2009年度GCOE地球惑星計測スクール

GPS Global Positioning System

Part 1

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Outline: Part 1

- **GPS/GNSS Basics and Principles**
  - GPS/GNSS Summary
  - GPS Signal and Receiver Architecture
  - Basic Measurement Models
  - Navigation Processing
  - Error Sources
  - DGPS and SBAS
- **Exercise: using GT 0.6.4**

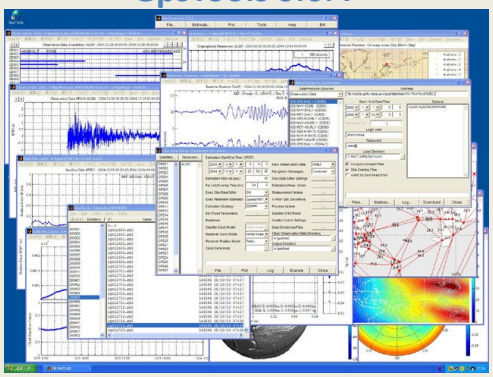
Outline: Part 2

- **Precise Positioning with GPS/GNSS**
  - Time and Coordinate Systems
  - Precise Measurement Models
  - Relative Positioning
  - Precise Point Positioning (PPP)
  - Applications
- **Exercise: PPP with GT 0.6.4**

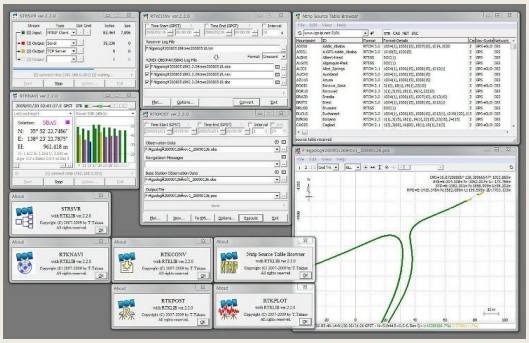
Self Introduction

?

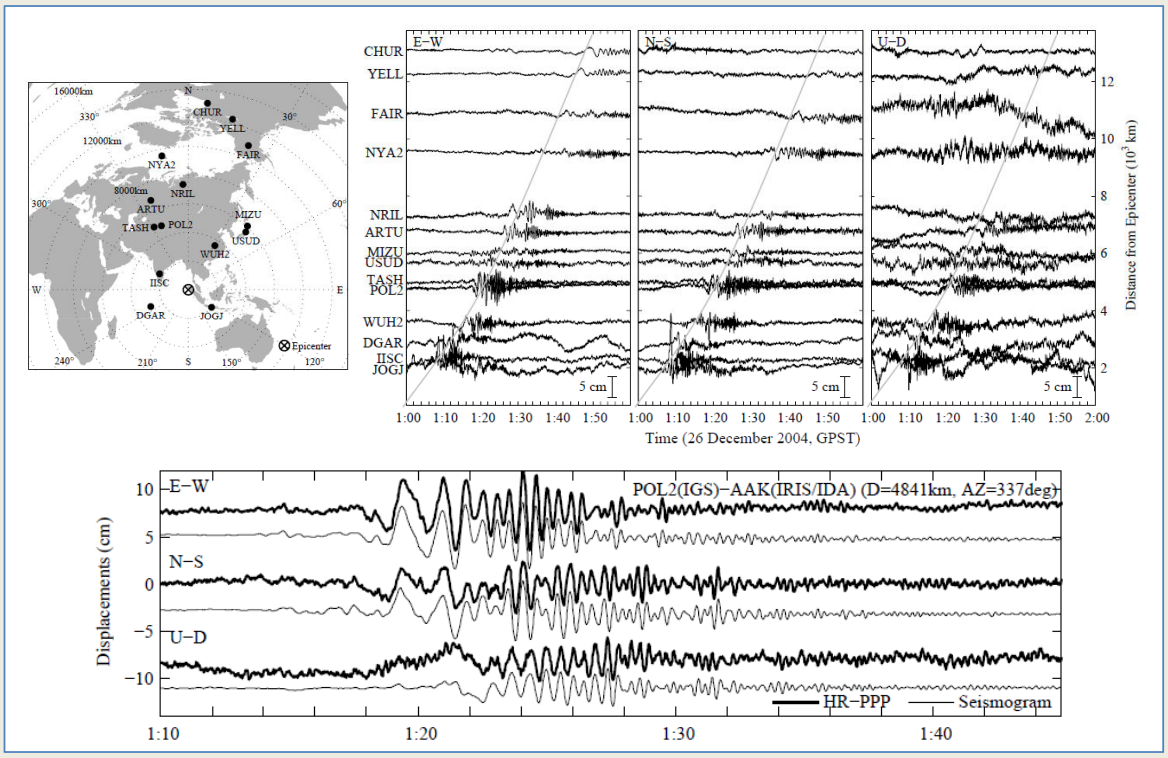
GpsTools 0.6.4



RTKLIB 2.2.1



GpsTools



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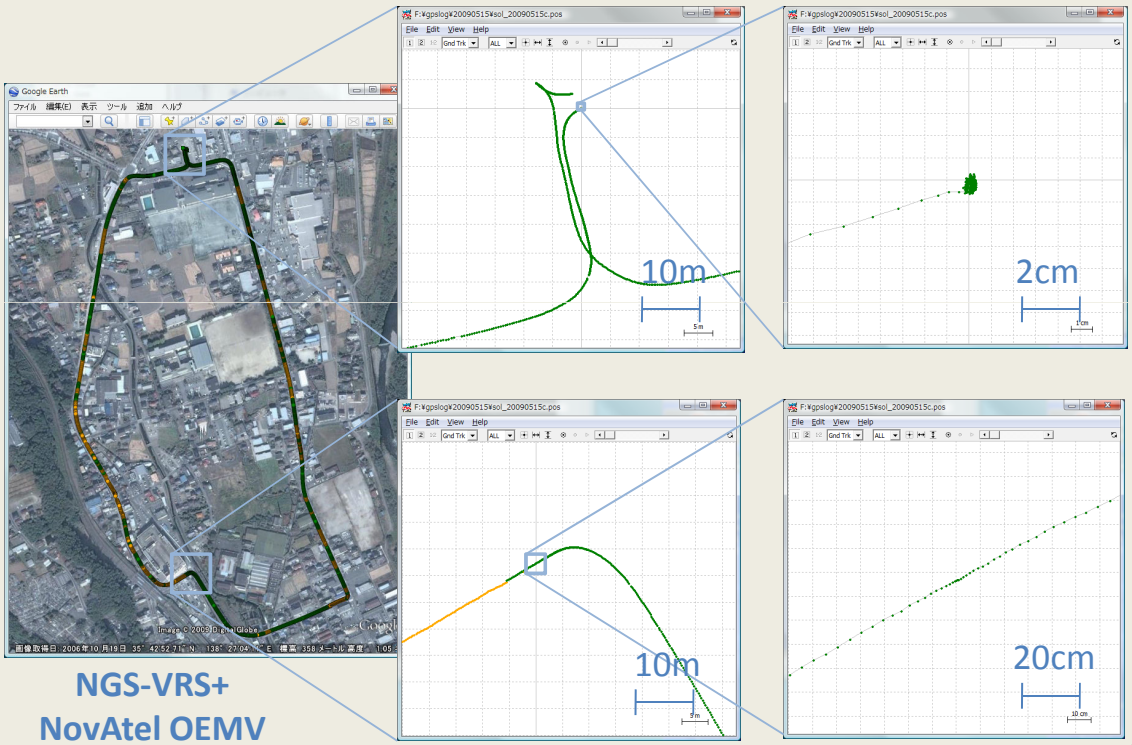


NGS-VRS+
NovAtel OEMV

GPS/GNSS Summary

GPS

- NAVSTAR GPS (Global Positioning System)
  - Satellite navigation system developed by US DoD
  - Operated by US Air Force GPS Wing
- History
  - 1978/2 First satellite launch
  - 1983 Freely available for civilian use
  - 1993 Fully operational (FOC)
  - 2000/5 S/A Termination
  - 2009/8 30 operational satellites

GNSS

- Global Navigation Satellite System
  - GPS (US)
  - GLONASS (Russia)
  - Galileo (EU)
  - Compass (China)
- Regional Navigation Satellite System
  - QZSS (Japan)
  - IRNSS (India)
- Satellite Based Augmentation System (SBAS)

GPS/GNSS Applications

Military Applications: ...

Civil Applications:

Air Navigation
- Nonprecision approach and landing
- Domestic en route
- Oceanic en route
- Terminal
- Remote areas
- Helicopter operations
- Aircraft attitude
- Collision avoidance
- Air Traffic Control

Land Navigation
- Vehicle monitoring
- Schedule improvement
- Minimal routing
- Law enforcement

Marine navigation
- Oceanic
- Coastal
- Harbor/approach
- Inland waterways

Static positioning and timing
- Offshore resource exploration
- Hydrographic surveying
- Aids to navigation
- Time transfer
- Land surveying
- Geographical information systems

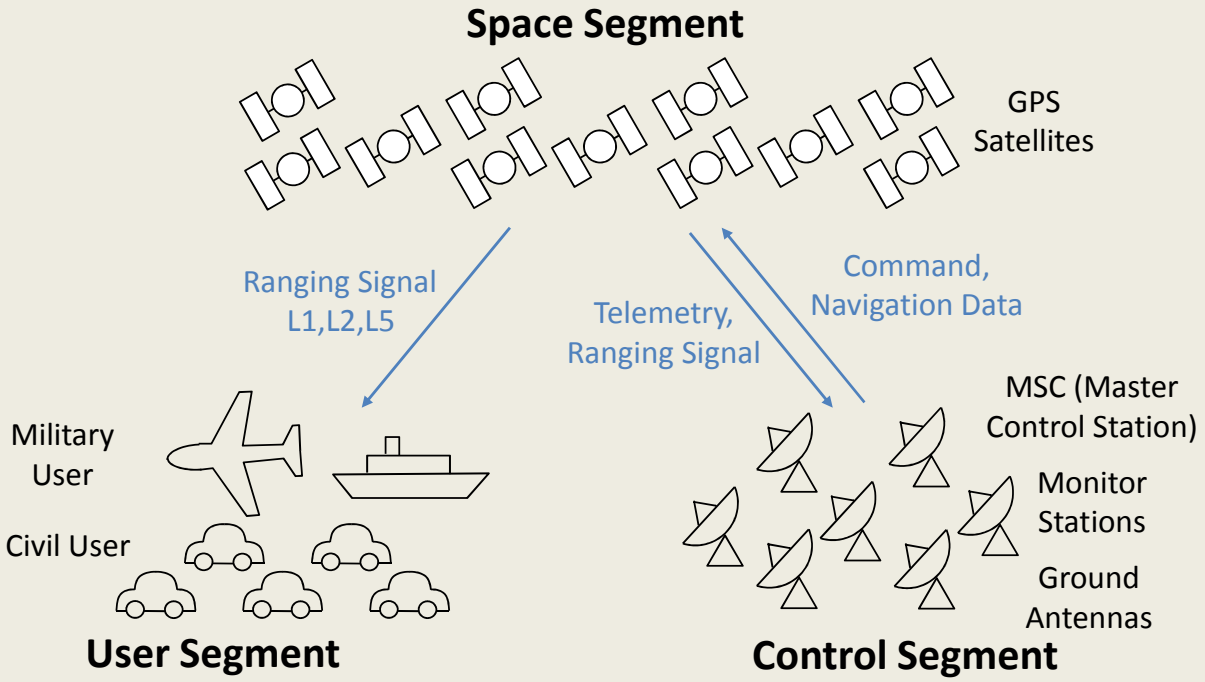
Space
- Launch
- In-flight/orbit
- Reentry/landing
- Attitude measurement

Search and Rescue
- Position reporting and monitoring
- Rendezvous
- Coordinated search
- Collision avoidance

...

