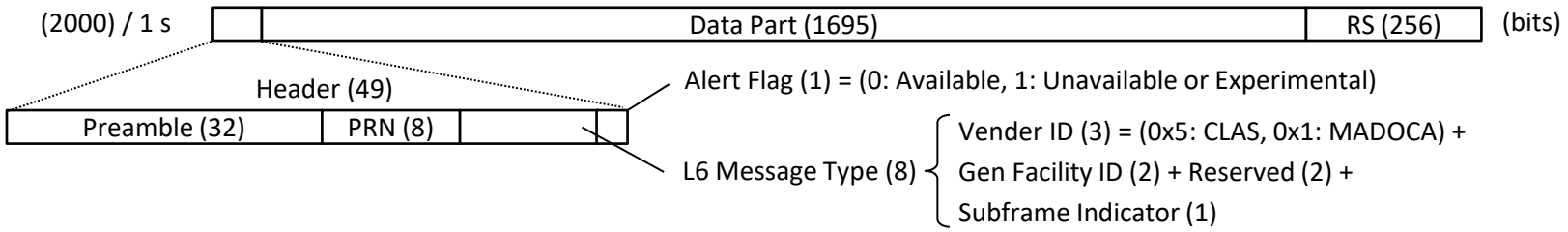
QZSS L6 Signal Specifications

Satellite	QZS-1 (Block I)		QZS-2, 3, 4 (Block II)	
Carrier Frequency	1278.75 MHz		1278.75 MHz	
Modulation	BPSK(5), TDM		BPSK(5), TDM	
Min Received Power	- 155.7 dBW		- 156.82 dBW	
Component	Data	Pilot	Data	Data
PRN number	193	193	194 ~ 197 (QZO) 199 ~ 201 (GEO)	204 ~ 207 (QZO) 209 ~ 211 (GEO)
Chip Rate	2.5575 Mcps	2.5575 Mcps	2.5575 Mcps	2.5575 Mcps
Code Length	4 ms	410 ms	4 ms	4 ms
Data	L6D	-	L6D	L6E
Data Modulation	CSK (8 bit/sym)	-	CSK (8 bit/sym)	CSK (8 bit/sym)
Symbol Rate	250 sym/s		250 sym/s	250 sym/s
Data Rate	2 kbps	-	2 kbps	2 kbps
Frame Length	2000 bits		2000 bits	2000 bits
Frame Rate	1 frame/s	-	1 frame/s	1 frame/s
FEC	RS(255,223)	-	RS(255,223)	RS(255,223)
Contents	CLAS	-	CLAS	MADOCA
Status	Available (2018/11/1~)	-	Available (2018/11/1~)	Experimental (2019/10)

IS-QZSS-L6-001, Quasi-Zenith Satellite System Interface Specification Centimeter Level Augmentation Service, Nov 5, 2018

QZSS L6 (2)

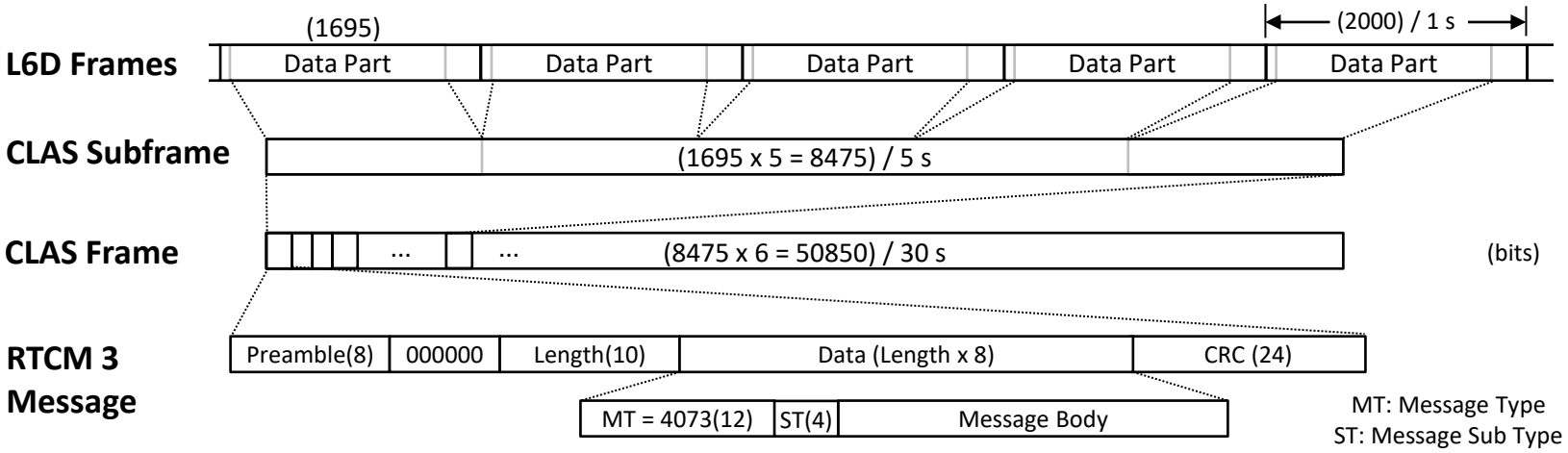
QZSS L6 L6D, L6E Frame



	CLAS	MADOCA
Positioning Mode	PPP-RTK	PPP
Service Area	Japanese Island	All areas covered by QZSS
Accuracy	H: < 6 cm, V: < 12 cm (static, 95%) H: < 12 cm, V: < 24 cm (kinematic, 95%)	Not Specified
TTFF/Convergence Time	< 60 s (95%)	Not Specified
Target GNSS (Signals) *1	GPS (L1C/A, L2P(Y), L2C, L5), GAL (E1, E5a), QZS (L1C/A, L2C, L5)	GPS (L1C/A, L2P(Y)), GLO (L1C/A, L2P), QZS-1 (?)
Corrections	Mask, Orbit Correction, Clock Correction, Code Bias, Phase Bias, URA, STEC Correction, Gridded Correction	Orbit Correction, High-Rate Clock Correction, Code Bias, URA
Format	Compact SSR (CSSR) on RTCM 3 Proprietary (MT 4073)	RTCM 3 SSR + Draft SSR (w/o preamble, length, CRC)

*1 in October 2019

Compact SSR (CSSR)

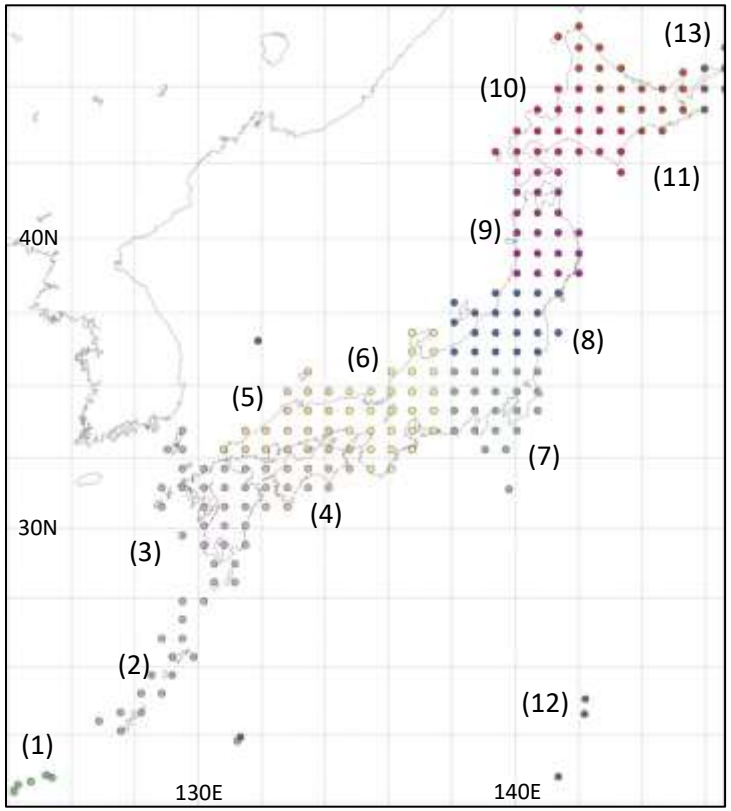


RTCM 3 Message	MT	ST	Contents
Compact SSR Mask	4073	1	Satellite Mask, Cell Mask, Signal Mask
Compact SSR Orbit Corrections	4073	2	IODE, Delta-Orbit (Radial, Along-Track, Cross-Track)
Compact SSR Clock Corrections	4073	3	Delta-Clock C0
Compact SSR Code Bias	4073	4	Code Bias
Compact SSR Phase Bias	4073	5	Phase Bias, Phase Discount. Indicator.
Compact SSR Code and Phase Bias	4073	6	Code Bias, Phase Bias, Phase Discount. Indicator, (N/W ID, N/W SV Mask)
Compact SSR URA	4073	7	URA
Compact SSR STEC Correction	4073	8	STEC Quality Indicator, Polynomial Coefficients, N/W ID, N/W SV Mask
Compact SSR Gridded Correction	4073	9	(Tropos. Hydro-Static Vertical Delay, Tropos. Wet Vertical Delay), STEC Residual Correction, N/W ID, N/W SV Mask, # of Grids
CLAS Service Information	4073	10	TBD
Compact SSR GNSS Combined Correction	4073	11	IODE, Delta-Orbit (Radial, Along-Track, Cross-Track), Delta-Clock C0

