2009年度GCOE地球惑星計測スクール

GPS Global Positioning System

Part 2

東京海洋大学 高須 知二

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Outline: Part 1

- **GPS/GNSS Basics and Principles**
  - GPS/GNSS Summary
  - GPS Signal and Receiver Architecture
  - Basic Observation Models
  - Navigation Processing
  - Error Sources
  - DGPS and SBAS
- **Exercise: using GT 0.6.4**

Outline: Part 2

- **Precise Positioning with GPS/GNSS**
  - Time and Coordinate Systems
  - Precise Measurement Models
  - Relative Positioning
  - Precise Point Positioning (PPP)
  - Applications
- **Exercise: PPP with GT 0.6.4**

Time and Coordinate Systems

Time Systems

- Time Systems
  - TAI: International Atomic Time
  - UTC: Coordinated Universal Time
  - Local Time (JST, EDT, ...)
  - UT0, UT1, UT2: Universal Time
  - GMST: Greenwich Mean Sidereal Time
  - GPS Time
  - GLONASS Time
  - ...

Time System Conversion

UTC-TAI (s)			
-25	1990/1/1-	-30	1996/1/1-
-26	1991/1/1-	-31	1997/7/1-
-27	1992/7/1-	-32	1999/1/1-
-28	1993/7/1-	-33	2006/1/1-
-29	1994/7/1-	-34	2009/1/1-

TAI to UTC:

$$t_{UTC} = t_{TAI} + (\underline{UTC - TAI})$$

UTC to UT1:

$$t_{UT1} = t_{UTC} + (\underline{UT1 - UTC})$$

UT1 to GMST:

$$GMST_{0h\,UT1} = 24110.54841 + 8640184.812866\,T'_u + 0.093104\,T'^{2}_u - 6.2\times10^{-6}\,T'^{3}_u$$

$$GMST = GMST_{0h\,UT1} + r(t_{UT1} - t_{0h\,UT1})$$

$$r = 1.002737909350795 + 5.9006\times10^{-11}\,T'_u - 5.9\times10^{-15}\,T'^{2}_u$$

$T'_u = d'_u / 36525$ $\quad d'_u$: number of days elapsed since 2000 Jan 1, 12h UT1

GPS Time to TAI:

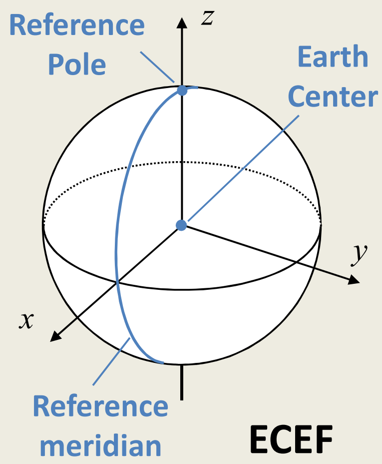
$$t_{TAI} \approx t_{GPST} + 19\,s$$

GPS Time to UTC:

$$t_{UTC} = t_{GPST} - (\underline{\Delta t_{LS}} + \underline{A_0} + \underline{A_1}(t_{GPST} - \underline{t_{ot}}))$$

Coordinate Systems

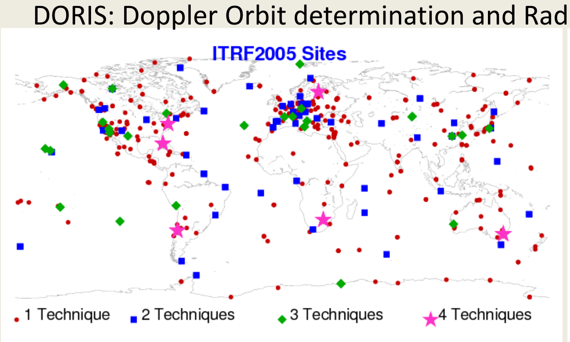
- ECEF: Earth-Centered Earth-Fixed
  - ITRF
  - WGS 84: US (GPS)
  - PZ90: Russia (GLONASS), ...
- ECI: Earth-Centered Inertial
  - ICRF: International Celestial Reference Frame
- ECI-ECEF Connection
  - Precession/Nutation Model
  - ERP: Earth Rotation Parameters

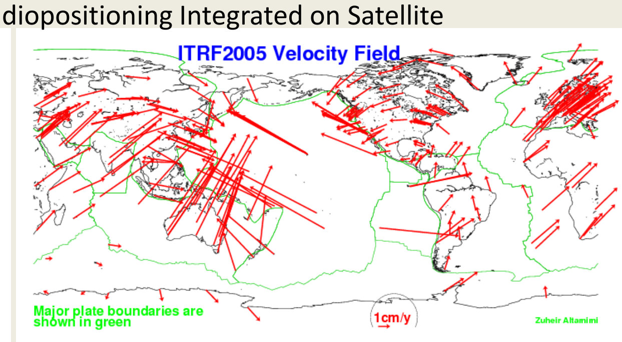


ITRF

- International Terrestrial Reference Frame
  - A "Realization" of ITRS Maintained by IERS
  - GPS, VLBI, SLR, DORIS Site Position/Velocity List
  - ITRF2005, ITRF2000, ITRF97, ITRF96, ...