(B.W.Parkinson, Introduction and Heritage of NAVSTAR, the Global Positioning System, 1994)

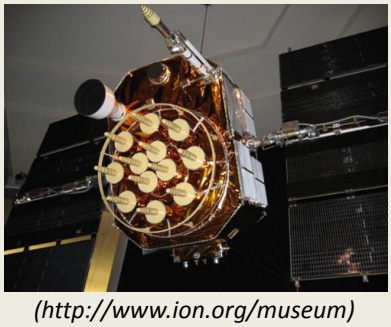
GPS System



GPS Space Segment

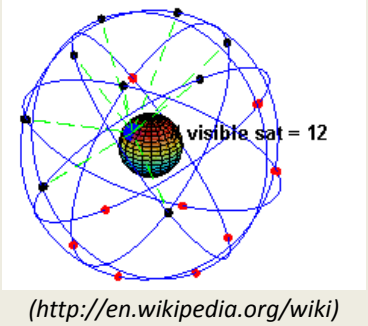
- Satellite Constellation
  - 6 Plane x 4 = 24 Satellites (Nominal)
  - Altitude: 20,100km
  - Inclination: 55°
  - Period: 1/2 Sidereal Day (11h 58' 2")

GPS Block II



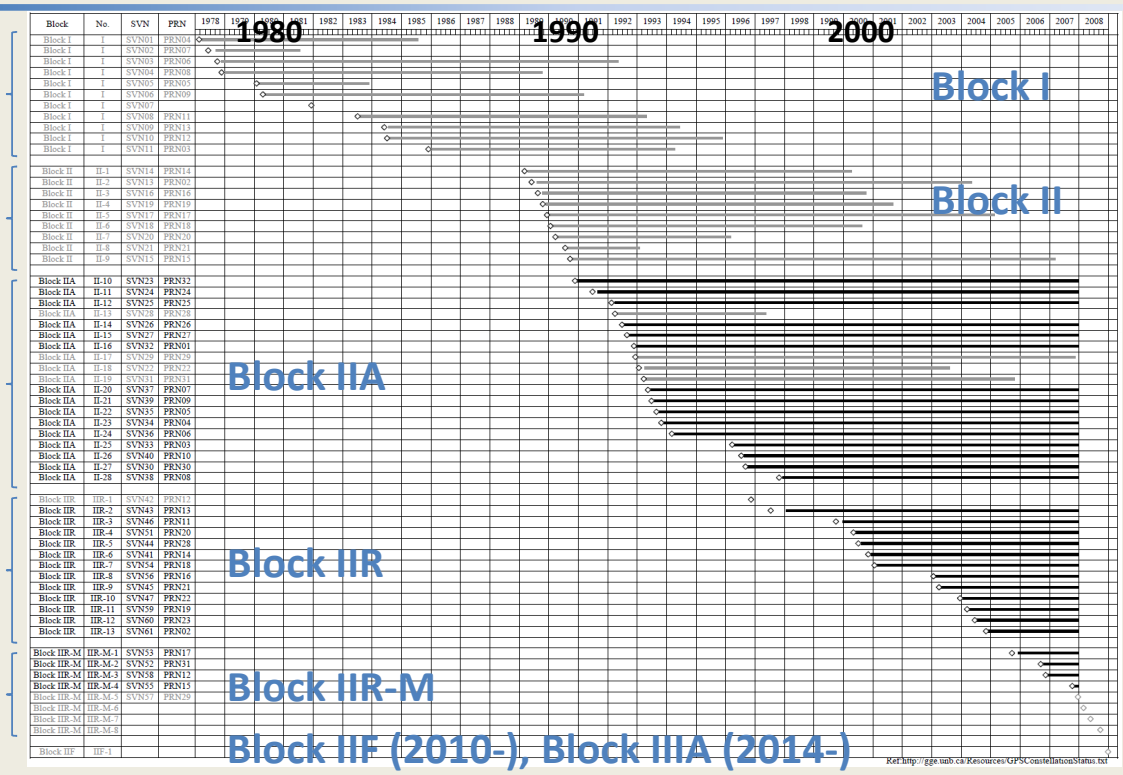
(http://www.ion.org/museum)

Satellite Orbit Planes

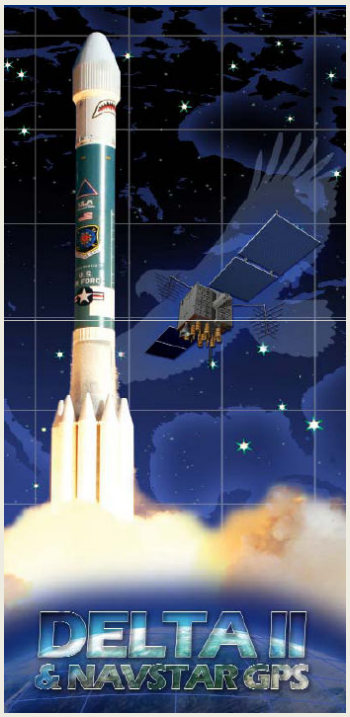


(http://en.wikipedia.org/wiki)

GPS Satellites

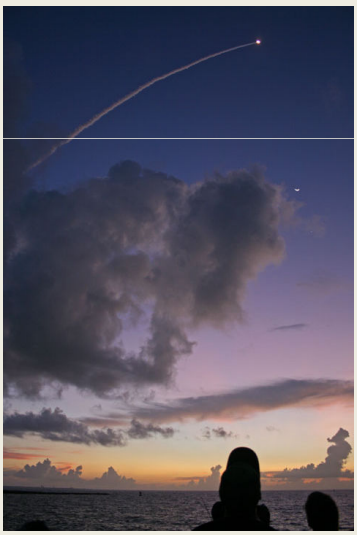


GPS Block IIR-M-8 Launch



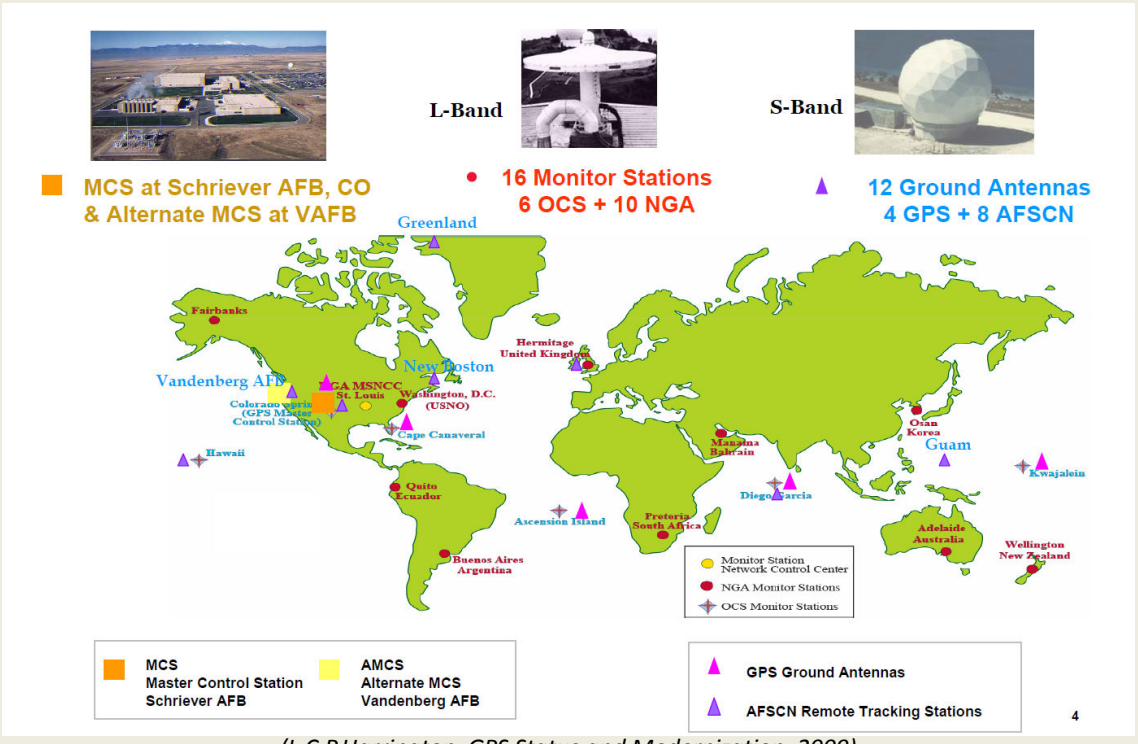
(United Launch Alliance, Delta II GPS IIR-21 Mission Booklet)

- 2009/8/17 10:35 UTC (6:35 a.m. EDT)
- US Cape Canaveral
- Launcher: Delta 2
- SVN50/PRN?
- Plane E/Slot 3
- Replace: IIA-26 (SVN40)



(Spaceflight Now: http://www.spaceflightnow.com)

GPS Ground Segment



(L.C.P.Harrington, GPS Status and Modernization, 2009)

GLONASS (ГЛОНАСС)

- Development: USSR and Russia
- Satellite Constellation:
  - 3 Plane x 8 = 24 Sats + 3 Spare (FOC)
  - Altitude: 19,100 km, Inclination: 64.5°
  - GLONASS, GLONASS-M (2003-), GLONASS-K (2010-)
- Signals:
  - L1C/A, L1P (FDMA: 1602+n x 0.5625 MHz)
  - L2C/A, L2P (FDMA: 1246+n x 0.4375 MHz)
  - L3 FDMA/CDMA (GLONASS-K-)

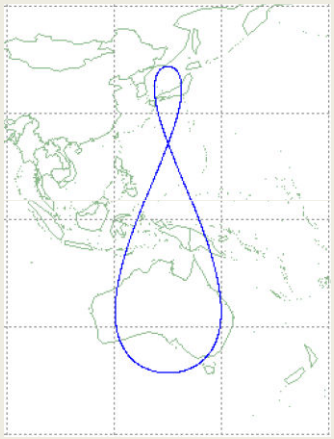
Galileo

- Development: EU and ESA
- Satellite Constellation:
  - 3 Plane x 9 = 27 Sats + 3 Spare (FOC)
  - Altitude: 23,200km, Inclination: 56°
  - Test Sats: GIOVE-A (2006), GIOVE-B (2008)
  - IOV: 2011 (4 Sats), FOC: 2013/E
- Signals:
  - E5a-I/Q (OS,CS), E5b-I/Q (OS,SOL,CS)
  - E6-A (PRS), E6-B/C (CS), E1A (PRS), E1-B/C (OS,SOL,CS)

QZSS

- Development: Japan, JAXA
- Satellite Constellation:
  - 1 sat (IOC), 3 sats (FOC)
  - Altitude: ~36,000km, Inclination: 43°
  - Eccentricity: 0.075
  - First Sat Launch: 2010
- Signals:
  - L1C/A, L1C, L2C, L5-I/Q: GPS Compatible
  - L1-SAIF, LEX: Augmentation