[1] IS-QZSS-L6-001 Quasi-Zenith Satellite System Interface Specification Centimeter Level Augmentation Service, November 5, 2018

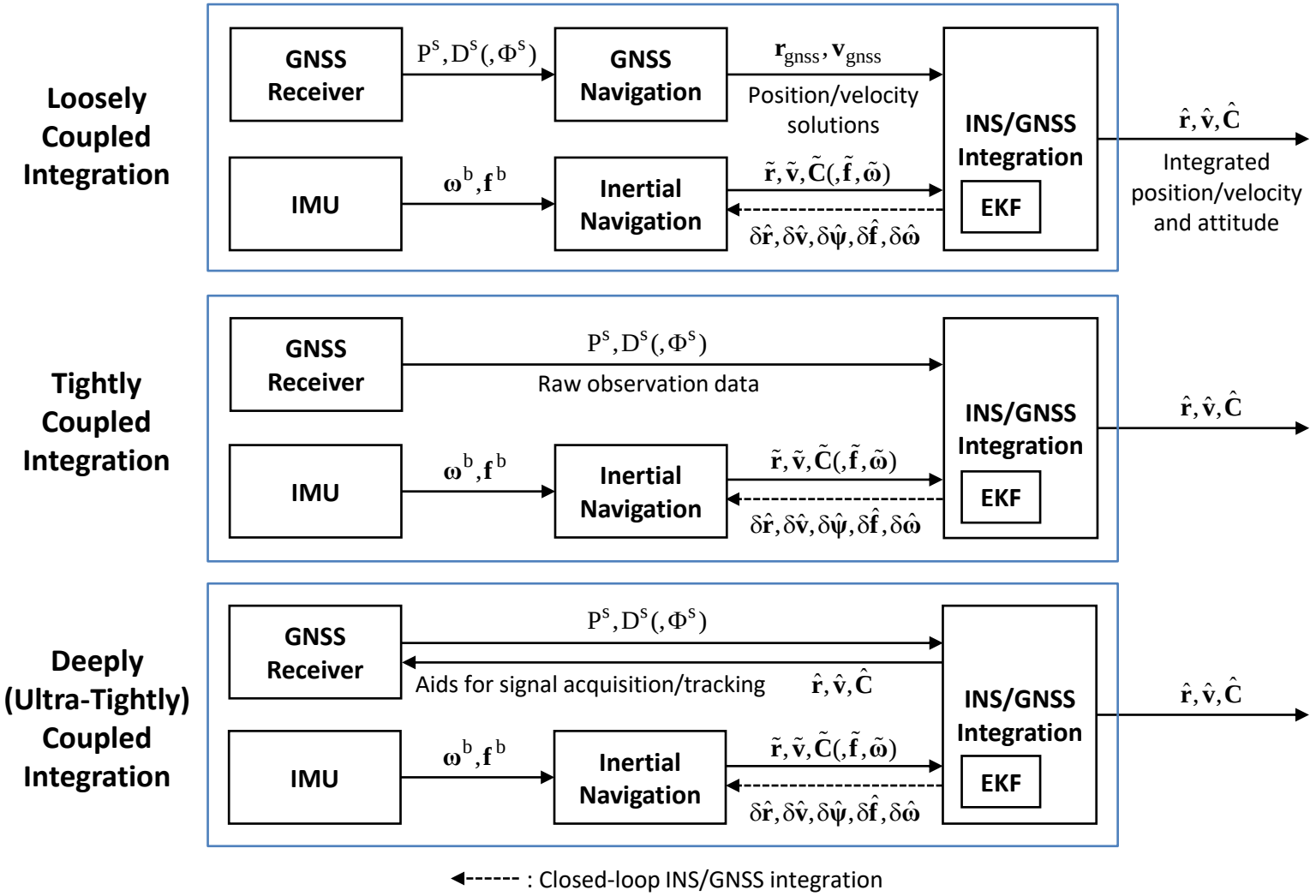
CSSR Networks and Grids

N/W ID	N/W	# of Grids	Area	
			Latitude (deg)	Longitude (deg)
1	Ishigaki	8	24.06 - 24.83 N	122.94 - 125.37 E
2	Okinawa	11	25.83 - 28.30 N	126.87 - 131.23 E
3	Kyusyu	32	28.84 - 34.77 N	128.84 - 131.47 E
4	Shikoku	15	32.62 - 34.23 N	132.13 - 134.76 E
5	Chugoku	15	34.23 - 36.39 N	130.82 - 134.11 E
6	Kansai	27	34.23 - 37.47 N	134.76 - 137.40 E
7	Kanto	22	33.11 - 36.39 N	138.05 - 140.69 E
8	Tohoku South	20	36.93 - 38.55 N	138.05 - 141.34 E
9	Tohoku North	18	39.09 - 41.24 N	140.03 - 142.00 E
10	Hokkaido West	23	41.78 - 43.94 N	139.37 - 143.32 E
11	Hokkaido East	19	42.86 - 45.55 N	141.34 - 145.29 E
12	Ogasawara	2	26.64 - 27.07 N	142.16 - 142.20 E
13	Hokkaido Island	13	43.40 - 45.55 N	145.95 - 149.24 E
14	Island	1	37.24 N	131.87 E
15	Island	1	25.96 N	131.31 E
16	Island	1	25.73 N	123.54 E
17	Island	1	24.77 N	141.34 E
18	Island	1	24.28 N	153.99 E
19	Island	1	20.44 N	136.09 E

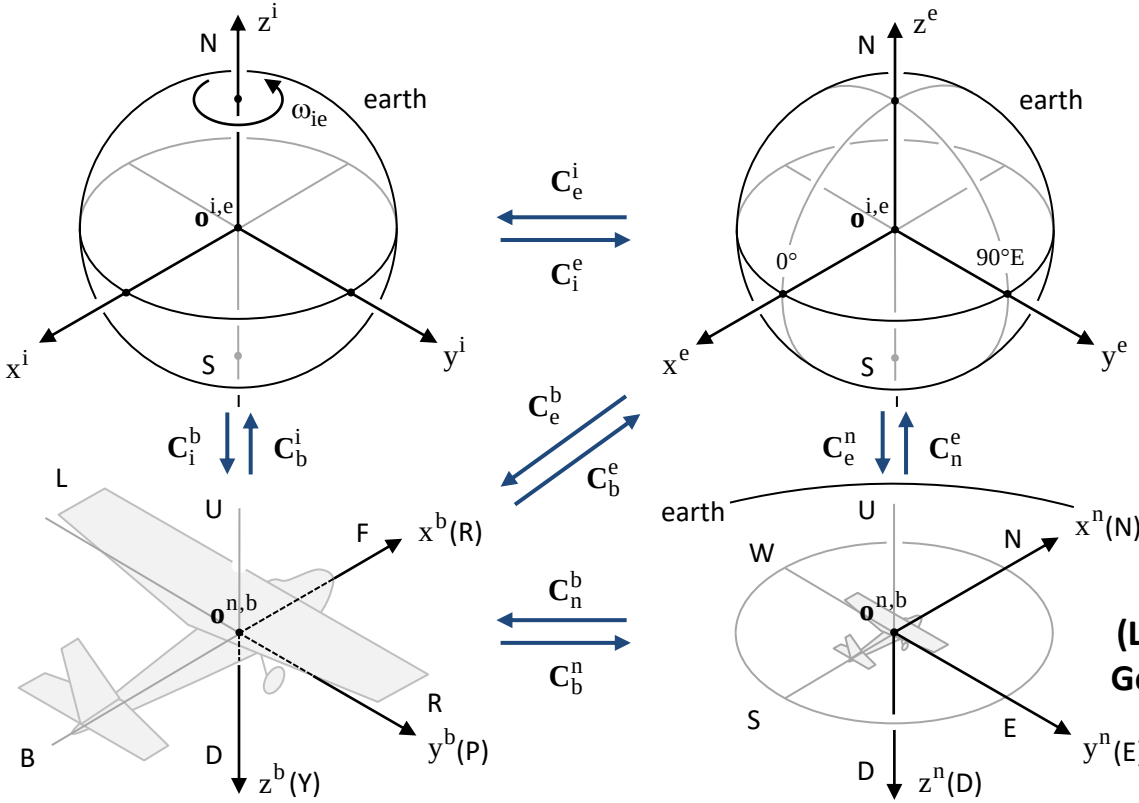


[1] IS-QZSS-L6-001 Quasi-Zenith Satellite System Interface Specification Centimeter Level Augmentation Service, November 5, 2018