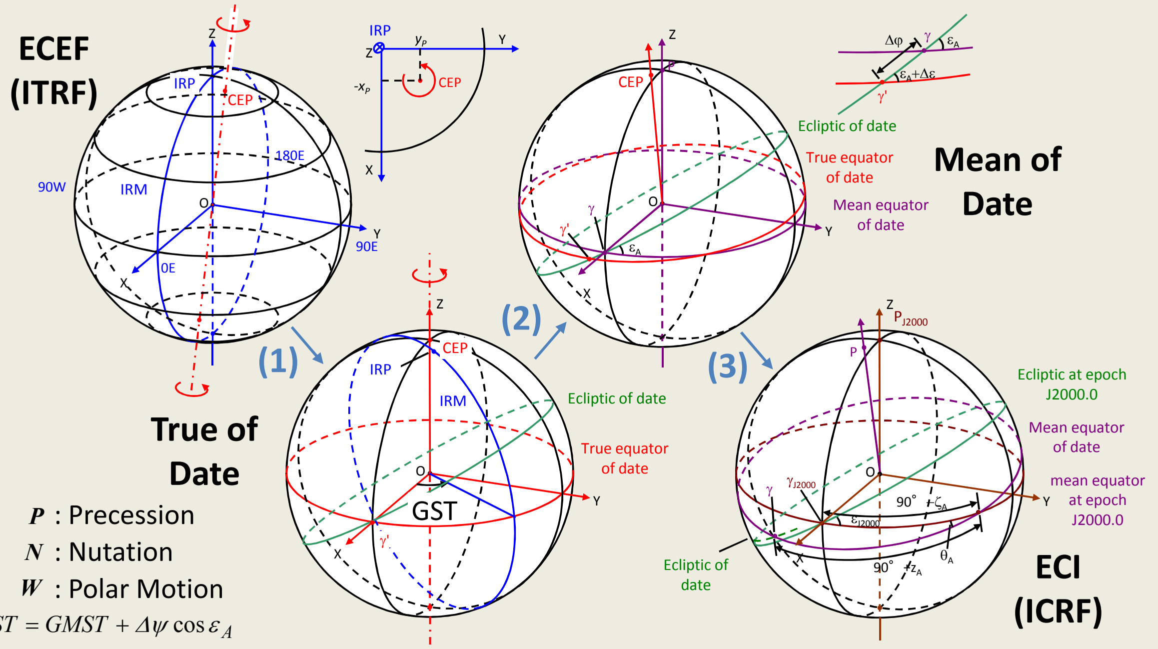
VLBI: Very Long Baseline Interferometry
SLR: Satellite Laser Ranging
ITRS: International Terrestrial Reference System
IERS: International Earth Rotation Service
DORIS: Doppler Orbit determination and Radiopositioning Integrated on Satellite





http://itrf.ensg.ign.fr/ITRF_solutions/2005/ITRF2005.php

ECEF to ECI Transformation



$\boldsymbol{P}$: Precession

$\boldsymbol{N}$: Nutation

$\boldsymbol{W}$: Polar Motion

$$GST = GMST + \Delta\psi \cos\varepsilon_A + 0''.00264 \sin\Omega + 0''.000063 \sin 2\Omega$$

$$\boldsymbol{r}_{eci} = \boldsymbol{PNR}_z(-GST)\boldsymbol{Wr}_{ecef}$$

$$= \underbrace{\boldsymbol{R}_Z(\zeta_A)\boldsymbol{R}_Y(-\theta_A)\boldsymbol{R}_Z(z_A)}_{(3)}\underbrace{\boldsymbol{R}_X(-\varepsilon_A)\boldsymbol{R}_Z(\Delta\varphi)\boldsymbol{R}_X(\varepsilon_A+\Delta\varepsilon)}_{(2)}\underbrace{\boldsymbol{R}_Z(-GST)}_{}\underbrace{\boldsymbol{R}_X(y_P)\boldsymbol{R}_Y(x_P)}_{(1)}\boldsymbol{r}_{ecef}$$