QZSS Ground Track



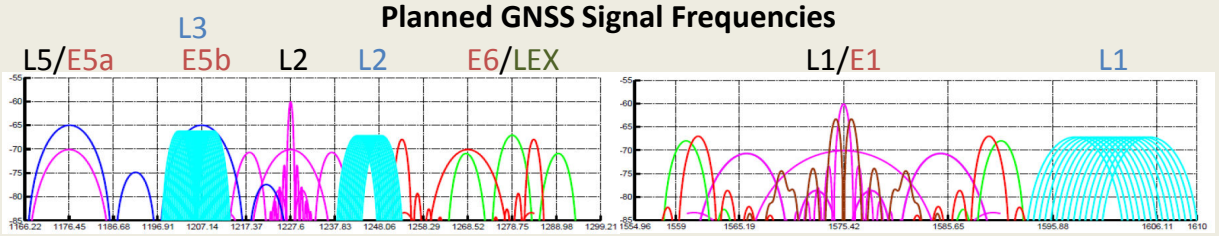
(IS-QZSS 1.1 Draft)

GNSS Constellation

Number of Planned GNSS Satellites

System	2009	2012	2015	2018
GPS	30	32	32	32
GLONASS	17 (+3)	24 (+3)	24 (+3)	24 (+3)
Galileo	0	15 (+3)	27 (+3)	27 (+3)
Compass	2	12	30	35
QZSS	0	1	3	3
IRNSS	0	7	7	7
SBAS	7	8	8	8
Total	56	99	131	136

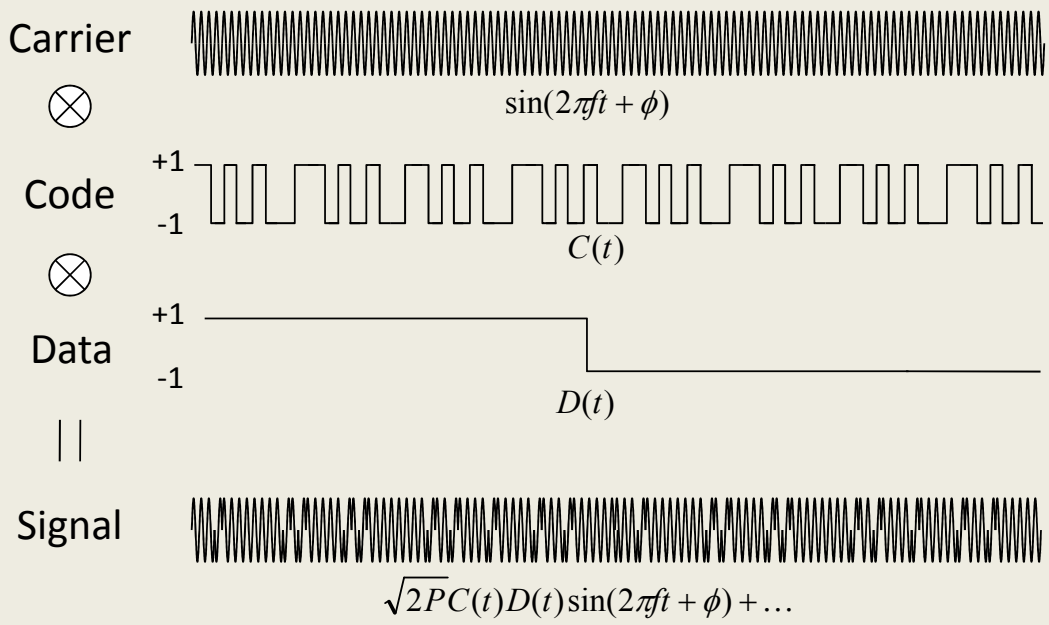
Planned GNSS Signal Frequencies



(Y.Yang, COMPASS: View on Compatibility and Interoperability, 2009)

GPS Signal and Receiver Architecture

GPS Signal 1



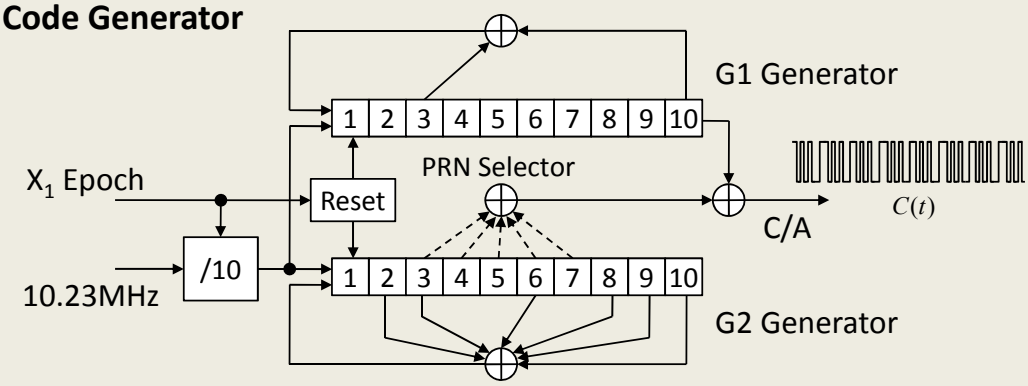
GPS Signal 2

Carrier Frequency		Code	Modulation	Data Rate	Satellite
L1	1575.42MHz	C/A	BPSK (1)	50 bps	
		P(Y)	BPSK (10)	50 bps	
		M-code	BOC (10,5)	?	IIRM-
		L1C-d	MBOC (6,1,1/11)	100 bps	IIIA-
		L1C-p	MBOC (6,1,1/11)	none	IIIA-
L2	1227.60MHz	P(Y)	BPSK (10)	50 bps	
		L2C	BPSK (1)	25 bps	IIRM-
		M-code	BOC (10,5)	?	IIRM-
L5	1176.45MHz	L5I	BPSK (10)	100 bps	IIF-
		L5Q	BPSK (10)	none	IIF-

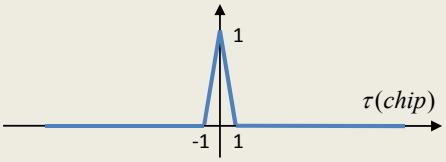
Military Signal, Planned

PRN Codes

C/A Code Generator

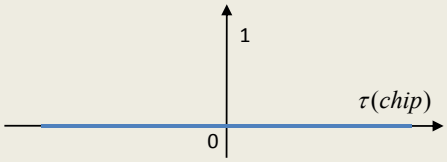


Auto-correlation function



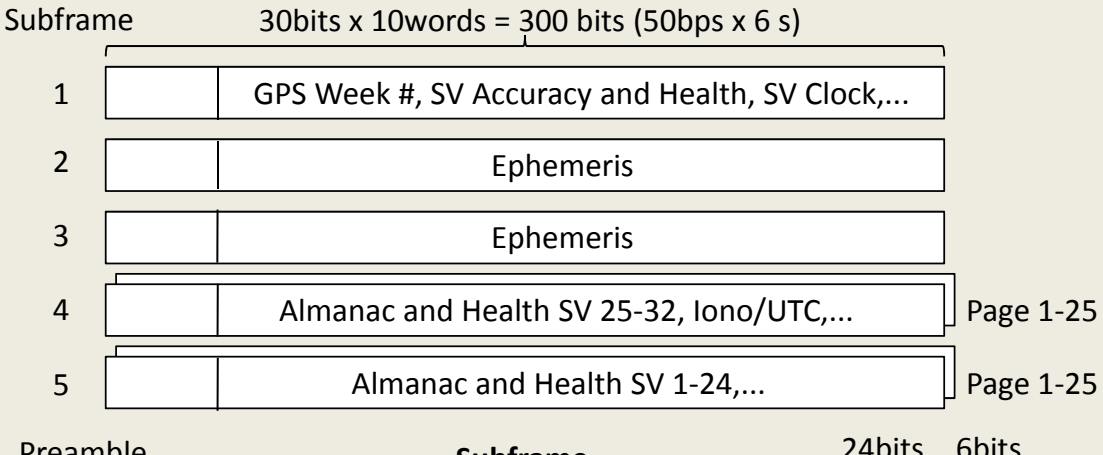
$$R(\tau) = \frac{1}{T}\int_0^T C^i(t)C^i(t-\tau)dt$$

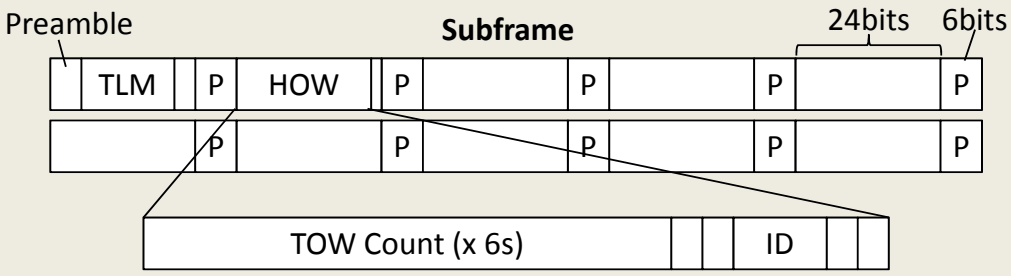
Cross-correlation function



$$R(\tau) = \frac{1}{T}\int_0^T C^i(t)C^j(t-\tau)dt \quad (i \neq j)$$

Navigation Data



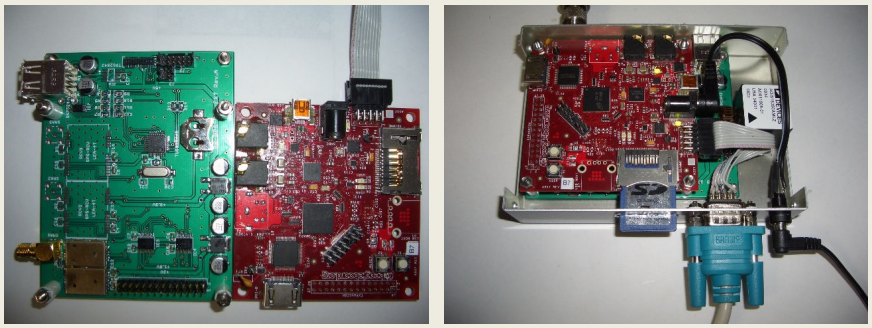


GPS Receivers

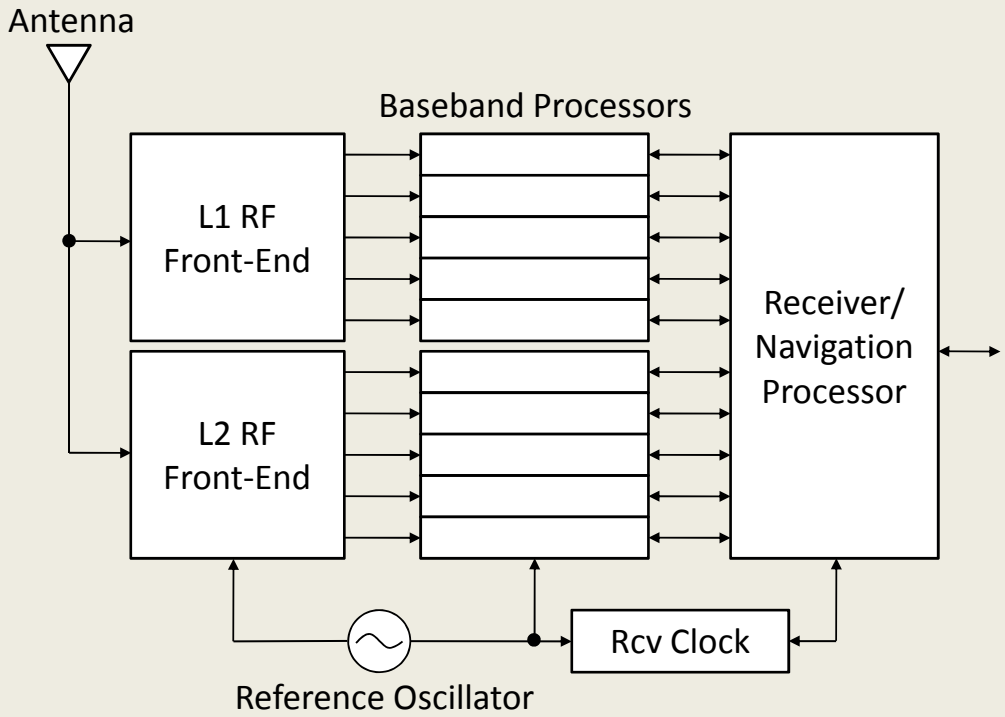
Receiver Products: $20 - $30,000

SiRF, u-blox, Garmin, Hemisphere, Trimble, Leica, Topcon, NovAtel, Javad, Magellan, ...