INS/GNSS Integration Architecture



Coordinate Frames



Frame Transformations

$$\mathbf{C}_i^e = \mathbf{R}_z(\omega_{ie}(t-t_0)), \mathbf{C}_e^n = \mathbf{R}_y(-\pi/2 - L_{en})\mathbf{R}_z(\lambda_{en}), \mathbf{C}_n^b = \mathbf{R}_x(\phi_{nb})\mathbf{R}_y(\theta_{nb})\mathbf{R}_z(\psi_{nb}),$$

$$\mathbf{C}_e^b = \mathbf{C}_n^b\mathbf{C}_e^n,\ \mathbf{C}_i^b = \mathbf{C}_e^b\mathbf{C}_i^e,\ \mathbf{C}_e^i = \mathbf{C}_i^{eT},\ \mathbf{C}_n^e = \mathbf{C}_e^{nT},\ \mathbf{C}_b^n = \mathbf{C}_n^{bT},\ \mathbf{C}_b^e = \mathbf{C}_e^{bT},\ \mathbf{C}_b^i = \mathbf{C}_i^{bT}$$

$\mathbf{C}_\beta^\alpha$: Transformation from β-frame to α-frame
$\mathbf{o}^{i,e}$: i- and e-frame origin (earth mass center)
$\mathbf{o}^{n,b}$: n- and b-frame origin

ω_{ie} : Earth rotation rate (rad/s)
L_{en} : Latitude of n- and b-frame origin (rad)
λ_{en} : Longitude of n- and b-frame origin (rad)

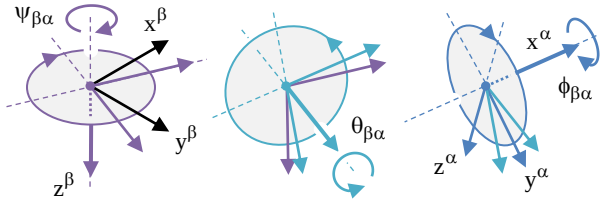
ϕ_{nb} : Roll angle of b- wrt n-frame (rad)
θ_{nb} : Pitch angle of b- wrt n-frame (rad)
ψ_{nb} : Yaw angle of b- wrt n-frame (rad)

Attitudes and Rotations

(1) Euler Angles/Attitude

$$\boldsymbol{\psi}_{\beta\alpha} = (\phi_{\beta\alpha}, \theta_{\beta\alpha}, \psi_{\beta\alpha})^{T}$$

$\phi_{\beta\alpha}, \theta_{\beta\alpha}, \psi_{\beta\alpha}$: Roll, Pitch and Yaw angles of α-frame wrt β-frame



(1) to (2) : $\mathbf{C}_{\beta}^{\alpha} = \mathbf{R}_{x}(\phi_{\beta\alpha})\mathbf{R}_{y}(\theta_{\beta\alpha})\mathbf{R}_{z}(\psi_{\beta\alpha})$

(1) to (2)

(2) to (1)

(2) to (3)

(4) to (2)

(2) DCM/CTM

$$\mathbf{C}_{\beta}^{\alpha} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

$\mathbf{C}_{\beta}^{\alpha}$: Transformation from β-frame to α-frame

(2) to (1) : $\boldsymbol{\psi}_{\beta\alpha} = \begin{pmatrix} \mathrm{ATAN2}(c_{23}, c_{33}) \\ -\arcsin(c_{13}) \\ \mathrm{ATAN2}(c_{12}, c_{11}) \end{pmatrix}$

$$\mu_{\beta\alpha} = \arccos(1/2(c_{11} + c_{22} + c_{33} - 1))$$

(2) to (3) : $\mathbf{e}_{\beta\alpha}^{\alpha/\beta} = \dfrac{1}{2\sin\mu_{\beta\alpha}} \begin{pmatrix} c_{23} - c_{32} \\ c_{31} - c_{13} \\ c_{12} - c_{21} \end{pmatrix}$

(3) Rotation Vector

$$\boldsymbol{\rho}_{\beta\alpha}^{\alpha/\beta} = \mu_{\beta\alpha}\mathbf{e}_{\beta\alpha}^{\alpha/\beta}$$

$\mu_{\beta\alpha}$: Rotation angle of α-frame wrt β-frame

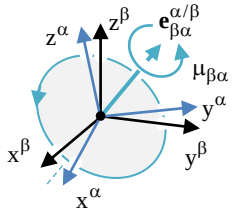
$\mathbf{e}_{\beta\alpha}^{\alpha/\beta}$: Unit vector of rotation axis

(3) to (4)

(4) Quaternion

$$\mathbf{q}_{\beta}^{\alpha} = (q_0, q_1, q_2, q_3)^{T}$$

$$(q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1)$$



(3) to (4) :

$$\mathbf{q}_{\beta}^{\alpha} = \begin{pmatrix} \cos(\mu_{\beta\alpha}/2) \\ \mathbf{e}_{\beta\alpha}^{\alpha/\beta}\sin(\mu_{\beta\alpha}/2) \end{pmatrix}$$

Note:
The $\mathbf{e}_{\beta\alpha}^{\alpha/\beta}$ is one of the eigenvectors of the $\mathbf{C}_{\beta}^{\alpha}$

(4) to (2) :

$$\mathbf{C}_{\beta}^{\alpha} = \begin{pmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_3q_0) & 2(q_1q_3 - q_2q_0) \\ 2(q_1q_2 - q_3q_0) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_1q_0) \\ 2(q_1q_3 + q_2q_0) & 2(q_2q_3 - q_1q_0) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{pmatrix}$$

$\mathbf{x}_{\beta\alpha}^{\gamma}$: **x** of α-frame wrt β-frame expressed in γ-frame axes

$$\mathbf{R}_{x}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \mathbf{R}_{y}(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \mathbf{R}_{z}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \boldsymbol{\Omega} = (\boldsymbol{\omega}\times) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$$

: Skew-symmetric matrix of angular rate vector $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)^{T}$

IMU (Inertial Measurement Unit)

Gyroscopes

Mechanical: RIG (rate integrating gyro), DTG (dynamically tuned gyro), ...

Vibratory: HRG (hemispherical resonator gyro), Wine grass resonator, ...

Optical: RLG (ring laser gyro), FOG (fiber optic gyro), ...

Accelerometers

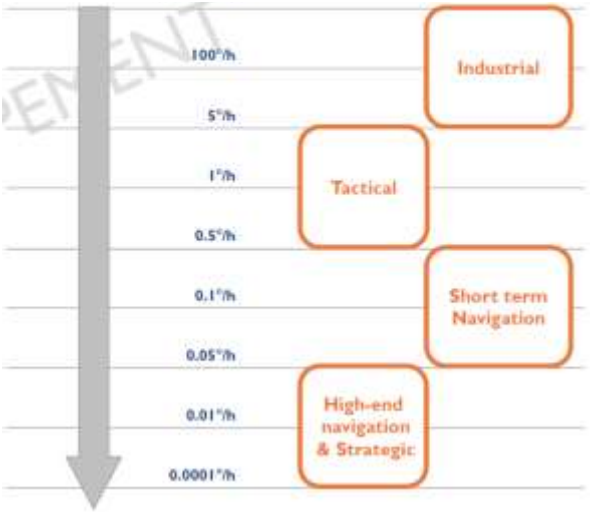
Mechanical: Force-feedback pendulous accelerometer, Vibrating beam, ...

Solid-state: Silicon sensors, Optical accelerometers, ...