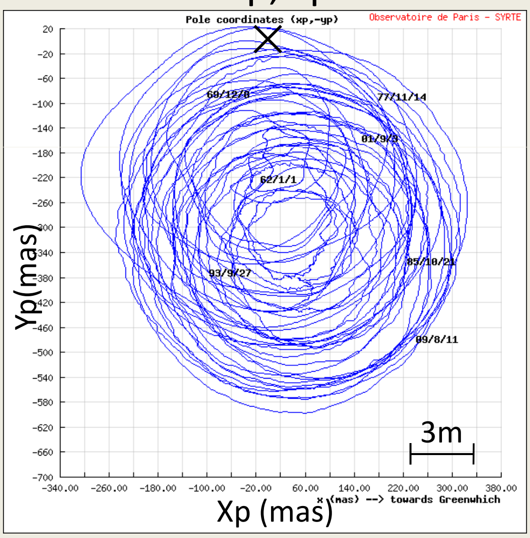
ERP: Earth Rotation Parameters

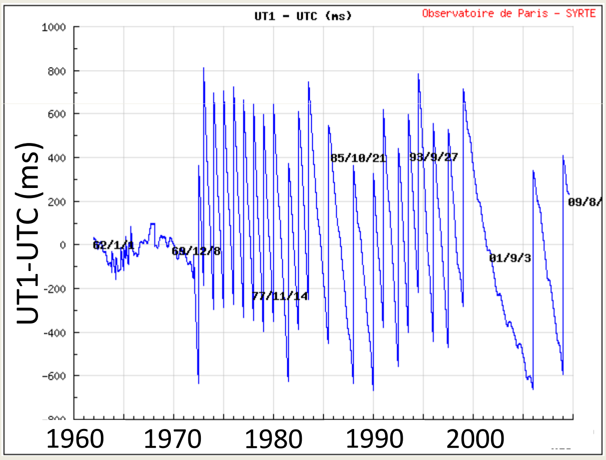
Polar Motion:

Xp, Yp

Earth Rotation Angle:

UT1-UTC

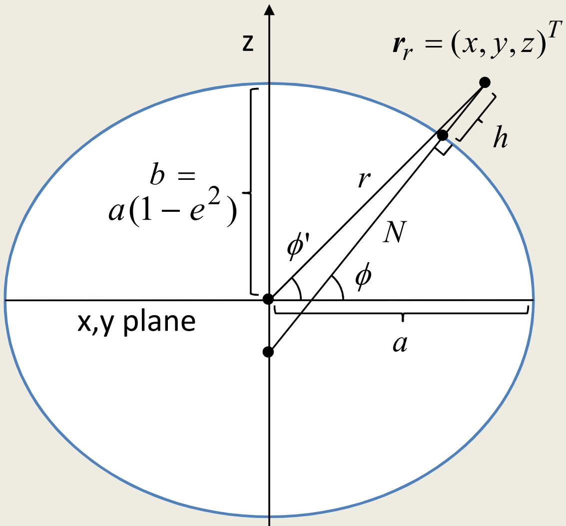




IERS C04 Series (1962/1/1-2009/8/11)

Ellipsoid and Datum

Ellipsoid:



ϕ' : Geocentric Latitude λ : Longitude
ϕ : Geodetic Latitude h : Ellipsoidal Height

	GRS 80	WGS 84
a (m)	6378137	6378137
f	1/298.257222 101	1/298.257223 563
GM (m^3/s^2)	3986005.000 x 10^8	3986004.418 x 10^8

Lat/Lon/Height to ECEF:

$$e^2 = f(2-f)$$

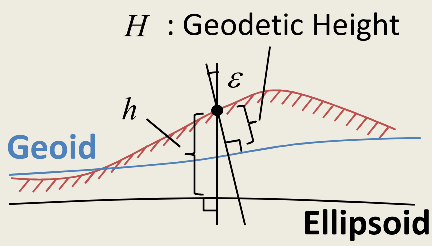
$$N = \frac{a}{\sqrt{1-e^2 \sin^2 \phi}}$$

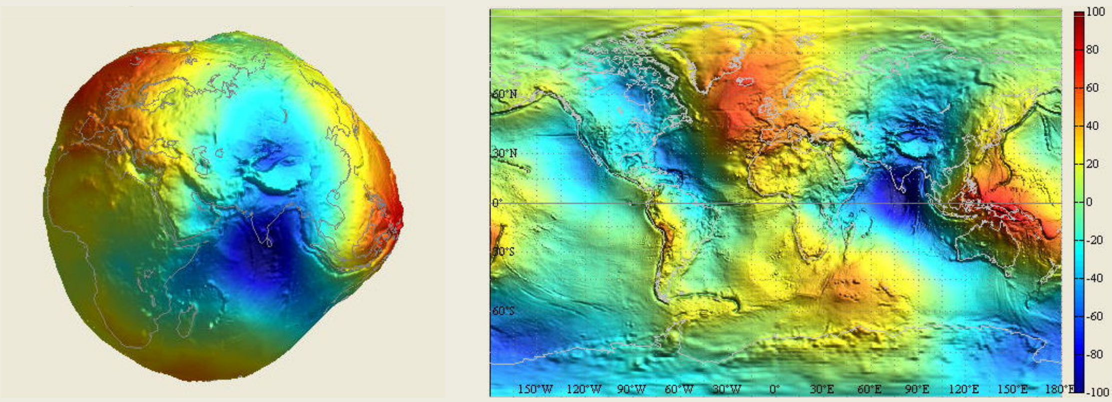
$$\boldsymbol{r}_r = \begin{pmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ (N(1+e^2)+h)\sin\phi \end{pmatrix}$$

Geoid

Geopotential:

$$V(r,\phi',\lambda) = \frac{GM}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n (\overline{C}_{nm}Y_{nmc} + \overline{S}_{nm}Y_{nms})\right\}$$