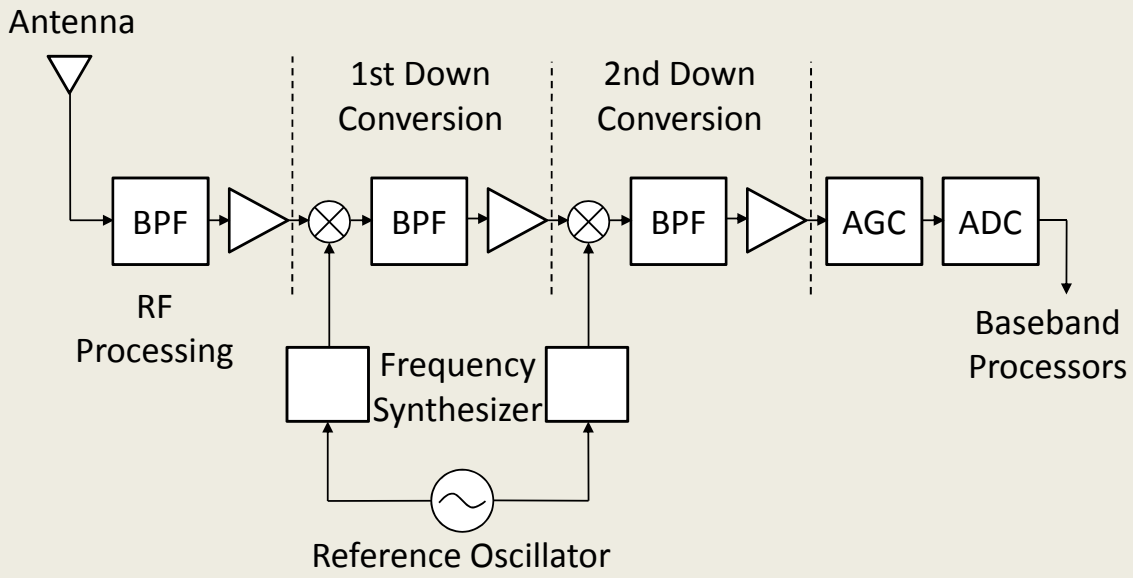
Handmade GPS receiver: $400



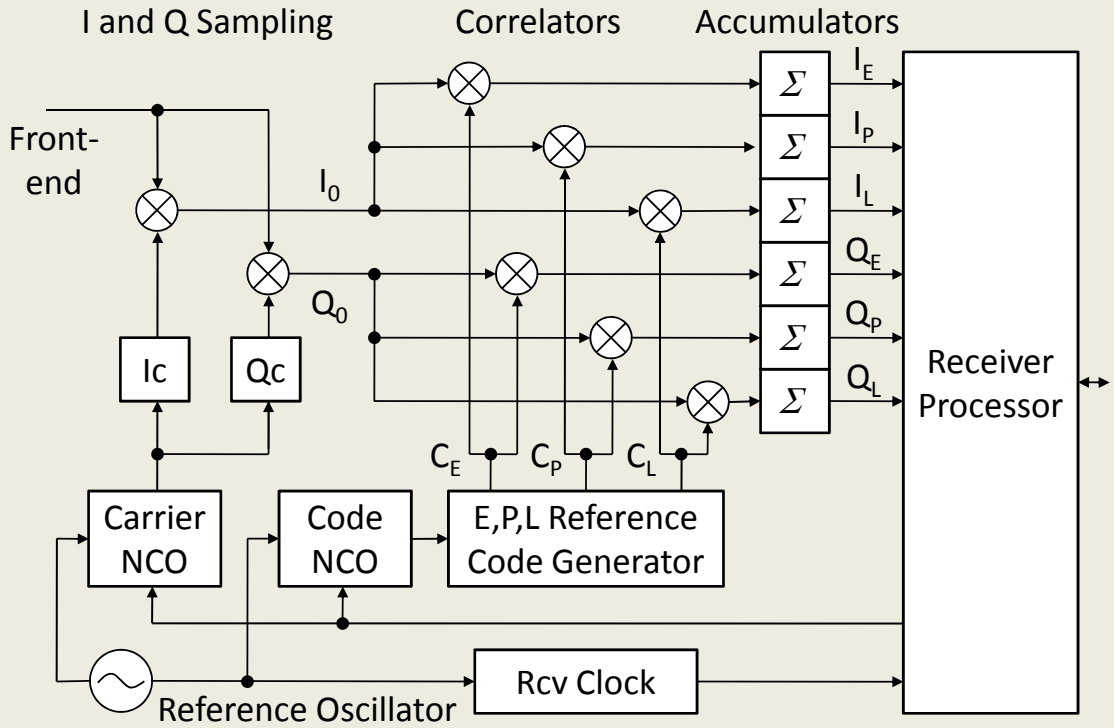
Receiver Architecture



RF Front-End



Baseband Processor



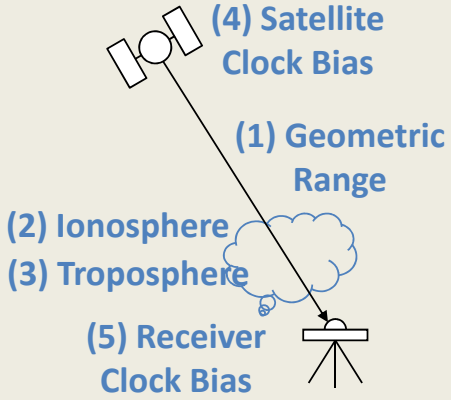
Receiver Processor

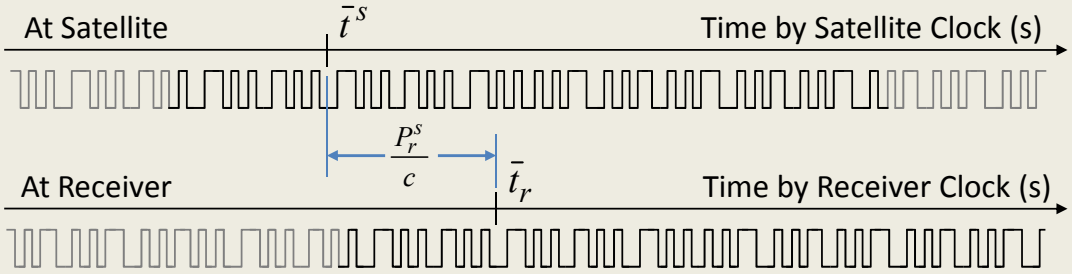
- Receiver Processor:
  - Acquisition : Search Doppler Shift and Code Phase
  - Code Tracking : DLL (Delay Lock Loop)
  - Carrier Tracking: FLL/PLL (Freq/Phase Lock Loop)
  - Navigation Data Decode
- Output to Navigation Processor:
  - Pseudorange
  - (Carrier-Phase, Doppler-Freq)
  - Navigation Data (Ephemeris, SV Clock Parameter, ...)

Basic Measurement Models

Pseudorange

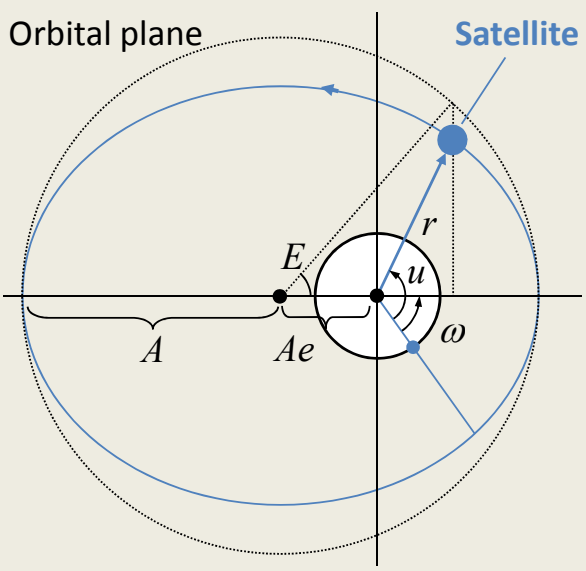
$$P_r^s \equiv c(\bar{t}_r - \bar{t}^s)$$
$$= c((t_r + dt) - (t^s + dT^s)) + \varepsilon_P$$
$$= c(t_r - t^s) + c(dt_r - dT^s) + \varepsilon_P$$
$$= \underbrace{(\rho_r^s}_{(1)} + \underbrace{I_r^s}_{(2)} + \underbrace{T_r^s)}_{(3)} + c(\underbrace{dt_r}_{(4)} - \underbrace{dT^s)}_{(5)} + \varepsilon_P$$

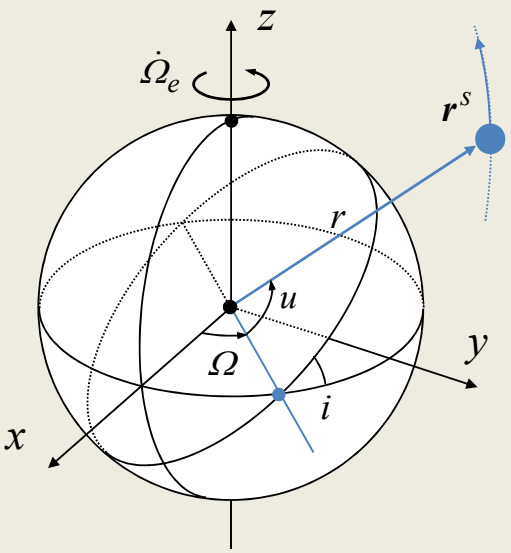




Ephemeris 1

$$M_0, \Delta n, e, \sqrt{A}, \Omega_0, i_0, \omega, \dot{\Omega}, IDOT, C_{uc}, C_{us}, C_{rc}, C_{rs}, C_{ic}, C_{is}, T_{oe}$$





Ephemeris 2

Satellite Position (ECEF):

$$t_k = t - t_{oe}$$

$$n = \sqrt{\mu / A^3} + \Delta n$$

$$M = M_0 + n t_k$$

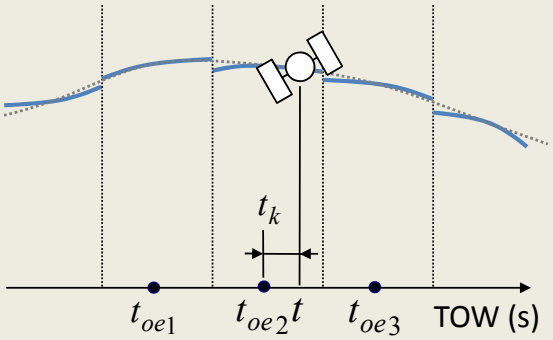
$$E = M + e \sin E \quad \text{: Kepler Equation}$$

$$\phi = ATAN2(\sqrt{1-e^2}\sin E, \cos E - e) + \omega$$

$$\begin{pmatrix} u \\ r \\ i \end{pmatrix} = \begin{pmatrix} \phi \\ A(1 - e\cos E) \\ i_0 + IDOT\ t_k \end{pmatrix} + \begin{pmatrix} C_{us} & C_{uc} \\ C_{rs} & C_{rc} \\ C_{is} & C_{ic} \end{pmatrix} \begin{pmatrix} \sin 2\phi \\ \cos 2\phi \end{pmatrix}$$

$$\Omega = \Omega_0 + (\dot{\Omega} - \omega_e) t_k - \omega_e t_{oe}$$

$$\boldsymbol{r}^s(t) = \boldsymbol{R}_z(-\Omega)\boldsymbol{R}_x(-i)(r\cos u, r\sin u, 0)^T$$



$$\boldsymbol{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

$$\boldsymbol{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\boldsymbol{R}_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