Apollo IMU [2]

IMU Vendors [1]

Gyro Bias Stability - IMU Grades [1]

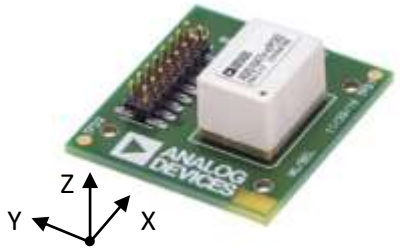


[1] Yole Development, High-end gyroscopes, accelerometers and IMUs for defense, aerospace & industrial, 2015

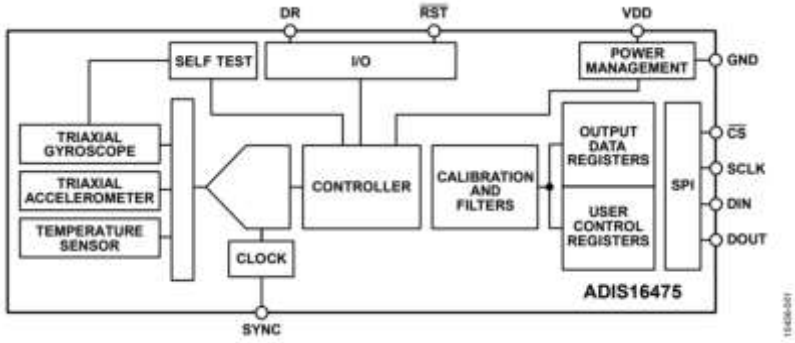
[2] https://en.wikipedia.org/wiki/Inertial_measurement_unit

MEMS IMU

ADI ADIS16475-1 [1]



Triaxial Gyro: +- 125 deg/s
Triaxial Accelerometer: +- 8 g
Size: 11 x 15 x 11 mm
Factory Calibrated Bias, Alignment
Sample Price: $499.87 (Digi-key)



Specifications [2]

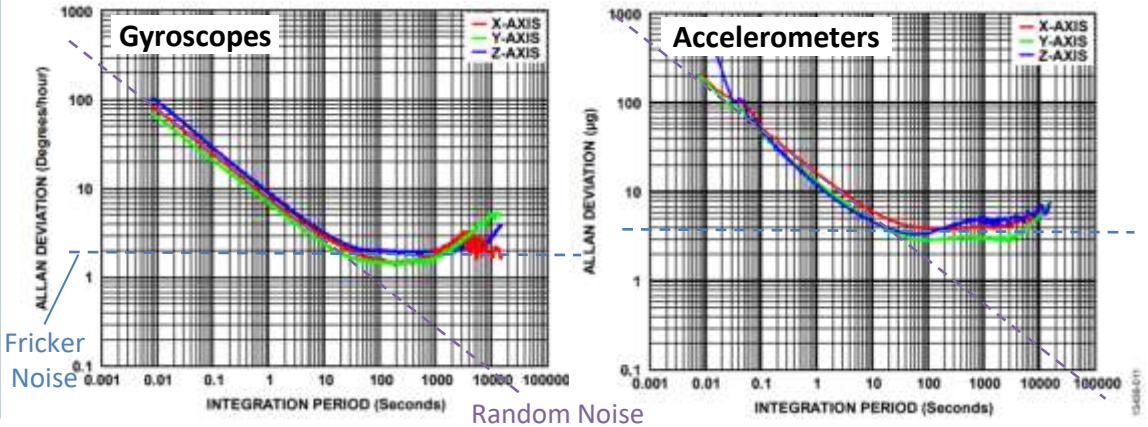
Gyroscopes

Misalignment Error	:	+- 0.1°
Nonlinearity	:	0.2 % FS
In-Run Bias Stability	:	2 °/hr
Angular Random Walk	:	0.15 $°/\sqrt{hr}$

Accelerometers

Misalignment Error	:	+- 0.05°
Nonlinearity (+- 2g)	:	0.25 % FS
In-Run Bias Stability	:	3.6 μg
Velocity Random Walk	:	0.012 $m/s/\sqrt{hr}$

Gyro/Accelerometer Bias Stability [2]



Expected Position Error due to IMU Noise (t = 60 s, v = 60 km/h)

Cross-Track: $(2\times\pi/180/3600\times 60/2)\times(60\times 10^{3}/3600\times 60)=0.291\ (\mathrm{m})$
(gyro stability (rad/s) × time (s) × distance (m))

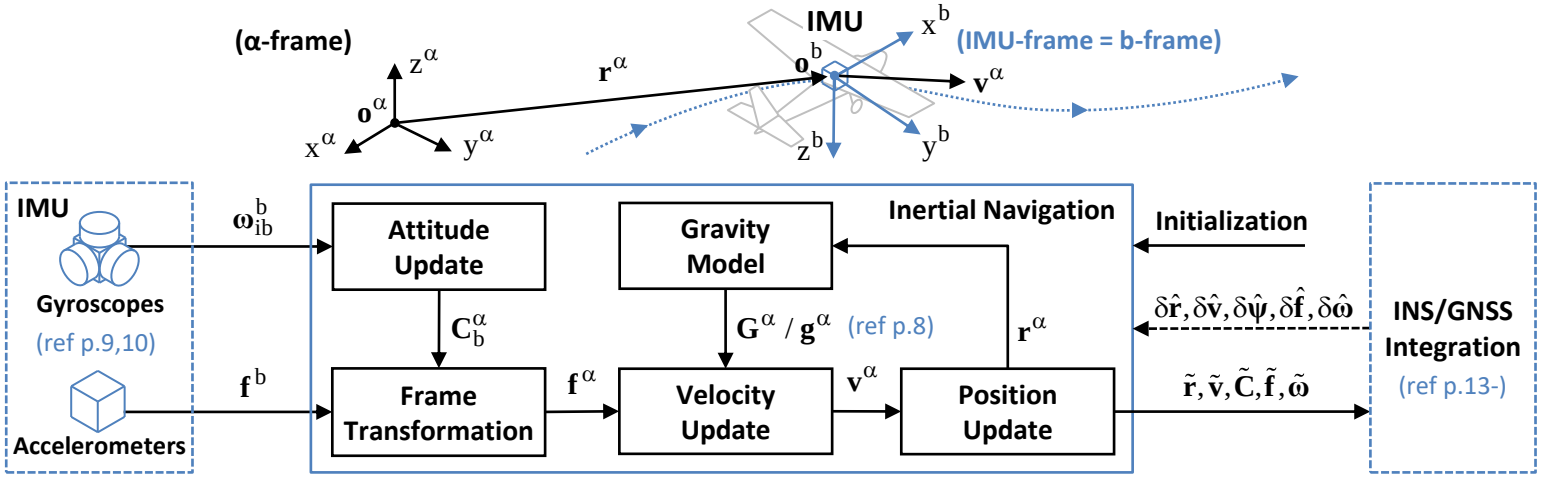
Along-Track: $(3.6\times 10^{-6}\times 9.8)\times 60^{2}/2=0.064\ (\mathrm{m})$
(accelerometer stability (m/s^2) × time (s))

[1] https://www.analog.com/en/products/adis16475.html

[2] Analog Devices, Precision, Miniature MEMs IMU ADIS16475 Data Sheet, Rev.C, 2019

Inertial Navigation

Strapdown Inertial Navigation



Attitude, velocity and position update in i-frame

$$\mathbf{C}_b^i(t+\tau) = \mathbf{C}_b^i(t)\exp\left(((\boldsymbol{\omega}_{ib}^b - \delta\boldsymbol{\omega})\times)\tau\right) \quad \text{(ref p.7)}$$

$$\mathbf{f}^i = \frac{1}{2}\left(\mathbf{C}_b^i(t) + \mathbf{C}_b^i(t+\tau)\right)(\mathbf{f}^b - \delta\mathbf{f})$$

$$\mathbf{v}^i(t+\tau) = \mathbf{v}^i(t) + \left(\mathbf{f}^i + \mathbf{G}^i(\mathbf{r}^i(t))\right)\tau \quad \text{(acceleration)}$$

$$\mathbf{r}^i(t+\tau) = \mathbf{r}^i(t) + \frac{1}{2}\left(\mathbf{v}^i(t) + \mathbf{v}^i(t+\tau)\right)\tau$$

Attitude, velocity and position update in e-frame

$$\mathbf{C}_b^e(t+\tau) = \mathbf{C}_b^e(t)\exp\left(((\boldsymbol{\omega}_{ib}^b - \delta\boldsymbol{\omega} - \mathbf{C}_b^e(t)^T\boldsymbol{\omega}_{ie}^e)\times)\tau\right)$$

(earth rotation)

$$\mathbf{f}^e = \frac{1}{2}\left(\mathbf{C}_b^e(t) + \mathbf{C}_b^e(t+\tau)\right)(\mathbf{f}^b - \delta\mathbf{f})$$

$$\mathbf{v}^e(t+\tau) = \mathbf{v}^e(t) + \left(\mathbf{f}^e + \mathbf{g}^e(\mathbf{r}^e(t)) - 2\boldsymbol{\omega}_{ie}^e \times \mathbf{v}^e(t)\right)\tau$$

(Coriolis force)

$$\mathbf{r}^e(t+\tau) = \mathbf{r}^e(t) + \frac{1}{2}\left(\mathbf{v}^e(t) + \mathbf{v}^e(t+\tau)\right)\tau$$

$\mathbf{r}^\alpha$: Position in α-frame(m)
$\mathbf{v}^\alpha$: Velocity in α-frame (m/s)
$\mathbf{C}_b^\alpha$: Transformation from b- to α-frame
$\boldsymbol{\omega}_{ib}^b$: Angular rate measurement (rad/s)
$\mathbf{f}^b$: Specific force measurement (m/s^2)
$\mathbf{f}^\alpha$: Specific force in α-frame (m/s^2)
$\delta\mathbf{f}$: Accelerometer bias (m/s2)
$\delta\boldsymbol{\omega}$: Gyro bias (rad/s)
$\mathbf{G}^\alpha$: Gravitational acceleration (m/s^2)
$\mathbf{g}^\alpha$: Acceleration due to gravity (m/s^2)
$\boldsymbol{\omega}_{ie}^e$: Earth rotation vector (rad/s)
τ : Update interval (s)