EGM96 Geoid Model

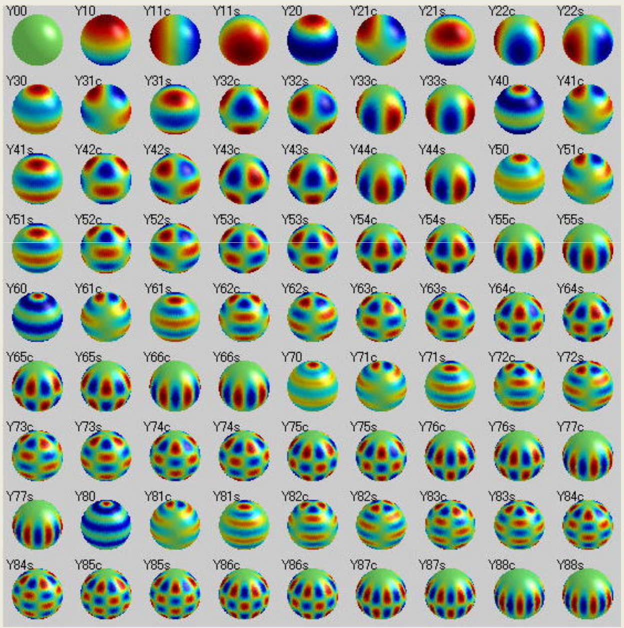
Spherical Harmonics

Spherical harmonic functions:

$$Y_{n0} = Y_{n0c}$$
$$Y_{nmc} = \overline{P}_{nm}(\sin\phi')\cos m\lambda$$
$$Y_{nms} = \overline{P}_{nm}(\sin\phi')\sin m\lambda$$

Legendre function:

$$\overline{P}_{nm} = N_{nm}P_{nm},\ P_{00}(x) = 1, P_{10}(x) = x$$
$$P_{n-1,n}(x) = 0,$$
$$P_{nn}(x) = (2n-1)(1-x^2)^{1/2}P_{n-1,n-1}(x)$$
$$P_{nm}(x) = \frac{(2n-1)xP_{n-1,m}(x) - (n+m-1)P_{n-2,m}(x)}{n-m}$$
$$N_{nm} = \begin{cases} \sqrt{2n+1} & (m=0) \\ \sqrt{\dfrac{2(2n+1)(n-m)!}{(n+m)!}} & (m>0) \end{cases}$$



Coordinates Transformation

Helmert Transformation (A to B):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_B = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + (1+D)\begin{pmatrix} 1 & -R_3 & R_2 \\ R_3 & 1 & -R_1 \\ -R_2 & R_1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_A$$

- T1, T2, T3 : Translation along coordinate axis
- D : Scale factor
- R1, R2, R3 : Rotation of coordinate axis (rad)

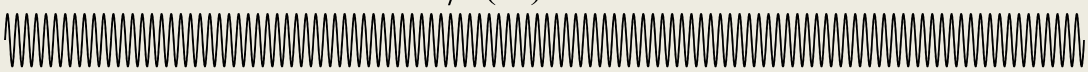
Coordinates		T1 (mm)	T2 (mm)	T3 (mm)	D (10^{-9})	R1 (mas)	R2 (mas)	R3 (mas)
A	B							
ITRF2005	ITRF2000	0.1	-0.8	-5.8	0.40	0.00	0.00	0.00
		-0.2/y	0.1/y	-1.8/y	0.08/y	0.00/y	0.00/y	0.00/y

(Epoch 2000.0)

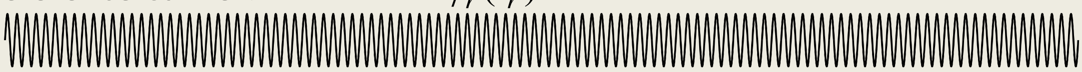
Precise Measurement Models

Carrier-Phase

Received Carrier Signal: $\phi^s(\bar{t}^s)$



Reference Carrier: $\phi_r(\bar{t}_r)$



Carrier Beat Signal: ϕ_r^s

$$\phi_r^s = \phi_r(t_r) - \phi^s(t^s) + N_r^s + \varepsilon_\phi \qquad (\phi_{r,0} = \phi_r(t_0), \phi_0^s = \phi^s(t_0))$$

$$= (f(t_r + dt_r - t_0) + \phi_{r,0}) - (f(t^s + dT^s - t_0) + \phi_0^s) + N_r^s + \varepsilon_\phi$$

$$= \frac{c}{\lambda}(t_r - t^s) + \frac{c}{\lambda}(dt_r - dT^s) + (\phi_{r,0} - \phi_0^s + N_r^s) + \varepsilon_\phi$$

$$\lambda\phi_r^s = c(t_r - t^s) + c(dt_r - dT^s) + \lambda(\phi_{r,0} - \phi_0^s + N_r^s) + \lambda\varepsilon_\phi$$

Carrier-Phase Model

Carrier-Phase Measurement Model:

$$\Phi_r^s \equiv \lambda\phi_r^s = \rho_r^s + c(dt_r - dT^s) - I_r^s + T_r^s + \lambda\underline{B_r^s} + \underline{d_r^s} + \varepsilon_\Phi$$

$\underline{B_r^s} = \phi_{r,0} - \phi_0^s + N_r^s$: Carrier-Phase Bias (cycles)