SV Clock Parameters

$$a_{f0}, a_{f1}, a_{f2}, T_{GD}, t_{oc}$$

Satellite Clock Bias:

$$dT(t) = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_{rel} + \Delta t_{GD}$$

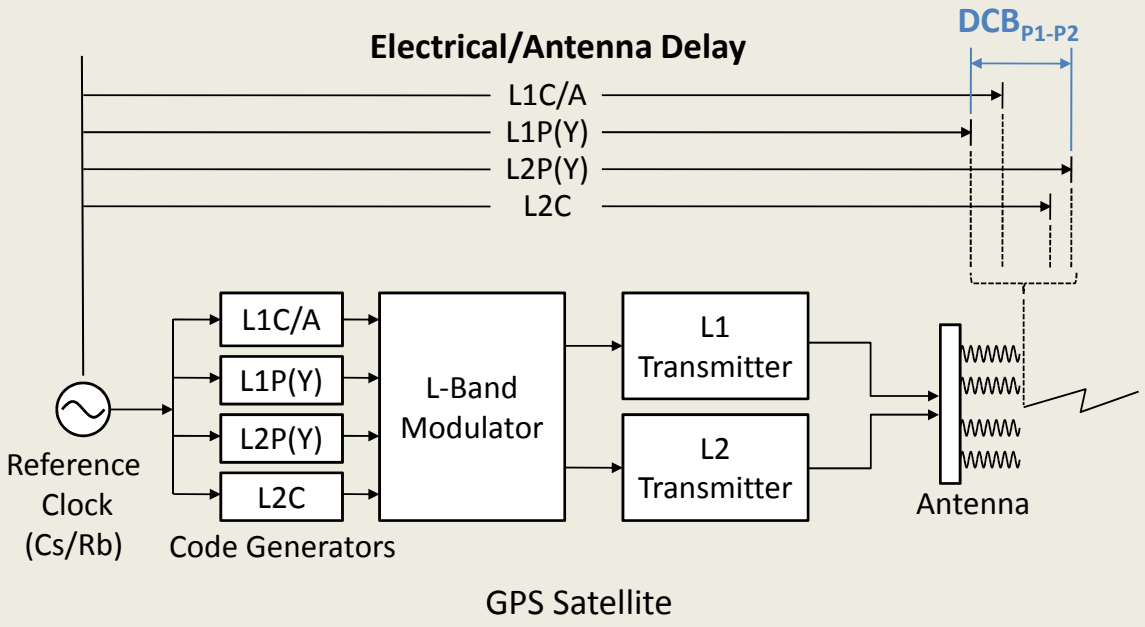
Relativity correction:

$$\Delta t_{rel} = \frac{-2\sqrt{\mu A}e\sin E}{c^2}$$

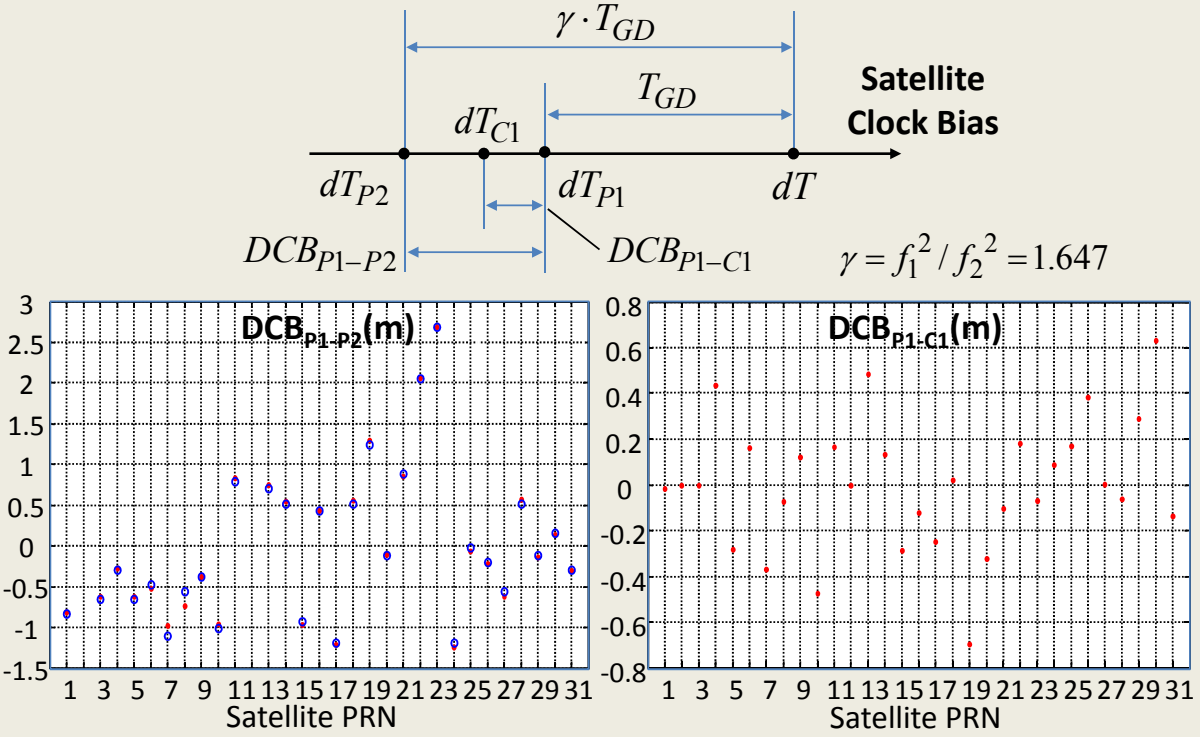
Differential code bias correction:

$$\Delta t_{GD} = \begin{cases} -T_{GD} & (L1) \\ -\gamma T_{GD} & (L2) \quad (\gamma = f_1^2 / f_2^2) \\ 0 & (LC) \end{cases}$$

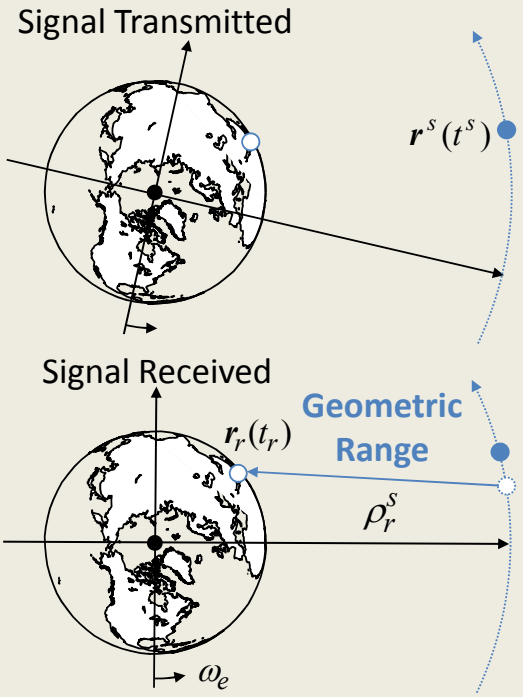
Differential Code Bias 1



Differential Code Bias 2



Geometric Range



Signal Transmission Time

$$t^s = \bar{t}_r - P_r^s / c - dT(t^s)$$

(1)
$$\rho_r^s = \left\| \boldsymbol{U}(t_r)\boldsymbol{r}_r(t_r) - \boldsymbol{U}(t^s)\boldsymbol{r}^s(t^s) \right\|$$

(2)
$$\rho_r^s \approx \left\| \boldsymbol{r}_r(t_r) - \boldsymbol{R}_z(\omega_e(t_r - t^s))\boldsymbol{r}^s(t^s) \right\|$$

(3)
$$\rho_r^s \approx \left\| \boldsymbol{r}_r(t_r) - \boldsymbol{R}_z(\omega_e \rho_r^s / c)\boldsymbol{r}^s(t^s) \right\|$$

(4)
$$\rho_r^s \approx \left\| \boldsymbol{r}_r(t_r) - \boldsymbol{r}^s(t^s) \right\| + \frac{\omega_e(x^s y_r - y^s x_r)}{c}$$

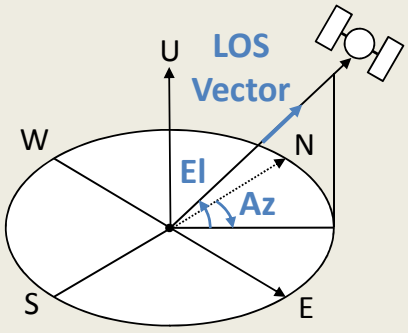
Sagnac Effect Correction

LOS Vector

Line-of-Sight (LOS) Vector:

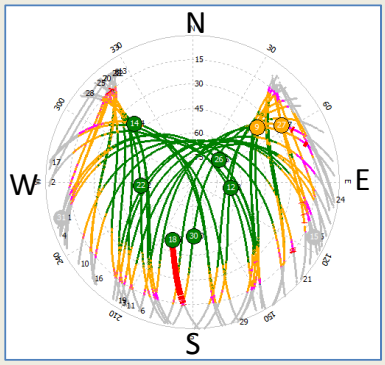
$$\boldsymbol{e}_r^s = \frac{\boldsymbol{r}^s - \boldsymbol{r}_r}{\left\| \boldsymbol{r}^s - \boldsymbol{r}_r \right\|}, \quad \boldsymbol{e}_{r,enu}^s = \boldsymbol{E}_{ecef \to enu}\boldsymbol{e}_r^s = (e_e, e_n, e_u)^T$$

$$\boldsymbol{E}_{ecef \to enu} = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{pmatrix}$$



Satellite Azimuth/Elevation Angle:

$$Az = \mathrm{ATAN2}(e_e, e_n)$$
$$El = \arcsin e_u$$

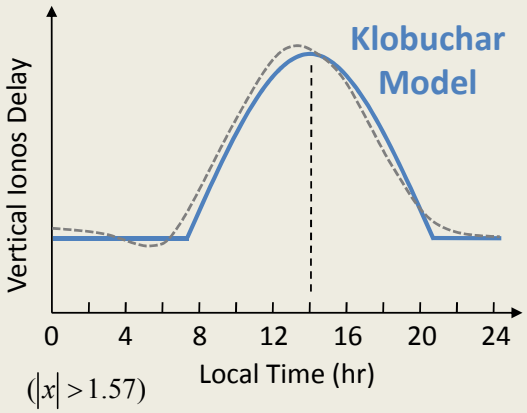


Ionospheric Model

$$\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3$$

Klobuchar Model:

$$\psi = 0.0137/(El + 0.11) - 0.022$$
$$\phi_i = \phi + \psi \cos Az$$
$$\lambda_i = \lambda + \psi \sin Az / \cos\phi_i$$
$$\phi_m = \phi_i + 0.064\cos(\lambda_i - 1.617)$$
$$t = 4.32\times10^4\,\lambda_i + t$$
$$F = 1.0 + 16.0\times(0.53 - El)^3$$
$$x = 2\pi(t - 50400)/\sum_{n=0}^{3}\beta_n{\phi_m}^n$$
$$I = \begin{cases} F\times5\times10^{-9} & (|x| > 1.57) \\ F\times\left(5\times10^{-9} + \sum_{n=1}^{4}\alpha_n{\phi_m}^n\times\left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)\right) & (|x| \le 1.57) \end{cases}$$