INS/GNSS Integration (1)

State Vector and their Propagation in e-frame

$$\mathbf{x}_{\text{ins}} = (\delta\mathbf{r}^{\mathrm{T}}, \delta\mathbf{v}^{\mathrm{T}}, \delta\boldsymbol{\psi}^{\mathrm{T}}, \delta\mathbf{f}^{\mathrm{T}}, \delta\boldsymbol{\omega}^{\mathrm{T}})^{\mathrm{T}}$$

$$\dot{\mathbf{x}}_{\text{ins}} = \begin{pmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \partial\mathbf{g}^{\mathrm{e}}/\partial\mathbf{r}^{\mathrm{e}} & -2\boldsymbol{\Omega}_{\mathrm{ie}}^{\mathrm{e}} & (-(\mathbf{C}_{\mathrm{b}}^{\mathrm{e}}\mathbf{f}^{\mathrm{b}})\times) & \mathbf{C}_{\mathrm{b}}^{\mathrm{e}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{\Omega}_{\mathrm{ie}}^{\mathrm{e}} & \mathbf{0} & \mathbf{C}_{\mathrm{b}}^{\mathrm{e}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \mathbf{x}_{\text{ins}} + \mathbf{q}_{\text{ins}}$$

$$\mathbf{r}^{\mathrm{e}} = \tilde{\mathbf{r}}^{\mathrm{e}} - \delta\mathbf{r},\ \mathbf{v}^{\mathrm{e}} = \tilde{\mathbf{v}}^{\mathrm{e}} - \delta\mathbf{v}$$

$$\mathbf{f}^{\mathrm{b}} = \tilde{\mathbf{f}}^{\mathrm{b}} - \delta\mathbf{f},\ \boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}} = \tilde{\boldsymbol{\omega}}_{\mathrm{ib}}^{\mathrm{b}} - \delta\boldsymbol{\omega}$$

$$\mathbf{C}_{\mathrm{b}}^{\mathrm{e}} = (\mathbf{I} - (\delta\boldsymbol{\psi}\times))\tilde{\mathbf{C}}_{\mathrm{b}}^{\mathrm{e}}$$

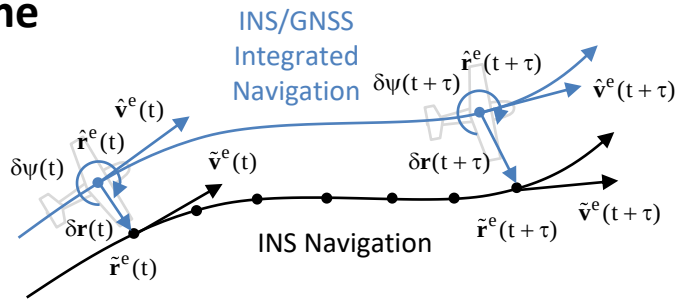
State Transition and Process Noise Matrix

$$\boldsymbol{\Phi}_{\text{ins}} \approx \begin{pmatrix} \mathbf{I} & \mathbf{I}\tau & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \partial\mathbf{g}^{\mathrm{e}}/\partial\mathbf{r}^{\mathrm{e}}\tau & \mathbf{I} - 2\boldsymbol{\Omega}_{\mathrm{ie}}^{\mathrm{e}}\tau & (-(\mathbf{C}_{\mathrm{b}}^{\mathrm{e}}\mathbf{f}^{\mathrm{b}})\times)\tau & \mathbf{C}_{\mathrm{b}}^{\mathrm{e}}\tau & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} - \boldsymbol{\Omega}_{\mathrm{ie}}^{\mathrm{e}}\tau & \mathbf{0} & \mathbf{C}_{\mathrm{b}}^{\mathrm{e}}\tau \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix}$$

(1st-order approx.) *

$$\mathbf{Q}_{\text{ins}} \approx \text{blkdiag}\left(\mathbf{0}, \sigma_{\mathrm{a}}^2\mathbf{I}, \sigma_{\mathrm{g}}^2\mathbf{I}, \sigma_{\mathrm{ab}}^2\mathbf{I}, \sigma_{\mathrm{gb}}^2\mathbf{I}\right)\tau$$

* To minimize the integration error in high-dynamic environment with fast attitude change, high-order approximation or finer step integration is desirable to generate a precise state transition matrix.



$\delta\mathbf{r}$: INS position error in e-frame (m)
$\delta\mathbf{v}$: INS velocity error in e-frame (m/s)
$\delta\boldsymbol{\psi}$: INS attitude error in e-frame (rad)
$\delta\mathbf{f}$: Accelerometer bias (m/s^2)
$\delta\boldsymbol{\omega}$: Gyro bias (rad/s)
$\tilde{\mathbf{r}}^{\mathrm{e}}$: INS position in e-frame (m)
$\tilde{\mathbf{v}}^{\mathrm{e}}$: INS velocity in e-frame (m/s)
$\tilde{\mathbf{C}}_{\mathrm{b}}^{\mathrm{e}}$: INS attitude in e-frame
$\tilde{\boldsymbol{\omega}}_{\mathrm{ib}}^{\mathrm{b}}$: INS angular rate measurement (rad/s)
$\tilde{\mathbf{f}}^{\mathrm{b}}$: INS specific force measurement (m/s^2)
$\mathbf{r}^{\mathrm{e}}$: Corrected position in e-frame (m)
$\mathbf{v}^{\mathrm{e}}$: Corrected velocity in e-frame (m/s)
$\mathbf{C}_{\mathrm{b}}^{\mathrm{e}}$: Corrected attitude in e-frame
$\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}}$: Corrected angular rate (rad/s)
$\mathbf{f}^{\mathrm{b}}$: Corrected specific force (m/s^2)
σ_{a} : Std-dev of accelerometer noise (m/s^2)
σ_{g} : Std-dev of gyro noise (rad/s)
σ_{ab} : Std-dev of accelerometer bias (m/s^2)
σ_{gb} : Std-dev of gyro bias (rad/s)
τ : Update interval of EKF (s)

INS/GNSS Integration (2)

Loosely Coupled Integration in e-frame

$$\mathbf{x} = \mathbf{x}_{ins} = (\delta\mathbf{r}^T, \delta\mathbf{v}^T, \delta\boldsymbol{\psi}^T, \delta\mathbf{f}^T, \delta\boldsymbol{\omega}^T)^T$$

Time Update of EKF

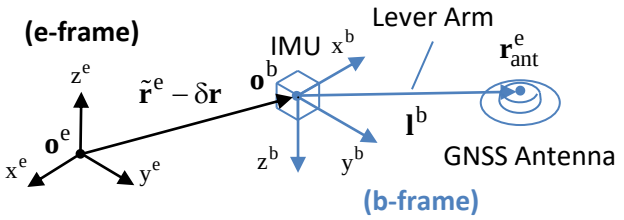
$$\hat{\mathbf{x}}_k^- = \boldsymbol{\Phi}_k \hat{\mathbf{x}}_{k-1}^+, \;\; \mathbf{P}_k^- = \boldsymbol{\Phi}_k \mathbf{P}_{k-1}^+ \boldsymbol{\Phi}_k^T + \mathbf{Q}_k \;\; (\boldsymbol{\Phi}_k = \boldsymbol{\Phi}_{ins}, \;\; \mathbf{Q}_k = \mathbf{Q}_{ins})$$

Measurement Update of EKF

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{y}_k - \mathbf{h}(\hat{\mathbf{x}}_k^-)), \mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-$$

$$\left\{ \begin{array}{l} \mathbf{y}_k = \begin{pmatrix} \mathbf{r}_{gnss} \\ \mathbf{v}_{gnss} \end{pmatrix}, \mathbf{R}_k = \begin{pmatrix} \mathbf{Q_r} & \\ & \mathbf{Q_v} \end{pmatrix} \\ \mathbf{h}(\mathbf{x}) = \begin{pmatrix} \mathbf{r}_{ant}^e \\ \mathbf{v}_{ant}^e \end{pmatrix}, \mathbf{H}_k = \begin{pmatrix} -\mathbf{I} & \mathbf{0} & (\delta\mathbf{r}_{ant}\times) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} & (\delta\mathbf{v}_{ant}\times) & \mathbf{0} & \mathbf{C}_b^e(\mathbf{l}^b\times) \end{pmatrix} \\ \mathbf{C}_b^e = (\mathbf{I} - (\delta\boldsymbol{\psi}\times))\tilde{\mathbf{C}}_b^e \\ \delta\mathbf{r}_{ant} = \mathbf{C}_b^e \mathbf{l}^b \\ \delta\mathbf{v}_{ant} \approx \mathbf{C}_b^e((\tilde{\boldsymbol{\omega}}_{ib}^b - \delta\boldsymbol{\omega}) \times \mathbf{l}^b) \\ \mathbf{r}_{ant}^e = \tilde{\mathbf{r}}^e - \delta\mathbf{r} + \delta\mathbf{r}_{ant} \\ \mathbf{v}_{ant}^e = \tilde{\mathbf{v}}^e - \delta\mathbf{v} + \delta\mathbf{v}_{ant} \end{array} \right.$$