N_r^s : Integer Ambiguity

$\phi_{r,0}$: Receiver Initial Phase

ϕ_0^s : Satellite Initial Phase

$$\underline{d_r^s} = -\boldsymbol{d}_{r,pco}{}^T \boldsymbol{e}_{r,enu}^s + \left(\boldsymbol{E}_{sat\to ecef}\boldsymbol{d}_{pco}^s\right)^T \boldsymbol{e}_r^s + d_{r,pcv} + d_{pcv}^s - \boldsymbol{d}_{disp}{}^T \boldsymbol{e}_{r,enu}^s + d_{pw} + d_{rel}$$

$\boldsymbol{d}_{r,pco}$: Receiver Antenna Phase Center Offset

$d_{r,pcv}$: Receiver Antenna Phase Center Variation

$\boldsymbol{d}_{pco}^s$: Satellite Antenna Phase Center Offset

d_{pcv}^s : Satellite Antenna Phase Center Variation

$\boldsymbol{d}_{disp}$: Site Displacement

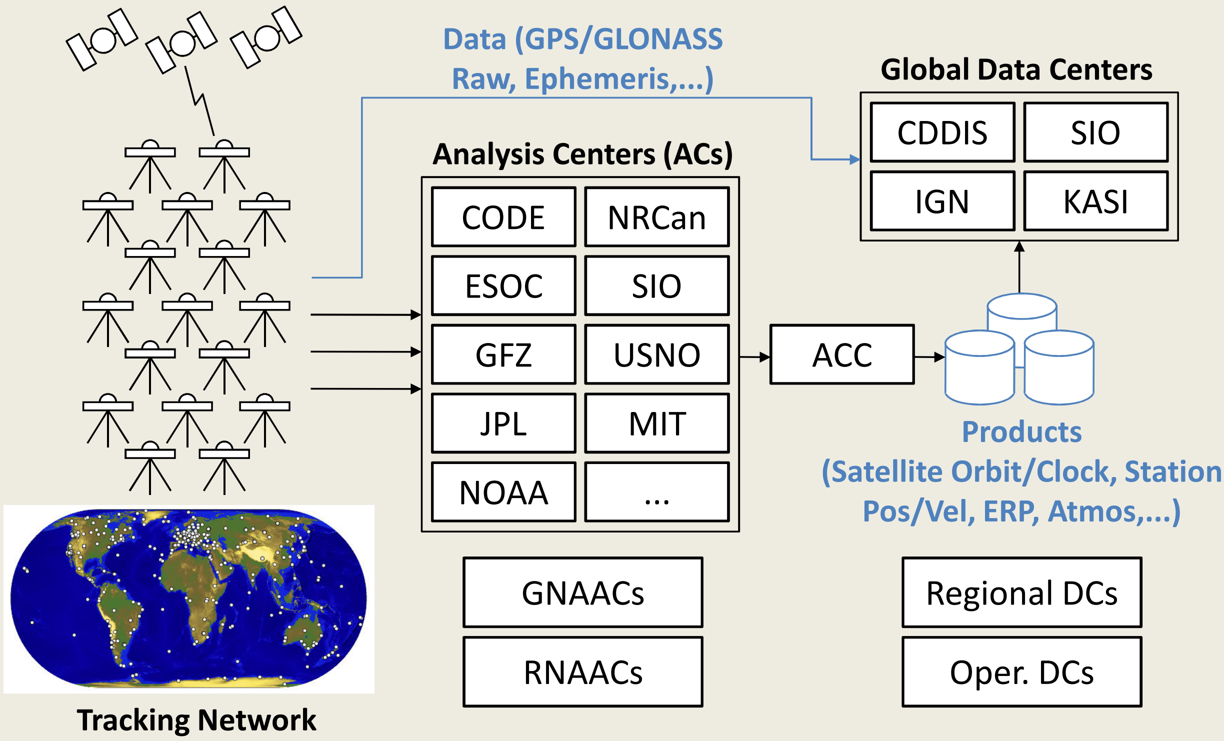
d_{pw} : Phase Wind-up Effect

d_{rel} : Relativistic Effect

Precise Ephemeris

- Precise Satellite Orbit and Clock
  - By Post-Processing or in Real-time
  - Observation Data of Tracking Stations World-Wide
- Format:
  - Orbit: NGS SP3
  - Clock: NGS SP3 or RINEX Clock Extension
- Contents:
  - Orbit: ECEF-Positions of Satellite Mass Center
  - Clock: Clock-biases wrt Time Scale Aligned to GPS Time

IGS: International GNSS Service



IGS Products

		Final (IGS)	Rapid (IGR)	Ultra-Rapid (IGU) Observed	Ultra-Rapid (IGU) Predicted	Broadcast
Accuracy	Orbit	~2.5cm	~2.5cm	~3cm	~5cm	~100cm
	Clock	~75ps RMS ~20ps STD	~75ps RMS ~25ps STD	~150ps RMS ~50ps STD	~3ns RMS ~1.5ns STD	~5ns RMS ~2.5ns STD
Latency		12-18 days	17-41 hours	3-9 hours	realtime	realtime
Updates		every Thursday	at 17 UTC daily	at 03, 09, 15, 21 UTC	at 03, 09, 15, 21 UTC	-
Sample Interval	Orbit	15min	15min	15min	15min	daily
	Clock	Sat: 30s Stn: 5min	5min	15min	15min	daily