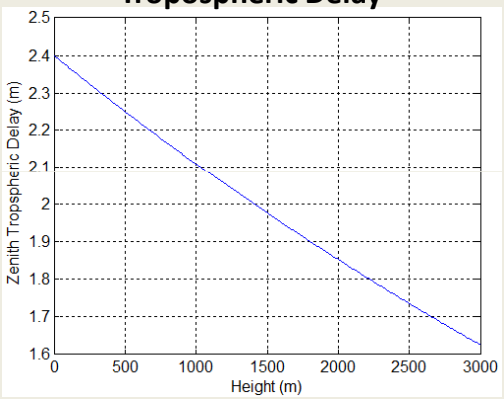
Troposphere Model

Standard Atmosphere:

$$p = 1013.25\times(1 - 2.2557\times10^{-5}H)^{5.2568}$$
$$T = 15.0 - 6.5\times10^{-3}H + 273.15$$
$$e = 6.108\times\exp\left\{\frac{17.15\mathrm{T} - 4684.0}{\mathrm{T} - 38.45}\right\}\times\frac{h_{rel}}{100}$$

p : Pressure (hPa)
H : Geopotential Height (m)
T : Temperature (K)
e : Partial Pressure of WV (hPa)
h_{rel} : Relative Humidity (%)

Tropospheric Delay



h_{rel}=50%, z=90°

Saastamoinen Model:

$$T_r^s = \frac{0.002277}{\cos z}\left\{p + \left(\frac{1255}{T} + 0.05\right)e - \tan^2 z\right\}$$

(z : Zenith Angle)

Navigation Processing

LSE: Least Square Estimation

Measurement Equation:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$$

$\boldsymbol{y}$: Measurement vector $\quad \boldsymbol{H}$: Design matrix

$\boldsymbol{x}$: Parameter vector $\quad \boldsymbol{v}$: Residual vector

$$J_{LS} = v_1^2 + v_2^2 + \ldots + v_m^2 = \boldsymbol{v}^T\boldsymbol{v} = (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^T(\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})$$

$$= \boldsymbol{y}^T\boldsymbol{y} - \boldsymbol{y}^T\boldsymbol{H}\boldsymbol{x} - \boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{y} + \boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{H}\boldsymbol{x} \rightarrow \min$$

$$\frac{\partial J_{LS}}{\partial \boldsymbol{x}} = \boldsymbol{0}^T - \boldsymbol{y}^T\boldsymbol{H} - (\boldsymbol{H}^T\boldsymbol{y})^T + (\boldsymbol{H}^T\boldsymbol{H}\boldsymbol{x})^T + \boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{H}$$

$$= -2\boldsymbol{y}^T\boldsymbol{H} + 2\boldsymbol{x}^T\boldsymbol{H}^T\boldsymbol{H} = \boldsymbol{0}$$

Normal Equation (NEQ):

$$\boldsymbol{H}^T\boldsymbol{H}\hat{\boldsymbol{x}} = \boldsymbol{H}^T\boldsymbol{y} \rightarrow \hat{\boldsymbol{x}} = (\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{y}$$

Weighted LSE:

$$\hat{\boldsymbol{x}} = (\boldsymbol{H}^T\boldsymbol{W}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{W}\boldsymbol{y} \quad (J_{WLS} = \boldsymbol{v}^T\boldsymbol{W}\boldsymbol{v} \rightarrow \min)$$

Non-linear LSE

Measurement Equation:

$$y = h(x) + v$$

$$h(x) = h(x_0) + H(x - x_0) + \ldots \quad \text{: Taylor Polynomial}$$

$$y \approx h(x_0) + H(x - x_0) + v$$
$$y - h(x_0) = H(x - x_0) + v$$
$$H^T H(\hat{x} - x_0) = H^T (y - h(x_0))$$

$$\hat{x} = x_0 + (H^T H)^{-1} H^T (y - h(x_0))$$

$$\left(H = \frac{\partial h(x)}{\partial x}\bigg|_{x=x_0} \right)$$

Partial Derivatives

Iterative Solution (Gauss-Newton):

$$\hat{x}_{i+1} = \hat{x}_i + (H^T H)^{-1} H^T (y - h(\hat{x}_i))$$
$$\hat{x} = \lim_{i \to \infty} \hat{x}_i$$

Numerical Solution of LSE

Solution	Matlab Notation	Exec. Time
NEQ+Gauss	x=(H'*H)¥(H'*y);	5.3 s
NEQ+LU-Decomp	x=inv(H'*H)*(H'*y);	6.0 s
NEQ+Cholesky	R=chol(H'*H); x=R¥(R'¥(H'*y));	**5.2 s**
QR-Decomp	[Q,R]=qr(H,0); x=R¥(Q'*y);	11.8 s
SVD	[U,D,V]=svd(H,0); x=V*(D¥(U'*y));	46.4 s
Pseudo-Inverse	x=pinv(H)*y;	51.9 s

size(H)=[5000,1000], Pentium 4 3.2GHz, Matlab 6.5.1, Windows XP Pro

Kalman Filter

Discrete Kalman Filter:

System Model: $\boldsymbol{x}_k = \boldsymbol{\Phi}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1} \quad (\boldsymbol{w}_{k-1} \approx \boldsymbol{N}(0, \boldsymbol{Q}_{k-1}))$

Measurement Model: $\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \quad (\boldsymbol{v}_k \approx \boldsymbol{N}(0, \boldsymbol{R}_k))$

Time Update (Projection):

$$\hat{\boldsymbol{x}}_k(-) = \boldsymbol{\Phi}_{k-1}\hat{\boldsymbol{x}}_k(+)$$
$$\boldsymbol{P}_k(-) = \boldsymbol{\Phi}_{k-1}\boldsymbol{P}_{k-1}(+)\boldsymbol{\Phi}_{k-1}^{\ T} + \boldsymbol{Q}_{k-1}$$

$\hat{\boldsymbol{x}}_k(-), \boldsymbol{P}_k(-)$

Measurement Update:

$$\boldsymbol{K}_k = \boldsymbol{P}_k(+)\boldsymbol{H}_k^{\ T}(\boldsymbol{H}_k\boldsymbol{P}_k(+)\boldsymbol{H}_k^{\ T} + \boldsymbol{R}_k)^{-1}$$
$$\hat{\boldsymbol{x}}_k(+) = \hat{\boldsymbol{x}}_k(-) + \boldsymbol{K}_k(\boldsymbol{y}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_k(-))$$
$$\boldsymbol{P}_k(+) = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_k(-)$$

$\hat{\boldsymbol{x}}_k(+), \boldsymbol{P}_k(+)$

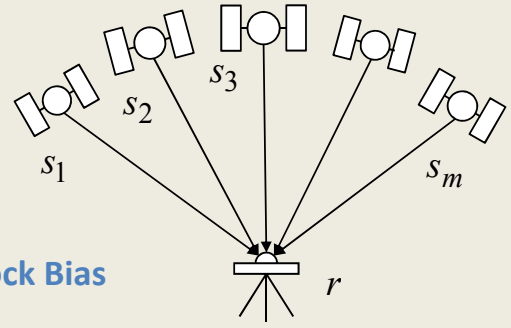
$\boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{y}_3, \ldots$ → $\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{x}}_2, \hat{\boldsymbol{x}}_3, \ldots$

Navigation Processing

$$\boldsymbol{x} = (\boldsymbol{r}_r^{\ T}, cdt)^T, \quad \boldsymbol{y} = (P_r^{s_1}, P_r^{s_2}, P_r^{s_3}, \ldots, P_r^{s_m})^T$$

$$\boldsymbol{h}(\hat{\boldsymbol{x}}) = \begin{pmatrix} \rho_r^{s_1} + c\hat{dt} - cdT^{s_1} + I_r^{s_1} + T_r^{s_1} \\ \rho_r^{s_2} + c\hat{dt} - cdT^{s_2} + I_r^{s_2} + T_r^{s_2} \\ \rho_r^{s_3} + c\hat{dt} - cdT^{s_3} + I_r^{s_3} + T_r^{s_3} \\ \vdots \\ \rho_r^{s_m} + c\hat{dt} - cdT^{s_m} + I_r^{s_m} + T_r^{s_m} \end{pmatrix} \quad \boldsymbol{H} = \begin{pmatrix} -\boldsymbol{e}_r^{s_1\,T} & 1 \\ -\boldsymbol{e}_r^{s_2\,T} & 1 \\ -\boldsymbol{e}_r^{s_3\,T} & 1 \\ \vdots & \vdots \\ -\boldsymbol{e}_r^{s_m\,T} & 1 \end{pmatrix}$$

$$\hat{\boldsymbol{x}}_0 = (0,0,0,0)^T$$
$$\hat{\boldsymbol{x}}_{i+1} = \hat{\boldsymbol{x}}_i + (\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T(\boldsymbol{y} - \boldsymbol{h}(\hat{\boldsymbol{x}}_i))$$
$$\hat{\boldsymbol{x}} = \lim_{i\to\infty}\hat{\boldsymbol{x}}_i = (\hat{\boldsymbol{r}}_r^{\ T}, c\hat{dt})^T$$



Single-Point Solution + Receiver Clock Bias

Partial Derivatives

Partial Derivatives of Range by Receiver Position:

$$\boldsymbol{r}_r \equiv (x_r, y_r, z_r)^T,\ \boldsymbol{r}^s \equiv (x^s, y^s, z^s)^T$$

$$\frac{\partial \rho_r^s}{\partial x_r} = \frac{\partial \sqrt{(x^s - x_r)^2 + (y^s - y_r)^2 + (z^s - z_r)^2}}{\partial x_r}$$

$$= \frac{1}{2}\left\{(x^s - x_r)^2 + (y^s - y_r)^2 + (z^s - z_r)^2\right\}^{-1/2} \frac{\partial (x^s - x_r)^2}{\partial x_r}$$

$$= \frac{-2(x^s - x_r)}{2\sqrt{(x^s - x_r)^2 + (y^s - y_r)^2 + (z^s - z_r)^2}} = \frac{-(x^s - x_r)}{\rho_r^s}$$

$$\frac{\partial \rho_r^s}{\partial \boldsymbol{r}_r} = \left(\frac{\partial \rho_r^s}{\partial x_r}, \frac{\partial \rho_r^s}{\partial y_r}, \frac{\partial \rho_r^s}{\partial z_r}\right) = -\left(\frac{x^s - x_r}{\rho_r^s}, \frac{y^s - y_r}{\rho_r^s}, \frac{z^s - z_r}{\rho_r^s}\right) = -\frac{(\boldsymbol{r}^s - \boldsymbol{r}_r)^T}{\rho_r^s} = -\boldsymbol{e}_r^{sT}$$