$\tilde{\mathbf{r}}^e$: INS position in e-frame (m)
$\tilde{\mathbf{v}}^e$: INS velocity in e-frame (m/s)
$\tilde{\mathbf{C}}_b^e$: INS attitude in e-frame
$\tilde{\boldsymbol{\omega}}_{ib}^b$: INS angular rate measurement (rad/s)
$\tilde{\mathbf{f}}^b$: INS specific force measurement (m/s²)
$\delta\mathbf{r}$: INS position error in e-frame (m)
$\delta\mathbf{v}$: INS velocity error in e-frame (m/s)
$\delta\boldsymbol{\psi}$: INS attitude error in e-frame (rad)
$\delta\mathbf{f}$: Accelerometer bias (m/s²)
$\delta\boldsymbol{\omega}$: Gyro bias (rad/s)
$\mathbf{l}^b$: Lever-arm vector in b-frame (m)
$\mathbf{r}_{ant}^e$: Antenna position in e-frame (m)
$\mathbf{v}_{ant}^e$: Antenna velocity in e-frame (m/s)
$\mathbf{r}_{gnss}$: GNSS position solution (ECEF) (m)
$\mathbf{v}_{gnss}$: GNSS velocity solution (ECEF) (m/s)
$\mathbf{Q_r}$: VC-matrix of GNSS position solution
$\mathbf{Q_v}$: VC-matrix of GNSS velocity solution

Integrated INS/GNSS Performance

Applanix POS LV [1]

PCS (POS computer system) + IMU +
DMI (Distance measurement instrument) +
GNSS (Trimble BD960 and Zephyr II antenna)
Price range: $100,000 ~ $200,000

INERTIAL MEASUREMENT UNIT (IMU) [2]

Type	Operational Temperature °C	Models Used In	Maximum Data Rate	Dimensions (L x W x H) mm	Weight kg
IIMU-7[1]	-54 to +71	POSLV 420	200 Hz	158 x 158 x 124	2.5
IMU-17[1]	-40 to +60	POSLV 210/220	100 Hz	158 x 158 x 124	2.5
IMU-21[1]	-40 to +60	POSLV 610/620	200 Hz	213 x 172 x 172	4.8
IMU-42[2]	-20 to +55	POSLV 210/220	200 Hz	158 x 158 x 124	2.6
IMU-80[2]	-20 to +55	POSLV 510/520	200 Hz	161 x 120 x 126	1.9
IMU-57[2]	-20 to +55	POSLV 610/620	200 Hz	179 x 126 x 127	2.6
IMU-64[2]	-20 to +55	POSLV 420	200 Hz	158 x 158 x 124	2.6
IMU-82[2]	-40 to +65	POSLV 210/220	200 Hz	158 x 158 x 124	2.3

PERFORMANCE SPECIFICATIONS - GNSS OUTAGE, 60 SECONDS* [2]

POS LV	220 PP	220 IARTK	220 DGPS	420 PP	420 IARTK	420 DGPS	510/520 PP	510/520 IARTK	510/520 DGPS	610/620 PP	610/620 IARTK	610/620 DGPS
X,Y Position (m)	0.240	0.690	0.880	0.120	0.340	0.450	0.100	0.300	0.420	0.100	0.280	0.410
Z Position (m)	0.130	0.350	0.610	0.100	0.270	0.560	0.070	0.100	0.530	0.070	0.100	0.510
Roll & Pitch (deg)	0.060	0.060	0.060	0.020	0.020	0.020	0.005	0.008	0.008	0.005	0.005	0.005
True Heading (deg)	0.030	0.070	0.070	0.020	0.030	0.030	0.015	0.020	0.020	0.015	0.020	0.020

* All accuracy values given as RMS. Assumes typical road vehicle dynamics for initialization.

[1] https://www.applanix.com/products/poslv.htm
[2] Applanix, POS LV - designed for integration, built for performance - Data Sheet, 2017

Appendix

GNSS Signals (1/3)

System	Carrier Freq (MHz)	Signal	I/Q	Min Received Power (dBW)	Modulation	Spreading Code				Navigation Data			Notes
						Length (chips)	Chip Rate (Mcps)	Overlay Code (chips)	Period (ms)	Data	Bit Rate (bps)	FEC	
GPS [1][2]	1575.42	L1C/A	Q	-158.5	BPSK(1)	1023	1.023	-	1	LNAV	50	-	
		L1P(Y)*1	I	-161.5	BPSK(10)	1week	10.23	-	?	LNAV	50	-	
		L1M	I	?	BOC(10,5)	?	5.115	?	?	?	?	?	Block IIR-M~
		L1C-D	I	-163.0	BOC(1,1)	10230	1.023	-	10	CNAV-2	50	1/2	GPS III~
		L1C-P	I	-158.25	TMBOC(6,1,4/33)	10230	1.023	1800	18000	-	-	-	
	1227.6	L2C/A	Q	-164.5	BPSK(1)	1023	1.023	-	1	LNAV	50	-	Block IIR-M~
		L2P(Y)*1	I	-164.5/-161.5	BPSK(10)	1week	10.23	-	?	LNAV	50	-	
		L2M	I	?	BOC(10,5)	?	5.115	?	?	?	?	?	Block IIR-M~
		L2C-M	Q/I	-160.0/-158.5	BPSK(1), TDM	10230	0.5115	-	20	LNAV	50	-	Block IIR-M~
										CNAV	25	1/2	
		L2C-L				767250	0.5115	-	1500	-	-	-	
	1176.45	L5-I	I	-157.9/-157.0	BPSK(10)	10230	10.23	10 (NH)	10	CNAV	50	1/2	Block IIF~
		L5-Q	Q	-157.9/-157.0	BPSK(10)	10230	10.23	20 (NH)	20	-	-	-	
GLONASS [3][4][5][6]	1602.0 + 0.5625K*2	L1C/A	I	-161.0	BPSK	511	0.511	-	1	NAV	50	-	
		L1P	Q	?	BPSK	5110000	5.11	-	1000	?	?	-	
	1600.995	L1OCd	Q	?	BPSK(1), TDM	1023	0.5115	2	4	NAV	125	1/2	GLO-K2~
		L1OCp				4092	2.046	4 (MS)	8	-	-	-	
	1246.0 + 0.4375K*2	L2C/A	I	-167.0	BPSK	511	0.511	-	1	NAV	50	-	
		L2P	Q	?	BPSK	5110000	5.11	-	1000	?	?	-	
	1248.06	L2CSI	Q	?	BPSK(1), TDM	?	0.5115	?	?	?	?	?	GLO-K2~
		L2OCp				10230	0.5115	50	1000	-	-	-	
	1202.025	L3OCd	I	?	BPSK(10)	10230	10.23	5 (BC)	5	NAV	100	1/2	GLO-K1~
		L3OCp	Q	?	BPSK(10)	10230	10.23	10 (NH)	10	-	-	-	

*1 AS ON, *2 K = {-7 ... +6}

[1] IS-GPS-200K, Navstar GPS space segment/navigation user interfaces - interface specification, 2019, [2] IS-GPS-800F, Navstar GPS space segment/user segment L1C interface - interface specification, 2019, [3] GLONASS interface control document - navigation radiosignal in bands L1, L2, version 5.1, 2008, [4] GLONASS interface control document - code division multiple access open service navigation signal in L1 frequency band, edition 1.0, 2016, [5] GLONASS interface control document - code division multiple access open service navigation signal in L2 frequency band, edition 1.0, 2016, [6] GLONASS interface control document - code division multiple access open service navigation signal in L3 frequency band, edition 1.0, 2016

GNSS Signals (2/3)