(2009/8, http://igscb.jpl.nasa.gov/)

Interpolation of Satellite Orbit

Lagrange Interpolation:

$$\boldsymbol{r}^s(t)=\frac{(t-t_2)(t-t_3)\ldots(t-t_{n+1})}{(t_1-t_2)(t_1-t_3)\ldots(t_1-t_{n+1})}\boldsymbol{r}^s(t_1)+\frac{(t-t_1)(t-t_3)\ldots(t-t_{n+1})}{(t_2-t_1)(t_2-t_3)\ldots(t_2-t_{n+1})}\boldsymbol{r}^s(t_2)$$
$$+\frac{(t-t_1)(t-t_2)\ldots(t-t_{n+1})}{(t_3-t_1)(t_3-t_2)\ldots(t_3-t_{n+1})}\boldsymbol{r}^s(t_3)+\ldots+\frac{(t-t_1)(t-t_2)\ldots(t-t_n)}{(t_{n+1}-t_1)(t_{n+1}-t_2)\ldots(t_{n+1}-t_n)}\boldsymbol{r}^s(t_{n+1})$$

Interpolation Error of 15-min Sample Orbit

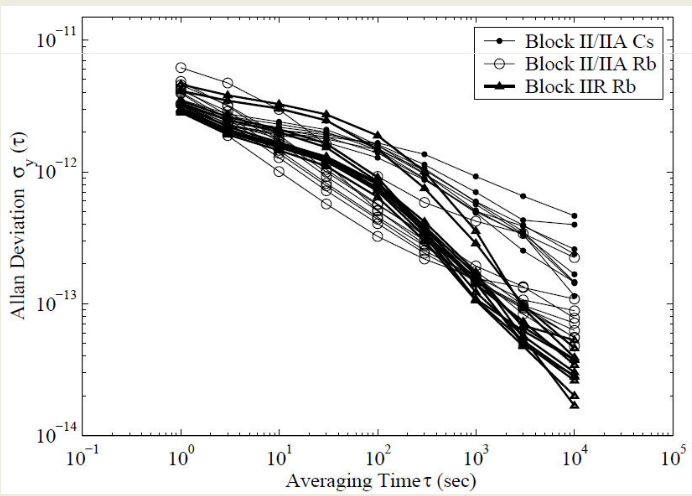
Degree of Polynomial	Position RMS Error (cm)			Velocity RMS Error (cm/s)		
	Radial	Along-Trk	Cross-Trk	Radial	Along-Trk	Cross-Trk
n=5	72.10	73.84	57.48	0.253	0.260	0.202
n=6	7.31	6.89	5.75	0.032	0.031	0.025
n=7	0.63	0.63	0.50	0.017	0.019	0.014
n=8	0.08	0.11	0.08	0.017	0.018	0.013
n=9	0.05	0.11	0.05	0.017	0.018	0.013
n=10	0.05	0.10	0.06	0.017	0.018	0.013
n=11	0.05	0.12	0.06	0.017	0.018	0.013

Interpolation of Satellite Clock

Linear Interpolation:

$$dT^s(t)=\frac{(t_2-t)\,dT^s(t_1)+(t-t_1)\,dT^s(t_2)}{t_2-t_1}\quad(t_1\le t<t_2)$$

Satellite Clock Stability:



Ionospheric Delay

Ionospheric Group Delay Model:

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{{Y_T}^2}{2(1-X-iZ)} \pm \sqrt{\frac{{Y_T}^4}{4(1-X-iZ)^2} + {Y_L}^2}} \approx 1 - X = 1 - f_N^2 / f^2 \ (L-band)$$

: Appleton-Hartree Formula

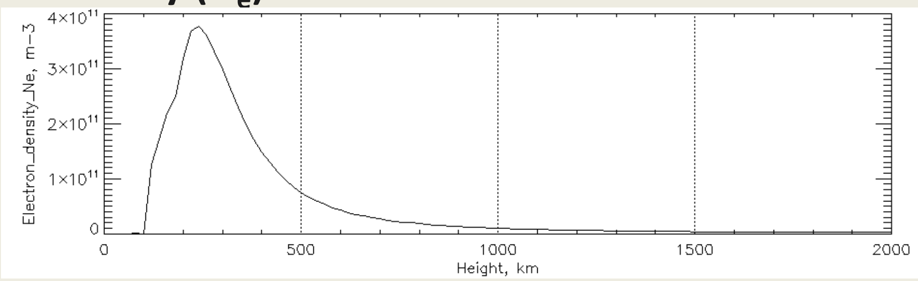
$$n = \sqrt{1 - f_N^2 / f^2} \approx 1 - f_N^2 / 2f^2 = 1 - 40.30 N_e / f^2$$