LOS Vector

Solution Convergence

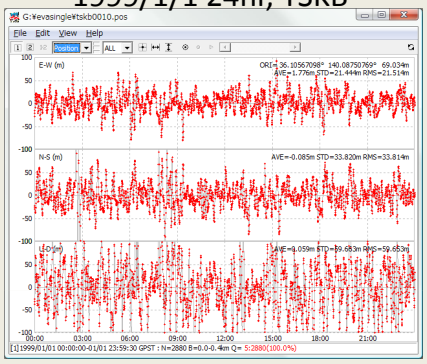
Estimated Parameters in LSE Iteration Loop

i	x (m)	y (m)	z (m)	cdt (m)
(0) X=	0.0000000	0.0000000	0.0000000	0.0000000
(1) X=	-4739338.8790644	3968053.3426383	4470195.0681293	1290751.6350707
(2) X=	-3990084.5939062	3334559.7805777	3763444.6383541	50195.3310677
(3) X=	-3957255.7455862	3310242.1098583	3737755.6233736	510.7878812
(4) X=	-3957205.2229884	3310203.7001970	3737718.0508664	432.5789153
(5) X=	-3957205.1820501	3310203.6651692	3737718.0078941	432.4910365
(6) X=	-3957205.1820116	3310203.6651363	3737718.0078537	432.4909539
(7) X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538
(8) X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538
(9) X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538
(10) X=	-3957205.1820116	3310203.6651363	3737718.0078536	432.4909538

2001/1/1 0:00:00, TKSB, processed by RTKLIB 2.2.1, n=8

Single-Point Solution

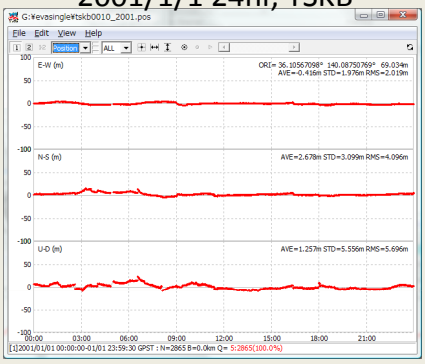
1999/1/1 24hr, TSKB



RMS Error:
E: 21.51m
N: 33.81m
U: 59.65m

100m

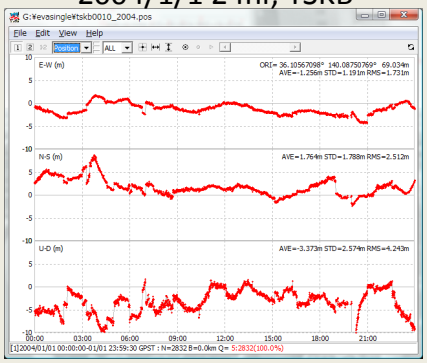
2001/1/1 24hr, TSKB



RMS Error:
E: 2.02m
N: 4.10m
U: 5.70m

100m

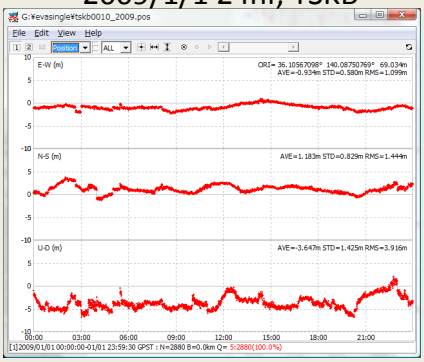
2004/1/1 24hr, TSKB



RMS Error:
E: 1.73m
N: 2.51m
U: 4.24m

10m

2009/1/1 24hr, TSKB



RMS Error:
E: 1.10m
N: 1.44m
U: 3.92m

10m

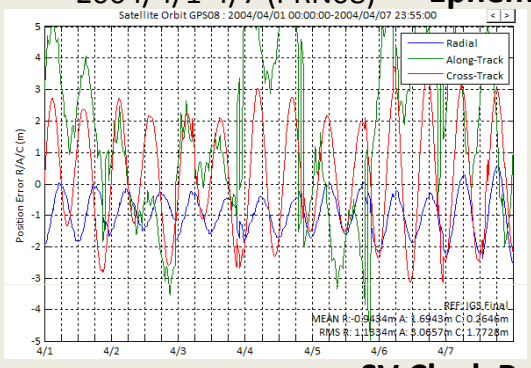
Error Sources

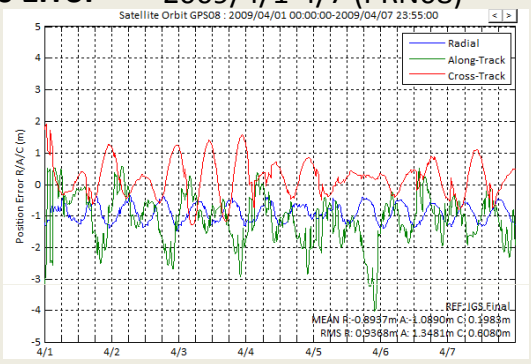
Error Sources

- Error sources of GPS positioning
  - Ephemeris
  - SV clock Parameter
  - Ionospheric Delay
  - Tropospheric Delay
  - Multipath
  - S/A: Selective Availability
  - Code Tracking Noise
- Satellites-Receiver Geometry

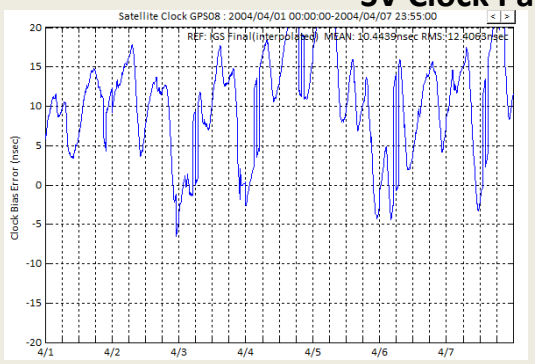
Ephemeris/SV Clock Parameter

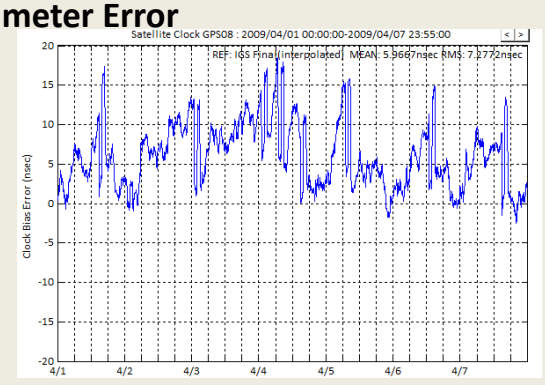
2004/4/1-4/7 (PRN08) **Ephemeris Error** 2009/4/1-4/7 (PRN08)



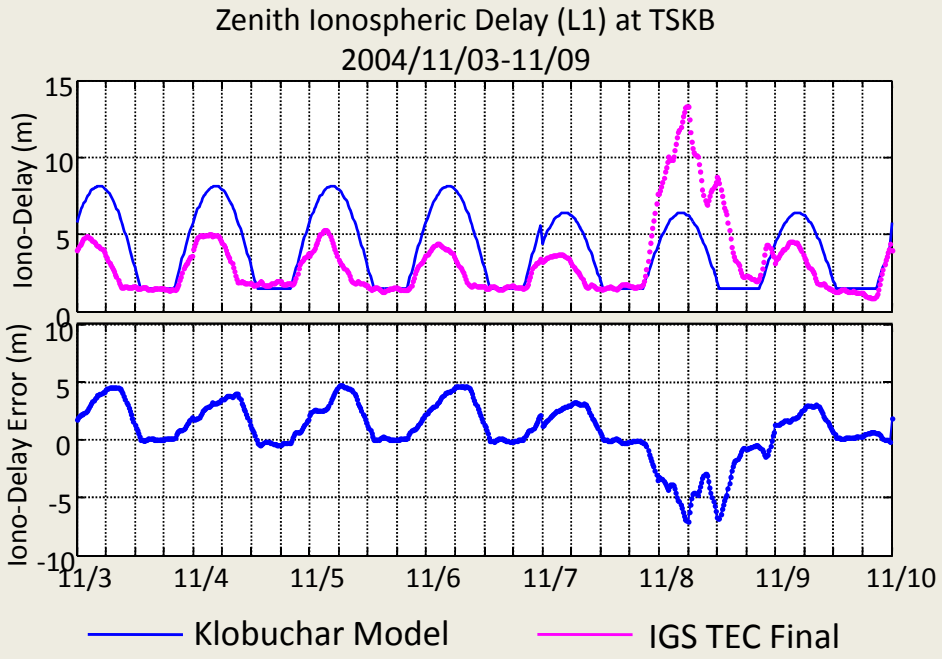


SV Clock Parameter Error





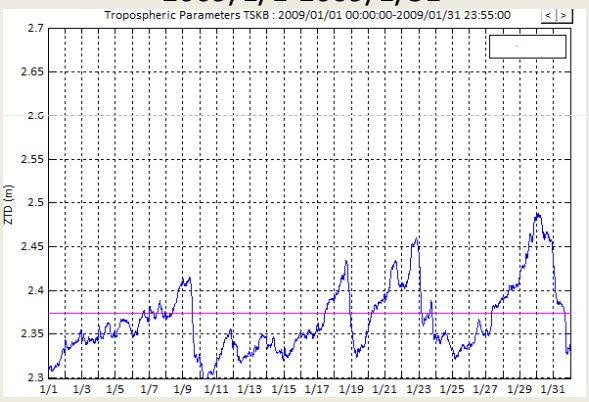
Ionospheric Error



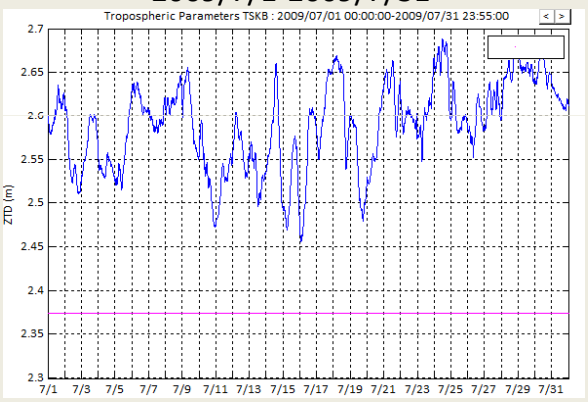
Tropospheric Error

ZTD (Zenith Total Delay) at TSKB

2009/1/1-2009/1/31



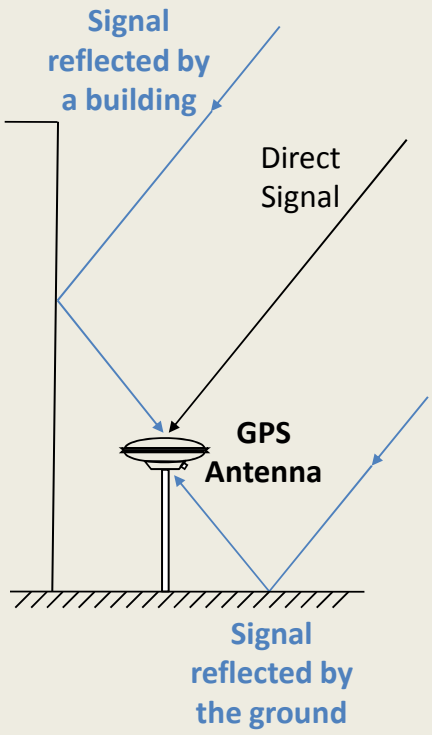
2009/7/1-2009/7/31



Saastamoinen Model

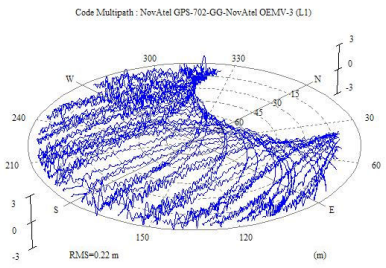
Estimated by PPP

Multipath

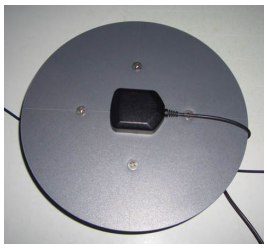


Geodetic-Grade Antenna

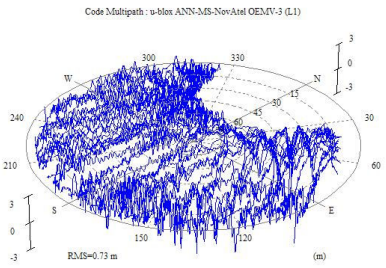
NovAtel GPS-702-GG



Consumer-Grade Antenna



u-blox ANN-MS



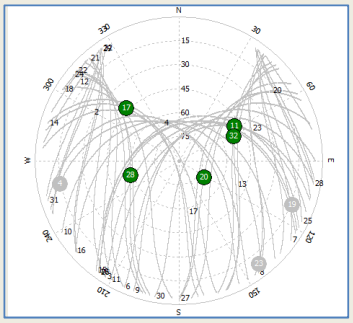
Satellites-Receiver Geometry

Dilution of Precision (DOP):

$$GDOP = \sqrt{q_{ee} + q_{nn} + q_{uu} + q_{tt}}$$
$$PDOP = \sqrt{q_{ee} + q_{nn} + q_{uu}}$$
$$HDOP = \sqrt{q_{ee} + q_{nn}}$$
$$VDOP = \sqrt{q_{uu}}$$