System	Carrier Freq (MHz)	Signal	I/Q	Min Received Power (dBW)	Modulation	Spreading Code				Navigation Data			Notes
						Length (chips)	Chip Rate (Mcps)	Overlay Code (chips)	Period (ms)	Data	Bit Rate (bps)	FEC	
Galileo [7]	1575.42	E1-A	Q	?	BOC(15,2.5)	?	2.5575	?	?	G/NAV	?	?	PRS
		E1-B	I	-157.0	CBOC(6,1,1/11)	4092	1.023	-	4	I/NAV	125	1/2	OS, SoL, CS
		E1-C	Q		CBOC(6,1,1/11)	4092	1.023	25	100	-	-	-	
	1176.45	E5a-I	I	-155.0	BPSK(10)	10230	10.23	20	20	F/NAV	50	1/2	OS, CS
		E5a-Q	Q		BPSK(10)	10230	10.23	100	100	-	-	-	
	1207.14	E5b-I	I	-155.0	BPSK(10)	10230	10.23	4	4	I/NAV	125	1/2	OS, SoL, CS
		E5b-Q	Q		BPSK(10)	10230	10.23	100	100	-	-	-	
	1191.795	E5a+b*3	-	(-152.0)	8-PSK(10)	10230	10.23	100	100	-	-	-	
	1278.75	E6-A	Q	?	BOC(10,5)	?	5.115	?	?	G/NAV	?	?	PRS
		E6-B	I	-155.0	BPSK(5)	5115	5.115	-	1	C/NAV	500	1/2	CS, HAS
		E6-C	Q		BPSK(5)	5115	5.115	100	100	-	-	-	
QZSS [8][9][10][11]	1575.42	L1C/A	I/Q	-158.5*4	BPSK(1)	1023	1.023	-	1	LNAV	50	-	
		L1C-D	I	-163.0*5	BOC(1,1)	10230	1.023	-	10	CNAV2	50	1/2	
		L1C-P	Q	-158.25	BOC(1,1)	10230	1.023	1800	18000	-	-	-	Block I
			I	-158.25*6	TMBOC(6,1,4/33)	10230	1.023	1800	18000	-	-	-	Block II
		L1S	I	-161.0/-158.5	BPSK(1)	1023	1.023	-	1	L1S	250	1/2	SLAS
	1227.6	L2C-M	I	-160.0/-158.5	BPSK(1), TDM	10230	0.5115	-	20	CNAV	25	1/2	
		L2C-L				767250	0.5115	-	1500	-	-	-	
	1176.45	L5-I	I	-157.9/-157.0	BPSK(10)	10230	10.23	10 (NH)	10	CNAV	50	1/2	
		L5-Q	Q	-157.9/-157.0	BPSK(10)	10230	10.23	20 (NH)	20	-	-	-	
		L5S-I	I	-157.0*7	BPSK(10)	10230	10.23	10 (NH)	10	L5S	250	1/2	
		L5S-Q	Q		BPSK(10)	10230	10.23	20 (NH)	20	-	-	-	
	1278.75	L6	I	-155.7	BPSK(5), TDM	10230	2.5575	-	4	L6D	2000	RS	
						1048575	2.5575	-	410	-	-	-	Block I
						10230	2.5575	-	4	L6E	2000	RS	Block II

*3 AltBOC *4 -164.0 dBW (SVID=7), *5 -167.2 dBW (SVID=7), *6 -162.4 dBW (SVID=7), *7 -162.6 dBW (SVID=3)

[7] European GNSS (Galileo) open service signal-in-space interface control document (OS SIS ICD), Issue 1, Revision 3, 2016, [8] Quasi-Zenith satellite system interface specification - satellite positioning, navigation and timing service (IS-QZSS-PNT-003), 2018, [9] Quasi-zenith satellite system interface specification - sub-meter level augmentation service (IS-QZSS-L1S-003), 2018, [10] Quasi-zenith satellite system interface specification - centimeter level augmentation service (IS-QZSS-L6-003), 2018, [11] Quasi-zenith satellite system interface specification - positioning technology verification service (IS-QZSS-TV-002), 2018,

GNSS Signals (3/3)

System	Carrier Freq (MHz)	Signal	I/Q	Min Received Power (dBW)	Modulation	Spreading Code				Navigation Data			Notes
						Length (chips)	Chip Rate (Mcps)	Overlay Code (chips)	Period (ms)	Data	Bit Rate (bps)	FEC	
BeiDou [12][13][14][15]	1561.098	B1I	I	-163.0	BPSK(2)	2046	2.046	20 (NH)	20	D1*10	50	BCH	BDS-2*12
								-	1	D2*11	500	BCH	
		B1Q	Q	?	BPSK(2)	?	2.046	?	?	?	?	?	
	1575.42	B1C-D	I	-159.0/-161.0	BOC(1,1)	10230	1.023	-	10	B-CNAV1	50	1/2	BDS-3
		B1C-P	Q		QMBOC(6,1,4/33)	10230	1.023	1800	18000	-	-	-	
		B1A-D*8	I	?	BOC(14,2)	?	?	?	?	?	50	?	
		B1A-P*8	Q			?	?	?	?	-	-	-	
	1207.14	B2I	I	?	BPSK(2)	2046	2.046	20 (NH)	20	D1*10	50	BCH	BDS-2
								-	1	D2*11	500	BCH	
		B2Q*8	Q	?	BPSK(10)	10230	10.23	?	?	?	?	?	
	1176.45	B2a-D	I	-156.0/-158.0	BPSK(10)	10230	10.23	5	5	B-CNAV2	25	1/2	BDS-3
		B2a-P	Q		BPSK(10)	10230	10.23	100	100	-	-	-	
	1207.14	B2b-I	I	?	BPSK(10)	10230	10.23	?	?	?	500	-	
		B2b-Q	Q	?	BPSK(10)	10230	10.23	?	?	?	500	-	
	1191.795	B2a+b*9	-	?	8-PSK(10)	10230	10.23	?	?	-	-	-	
	1268.52	B3I	I	-163.0	BPSK(10)	10230	10.23	-	1	D1*10	50	BCH	BDS-2*12
										D2*11	500	BCH	
		B3Q*8	Q	?	BPSK(10)	?	10.23	?	?	?	?	?	
		B3A-D*8	I	?	BPSK(10)	?	10.23	?	?	?	50	?	BDS-3
		B3A-P*8	Q		BPSK(10)	?	10.23	?	?	-	-	-	
SBAS	1575.42	L1C/A	I	-	BPSK(1)	1023	1.023	-	1	SBAS	250	1/2	
	1176.45	L5-I	I	-	BPSK(10)	10230	10.23	10 (NH)	10	SBAS	250	1/2	
		L5-Q	Q	-	BPSK(10)	10230	10.23	20 (NH)	20	-	-	-	

*8 Authorized signal, *9 ACE-BOC, *10 IGSO/MEO satellites, *11 GEO satellites *12 B1I and B3I signals are also transmitted by BDS-3

[12] BeiDou navigation satellite system signal in space interface control document - open service signal B1I, version 3, 2019, [13] BeiDou navigation satellite system signal in space interface control document - open service signal B1C, version 1.0, 2017, [14] BeiDou navigation satellite system signal in space interface control document - open service signal B2a, version 1.0, 2017, [15] BeiDou navigation satellite system signal in space interface control document - open service signal B3I, version 1.0, 2018

GNSS Signal ID (1/4)

System	Carrier Frequency (MHz)	Channel or Code	RINEX *								RTCM3 MSM[9]	BINEX[10]
			2.10[1]	2.11[2]	2.12[3]	3.00[4]	3.01[5]	3.02[6]	3.03[7]	3.04[8]		
GPS	L1/ 1575.42	C/A	C1	C1	CA	1C	1C	1C	1C	1C	2	1
		L1C(D)	-	-	CB	1S	1S	1S	1S	1S	30	6
		L1C(P)	-	-	CB	1L	1L	1L	1L	1L	31	6
		L1C(D+P)	-	-	CB	1X	1X	1X	1X	1X	32	6
		P (AS off)	P1	P1	P1	1P	1P	1P	1P	1P	3	2
		Z-tracking and similar (AS on)	P1	P1	P1	1W	1W	1W	1W	1W	4	3
		Y	-	-	-	1Y	1Y	1Y	1Y	1Y	-	4
		M	-	-	-	1M	1M	1M	1M	1M	-	5
		codeless	-	-	-	1N	1N	1N	1N	1N	-	7
	L2/ 1227.60	C/A	-	-	-	2C	2C	2C	2C	2C	8	11
		L1(C/A)+(P2-P1) (semi-codeless)	P2	P2	C2	2D	2D	2D	2D	2D	-	12
		L2C(M)	-	C2	CC	2S	2S	2S	2S	2S	15	13
		L2C(L)	-	C2	CC	2L	2L	2L	2L	2L	16	14
		L2C(M+L)	-	C2	CC	2X	2X	2X	2X	2X	17	15
		P (AS off)	P2	P2	P2	2P	2P	2P	2P	2P	9	16
		Z-tracking and similar (AS on)	P2	P2	P2	2W	2W	2W	2W	2W	10	17
		Y	-	-	-	2Y	2Y	2Y	2Y	2Y	-	18
		M	-	-	-	2M	2M	2M	2M	2M	-	19
		codeless	-	-	-	2N	2N	2N	2N	2N	-	20
	L5/ 1176.45	I	-	C5	C5	5I	5I	5I	5I	5I	22	24
		Q	-	C5	C5	5Q	5Q	5Q	5Q	5Q	23	25
		I+Q	-	C5	C5	5X	5X	5X	5X	5X	24	26
GLONASS	G1/ 1602+K*9/16	C/A	C1	C1	CA	1C	1C	1C	1C	1C	2	1
		P	P1	P1	P1	1P	1P	1P	1P	1P	3	2
	G1a/ 1600.995	L1OCd	-	-	-	-	-	-	-	4A	-	-
		L1OCp	-	-	-	-	-	-	-	4B	-	-
		L1OCd+L1OCp	-	-	-	-	-	-	-	4X	-	-