$$f_N^2 = \frac{N_e e^2}{4\pi^2 \varepsilon_0 m_e}$$: plasma frequency

$$I_r^s \approx \int 40.30 N_e / f^2 dl = 40.30 \times 10^{16} TEC / f^2$$

TEC: Total Electron Content

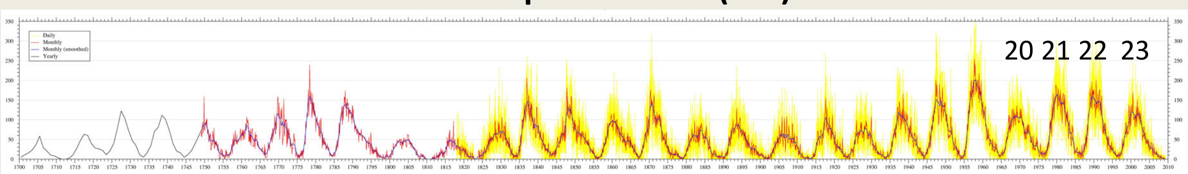
Electron Density (N_e):



IRI-2007 model: 2009/7/31 0:00 Tokyo (http://modelweb.gsfc.nasa.gov/models/iri.html)

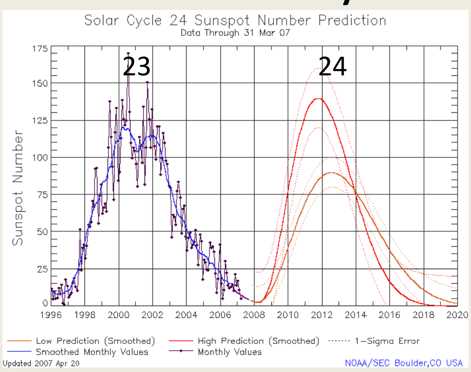
Solar Cycle

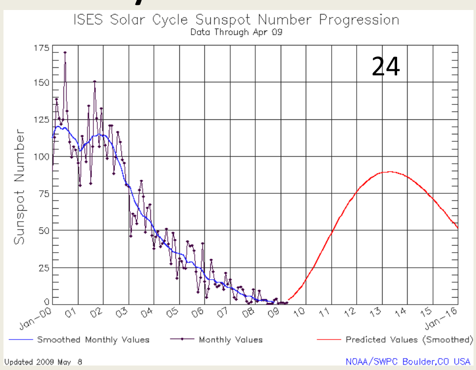
International Sunspot Number (ISN): 1700-2009



by SIDC (Solar Influences Data Analysis Center) in Belgium (http://sidc.oma.be)

Solar Cycle Prediction: Cycle 24





by NOAA SWPC (Space Weather Prediction Center) (http://www.swpc.noaa.gov/SolarCycle)

LC: Linear Combination

$$C = a\Phi_1 + b\Phi_2 + cP_1 + dP_2 (\Phi_1 = \lambda_1\phi_1, \Phi_2 = \lambda_2\phi_2)$$

	LC	Coefficients				Wave Length (cm)	Ionos Effect wrt L1	Typical Noise (cm)
		a	*b*	*c*	*d*			
L1	L1 Carrier-Phase	1	0	0	0	19.0	1.0	0.3
L2	L2 Carrier-Phase	0	1	0	0	24.4	1.6	0.3
LC/L3	Iono-Free Phase	C_1	C_2	0	0	-	0.0	0.9
LG/L4	Geometry-Free Phase	1	-1	0	0	-	0.6	0.4
WL	Wide-Lane Phase	λ_W/λ_1	$-\lambda_W/\lambda_2$	0	0	86.2	1.3	1.7
NL	Narrow-Lane Phase	λ_N/λ_1	λ_N/λ_2	0	0	10.7	1.3	1.7
MW	Melbourne-Wübbena	λ_W/λ_1	$-\lambda_W/\lambda_2$	λ_N/λ_1	λ_N/λ_2	86.2	0.0	21
MP1	L1-Multipath	$2C_2-1$	$-2C_2$	1	0	-	0.0	30
MP2	L2-Multipath	$-2C_1$	$2C_1-1$	0	1	-	0.0	30

$$C_1 = f_1^2/(f_1^2 - f_2^2),\ C_2 = -f_2^2/(f_1^2 - f_2^2),\ \lambda_W = 1/(1/\lambda_1 - 1/\lambda_2),\ \lambda_N = 1/(1/\lambda_1 + 1/\lambda_2)$$

Single Layer Model

Ionospheric Delay Model:

$$I = \frac{40.30\times 10^{16}}{f^2} TEC \approx \frac{1}{\cos z'} \frac{40.30\times 10^{16}}{f^2} \times VTEC(t, \phi_{pp}, \lambda_{pp})$$