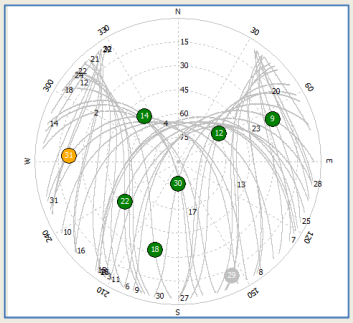
$$\boldsymbol{Q} = (\boldsymbol{H}^T \boldsymbol{H})^{-1} = \begin{pmatrix} q_{ee} & q_{en} & q_{eu} & q_{et} \\ q_{ne} & q_{nn} & q_{nu} & q_{nt} \\ q_{ue} & q_{un} & q_{uu} & q_{ut} \\ q_{te} & q_{tn} & q_{tu} & q_{tt} \end{pmatrix} \quad \boldsymbol{H} = \begin{pmatrix} -\boldsymbol{e}_{r,enu}^{s_1\,T} & 1 \\ -\boldsymbol{e}_{r,enu}^{s_2\,T} & 1 \\ \vdots & \vdots \\ -\boldsymbol{e}_{r,enu}^{s_m\,T} & 1 \end{pmatrix}$$

# of satellites = 5



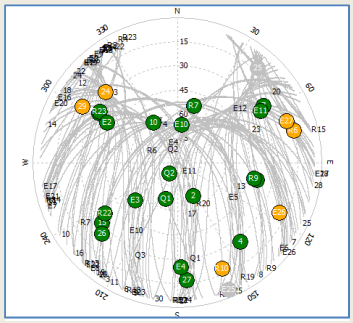
GDOP=33.4 PDOP=25.9
HDOP=8.1 VDOP=24.7

# of satellites = 7



GDOP=2.5 PDOP=2.1
HDOP=1.2 VDOP=1.8

# of satellites = 27



GDOP=1.2 PDOP=1.0
HDOP=0.5 VDOP=0.9

Positioning Error Budget

Error Source	Single-Point		DGPS (BL=100km)		SBAS	
Ephemeris Error	1.0 m		0.1 m		0.1 m	
SV Clock Param Error			0.0 m			
Ionospheric Error	1.5 m		0.2 m		0.2 m	
Tropospheric Error	0.3 m		0.1 m		0.3 m	
Multipath	1.0 m		1.2 m		1.0 m	
S/A	0.0 m		0.0 m		0.0 m	
Rcv Tracking Noise	0.3 m		0.3 m		0.3 m	
UERE	2.1 m		1.3 m		1.1 m	
HDOP/VDOP	1.5	2.5	1.5	2.5	1.5	2.5
Horizontal/Vertical RMS Error	**3.2 m**	**5.3 m**	**2.0 m**	**3.3 m**	**1.7 m**	**2.8 m**

DGPS and SBAS

DGPS

- Differential GPS
  - Fixed Reference Stations at Known Position
  - Generate Correction Messages
  - Broadcast Correction Messages to User
  - Eliminate Most of Errors of Positioning
- Service of DGPS
  - Space Based DGPS: OmniSTAR, SkyFix, StarFix
  - Maritime DGPS: Marine Beacons
  - National DGPS: VHF-band, Cellular Network, Internet
  - GDGPS: StarFire

RTCM SC-104

RTCM 2.3 Messages		RTCM 3.1 Messages	
Type	Message	Type	Message
1	Differential GPS Corrections	1001	L1-Only GPS RTK Observables
3	GPS Reference Station Parameters	1002	Extended L1-Only GPS RTK Observables
10	P-Code Differential Corrections	1003	L1&L2 GPS RTK Observables
11	C/A-Code L1, L2 Delta Corrections	1004	Extended L1&L2 GPS RTK Observables
17	GPS Ephemerides	1005	Stationary RTK Reference Station ARP
18	RTK Uncorrected Carrier Phase	1006	Stationary RTK Ref. Stn. ARP with Hgt.
19	RTK Uncorrected Pseudorange	1007	Antenna Descriptor
20	RTK Carrier Phase Corrections	1008	Antenna Descriptor & Serial Number
21	RTK Pseudorange Corrections	1013	System Parameters
22	Extended Reference Station Parameter	1014	Network Auxiliary Station Data
23	Antenna Type Definition Record	1015	GPS Ionospheric Correction Differences
24	Antenna Reference Point (ARP)	1016	GPS Geometric Correction Differences
59	Proprietary Messages	1019	GPS Ephemerides

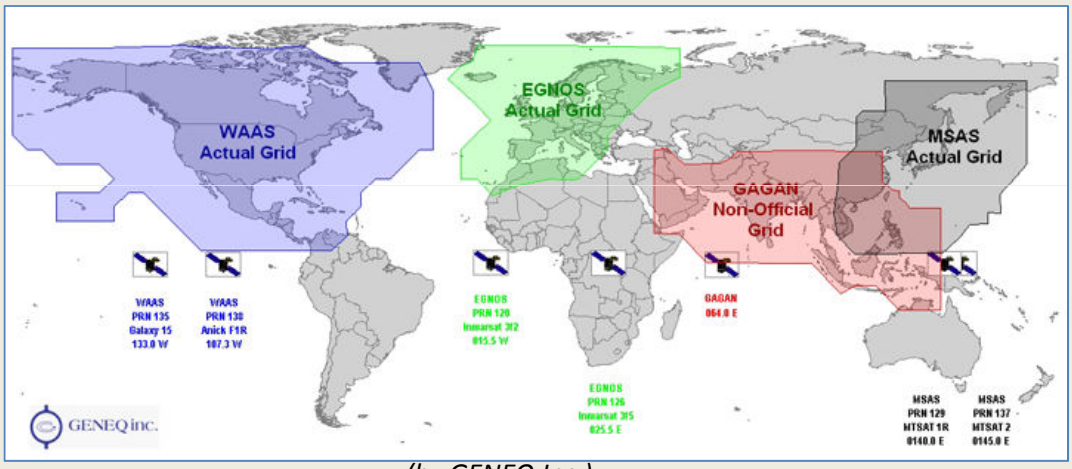
RTCM: The Radio Technical Commission for Marine Service

SBAS

SBAS: Satellite Based Augmentation Systems

System	Development	Operation	GEO Satellite		
			PRN	Name	Location
WAAS	US, DOT, FAA	2003/7-	135	Galaxy 15	133W
			138	Anik F1R	107.3W
EGNOS	ESA, EC, Eurocontrol	2005/7- (IOC)	120	Inmarsat-3 AOR-E	15.5W
			124	Artemis	21.5E
			126	Inmarsat-3 IOR-W	25E
MSAS	Japan, JCAB	2007/9-	129	MTSAT-IR	140E
			137	MTSAT-II	145E
GAGAN	India, AAI, ISRO	2011-	127	GSAT-4?	?

SBAS Coverage Map

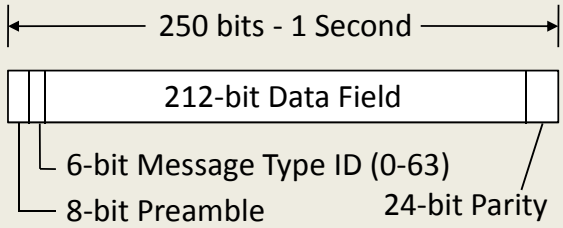


(by GENEQ Inc.)

SBAS Messages

RTCA/DO-229C

Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment
(Nov 28,2001)



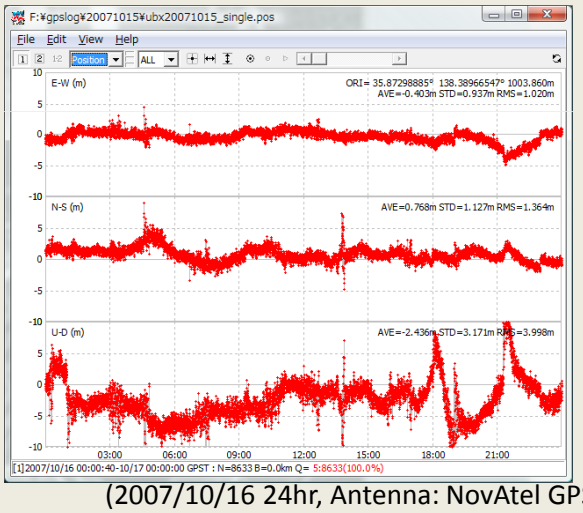
RTCA: Radio Technical Commission for Aeronautics

Type	Message
0	For WAAS Testing
1	PRN Mask assignment
2-5	Fast Corrections
6	Integrity Information
7	Fast Correction Degration Factor
9	GEO Navigation Messages
10	Degradation Parameters
12	WAAS Network Time/UTC Offset
17	GEO Satellite Almanac
18	Ionospheric Grid Mask
24	Mixed Fast/Long Term Satellite Correct.
25	Long Term Satellite Error Corrections
26	Ionospheric Delay Corrections
27	WAAS Service Messages

SBAS Performance

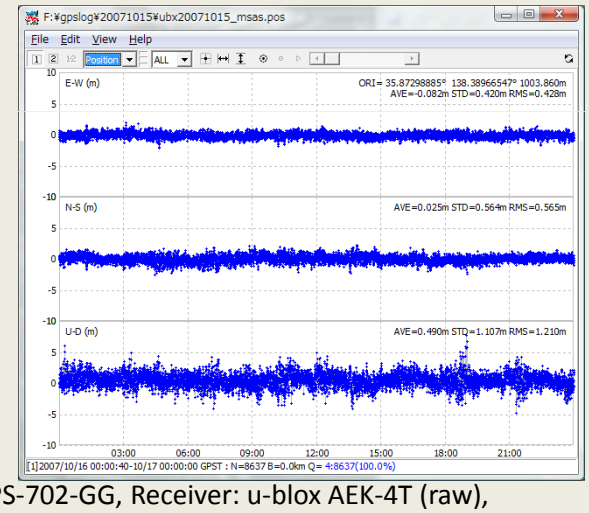
Single-Point

RMS Error:
E: 1.02m N: 1.36m U: 4.00m



MSAS

RMS Error:
E: 0.43m N: 0.57m U: 1.21m



(2007/10/16 24hr, Antenna: NovAtel GPS-702-GG, Receiver: u-blox AEK-4T (raw), Processing S/W: RTKLIB 2.1.0, All Corrections=ON, Ranging=ON)

Exercise 1

Using GT 0.6.4