GNSS Signal ID (2/4)

System	Carrier Frequency (MHz)	Channel or Code	RINEX *								RTCM3 MSM[9]	BINEX[10]
			2.10[1]	2.11[2]	2.12[3]	3.00[4]	3.01[5]	3.02[6]	3.03[7]	3.04[8]		
GLONASS (cont.)	G2/ 1246+K*7/16	C/A (GLONASS-M)	-	C2	CD	2C	2C	2C	2C	2C	8	11
		P	P2	P2	P2	2P	2P	2P	2P	2P	9	12
	G2a/ 1248.06	L2CSI	-	-	-	-	-	-	-	6A	-	-
		L2OCp	-	-	-	-	-	-	-	6B	-	-
		L2CSI+L2OCp	-	-	-	-	-	-	-	6X	-	-
	G3/ 1202.025	I	-	-	-	-	-	3I	3I	3I	-	14
		Q	-	-	-	-	-	3Q	3Q	3Q	-	15
		I+Q	-	-	-	-	-	3X	3X	3X	-	16
Galileo	E1/ 1575.42	A PRS	-	C1	C1	1A	1A	1A	1A	1A	3	1
		B I/NAV OS/CS/SoL	-	C1	C1	1B	1B	1B	1B	1B	4	2
		C	-	C1	C1	1C	1C	1C	1C	1C	2	3
		B+C	-	C1	C1	1X	1X	1X	1X	1X	5	4
		A+B+C	-	C1	C1	1Z	1Z	1Z	1Z	1Z	6	5
	E5a/ 1176.45	I F/NAV OS	-	C5	C5	5I	5I	5I	5I	5I	22	7
		Q no data	-	C5	C5	5Q	5Q	5Q	5Q	5Q	23	8
		I+Q	-	C5	C5	5X	5X	5X	5X	5X	24	9
	E5b/ 1207.14	I I/NAV OS/CS/SoL	-	C7	C7	7I	7I	7I	7I	7I	14	11
		Q no data	-	C7	C7	7Q	7Q	7Q	7Q	7Q	15	12
		I+Q	-	C7	C7	7X	7X	7X	7X	7X	16	13
	E5a+b/ 1191.795	I	-	C8	C8	8I	8I	8I	8I	8I	18	15
		Q	-	C8	C8	8Q	8Q	8Q	8Q	8Q	19	16
		I+Q	-	C8	C8	8X	8X	8X	8X	8X	20	17
	E6/ 1278.75	A PRS	-	C6	C6	6A	6A	6A	6A	6A	9	19
		B C/NAV CS	-	C6	C6	6B	6B	6B	6B	6B	10	20
		C no data	-	C6	C6	6C	6C	6C	6C	6C	8	21
		B+C	-	C6	C6	6X	6X	6X	6X	6X	11	22
		A+B+C	-	C6	C6	6Z	6Z	6Z	6Z	6Z	12	23

GNSS Signal ID (3/4)

System	Carrier Frequency (MHz)	Channel or Code	RINEX *								RTCM3 MSM[9]	BINEX[10]
			2.10[1]	2.11[2]	2.12[3]	3.00[4]	3.01[5]	3.02[6]	3.03[7]	3.04[8]		
BeiDou	B1-2/ 1561.098	I	-	-	C2	-	2I	1I	2I	2I	2	1
		Q	-	-	C2	-	2Q	1Q	2Q	2Q	3	2
		I+Q	-	-	C2	-	2X	1X	2X	2X	4	3
	B1/ 1575.42 (BDS-3)	Data	-	-	-	-	-	-	-	1D	-	13
		Pilot	-	-	-	-	-	-	-	1P	-	14
		Data+Pilot	-	-	-	-	-	-	-	1X	-	15
		B1A	-	-	-	-	-	-	-	1A	-	-
		Codeless	-	-	-	-	-	-	-	1N	-	-
	B2a/ 1176.45 (BDS-3)	Data	-	-	-	-	-	-	-	5D	-	17
		Pilot	-	-	-	-	-	-	-	5P	-	18
		Data+Pilot	-	-	-	-	-	-	-	5X	-	19
	B2b/ 1207.14 (BDS-2)	I	-	-	C7	-	7I	7I	7I	7I	14	5
		Q	-	-	C7	-	7Q	7Q	7Q	7Q	15	6
		I+Q	-	-	C7	-	7X	7X	7X	7X	16	7
	B2b/ 1207.14 (BDS-3)	Data	-	-	-	-	-	-	-	7D	-	-
		Pilot	-	-	-	-	-	-	-	7P	-	-
		Data+Pilot	-	-	-	-	-	-	-	7Z	-	-
	B2a+b/ 1191.795	Data	-	-	-	-	-	-	-	8D	-	-
		Pilot	-	-	-	-	-	-	-	8P	-	-
		Data+Pilot	-	-	-	-	-	-	-	8X	-	-
	B3/ 1268.52	I	-	-	C6	-	6I	6I	6I	6I	8	9
		Q	-	-	C6	-	6Q	6Q	6Q	6Q	9	10
		I+Q	-	-	C6	-	6X	6X	6X	6X	10	11
		B3A	-	-	-	-	-	-	-	6A	-	-

GNSS Signal ID (4/4)

System	Carrier Frequency (MHz)	Channel or Code	RINEX *								RTCM3 MSM[9]	BINEX[10]
			2.10[1]	2.11[2]	2.12[3]	3.00[4]	3.01[5]	3.02[6]	3.03[7]	3.04[8]		
QZSS	L1/ 1575.42	C/A	-	-	-	-	-	1C	1C	1C	2	1
		L1C(D)	-	-	-	-	-	1S	1S	1S	30	2
		L1C(P)	-	-	-	-	-	1L	1L	1L	31	3
		L1C(D+P)	-	-	-	-	-	1X	1X	1X	32	4
		L1-SAIF/L1S	-	-	-	-	-	1Z	1Z	1Z	6	30
	L2/ 1227.60	L2C(M)	-	-	-	-	-	2S	2S	2S	15	8
		L2C(L)	-	-	-	-	-	2L	2L	2L	16	9
		L2C(M+L)	-	-	-	-	-	2X	2X	2X	17	10
	L5/ 1176.45	I	-	-	-	-	-	5I	5I	5I	22	14
		Q	-	-	-	-	-	5Q	5Q	5Q	23	15
		I+Q	-	-	-	-	-	5X	5X	5X	24	16
		L5D (L5S)	-	-	-	-	-	-	-	5D	-	-
		L5P (L5S)	-	-	-	-	-	-	-	5P	-	-
		L5(D+P) (L5S)	-	-	-	-	-	-	-	5Z	-	-
	L6/ 1278.75	L6D	-	-	-	-	-	6S	6S	6S	9	20
		L6P	-	-	-	-	-	6L	6L	6L	10	21
		L6(D+P)	-	-	-	-	-	6X	6X	6X	11	22
		L6E	-	-	-	-	-	-	-	6E	-	-
		L6(D+E)	-	-	-	-	-	-	-	6Z	-	-
SBAS	L1/ 1575.42	C/A	C1	C1	C1	1C	1C	1C	1C	1C	2	1
	L5/ 1176.45	I	-	C5	C5	5I	5I	5I	5I	5I	22	7
		Q	-	C5	C5	5Q	5Q	5Q	5Q	5Q	23	8
		I+Q	-	C5	C5	5X	5X	5X	5X	5X	24	9

* observation code of pseudorange (RINEX 2), band/frequency + attribute (RINEX 3)

[1] RINEX: The receiver independent exchange format version 2.10, Dec 2007, [2]RINEX: The receiver independent exchange format version 2.11, June 2007, [3] RINEX: The receiver independent exchange format version 2.12, June 2009, [4] RINEX - The receiver independent exchange format version 3.00, Nov 2007, [5] RINEX - The receiver independent exchange format version 3.01, June 2009, [6] RINEX - The receiver independent exchange format version 3.02, April 2013, [7] RINEX - The receiver independent exchange format version 3.03, July 2015, [8] RINEX - The receiver independent exchange format version 3.04, Nov 2018 , [9] RTCM standard 10403.3, Differential GNSS (global navigation satellite system) services - version 3, Oct 2016, [10] BINEX: Binary exchange format - BINEX record 0x7f-05: GNSS observable prototyping (https://www.binex.unavco.org/binex.html)