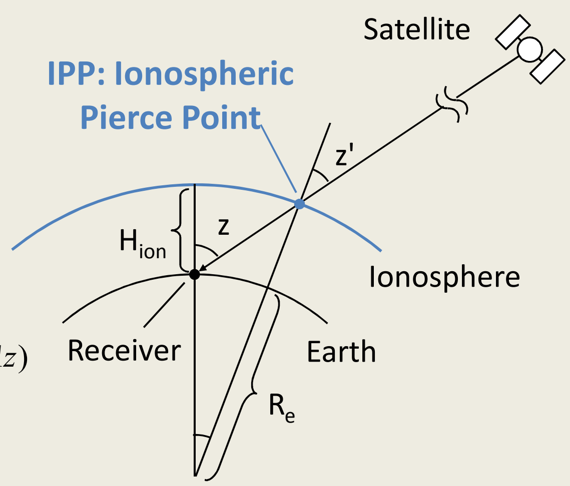
IPP Position/Slant Factor:

$$z = \pi/2 - El$$

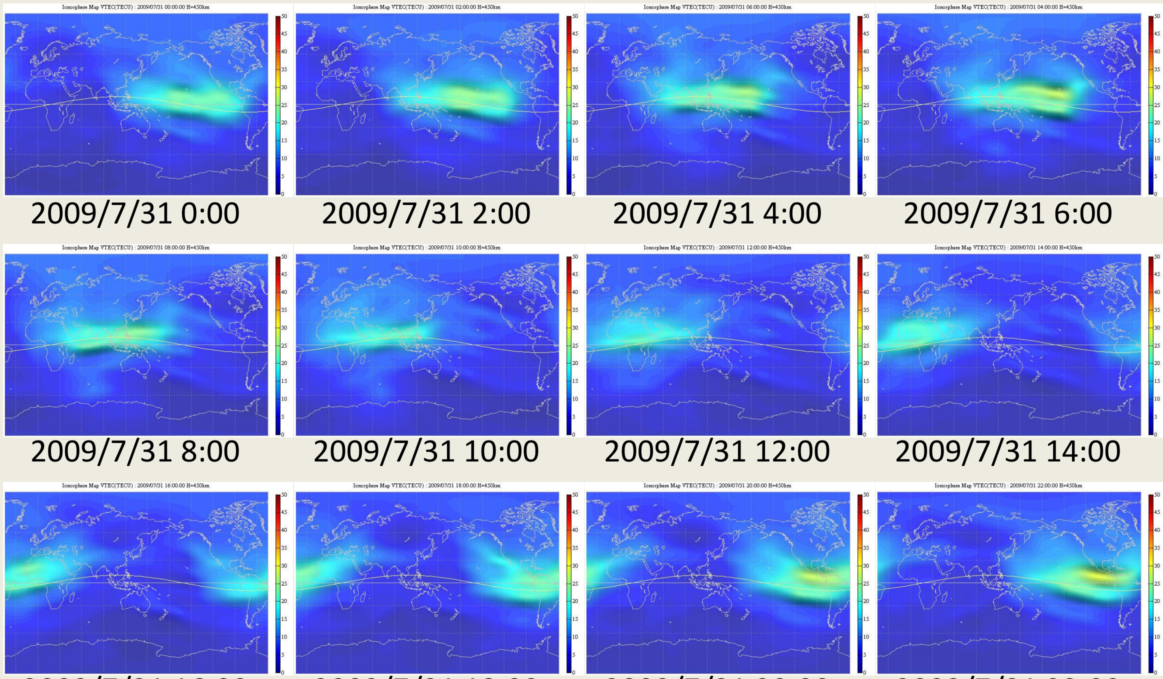
$$z' = \arcsin \frac{R_e \sin z}{R_e + H_{ion}}, \alpha = z - z'$$

$$\phi_{pp} = \arcsin(\cos\alpha \sin\phi + \sin\alpha\cos\phi\cos Az)$$

$$\lambda_{pp} = \lambda + \arcsin \frac{\sin\alpha \sin Az}{\phi_{pp}}$$



Ionospheric TEC Grid



2009/7/31 0:00 | 2009/7/31 2:00 | 2009/7/31 4:00 | 2009/7/31 6:00

2009/7/31 8:00 | 2009/7/31 10:00 | 2009/7/31 12:00 | 2009/7/31 14:00

2009/7/31 16:00 | 2009/7/31 18:00 | 2009/7/31 20:00 | 2009/7/31 22:00

(IGS TEC Final, GPS Time)

Tropospheric Delay

Tropospheric Delay Model:

$$T = m_h(El)ZHD + m_w(El)ZWD$$

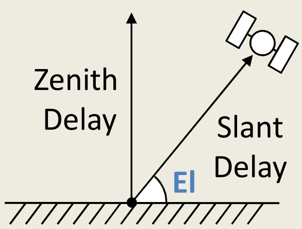
$$ZHD = \frac{0.0022768\,p}{1 - 0.00266\cos 2\phi - 2.8\times 10^{-7} H}$$

: Zenith Hydrostatic Delay (m)

ZWD : Zenith Wet Delay (m)

$m_h(El)$: Hydrostatic Mapping Function

$m_w(El)$: Wet Mapping Function



ZWD to PWV (Precipitable Water Vapor):

$$T_m = 70.2 + 0.72T$$

$$PWV = \frac{1\times 10^5}{R_v\left(k_2 - k_1\frac{m_v}{m_d} + \frac{k_3}{T_m}\right)} ZWD$$

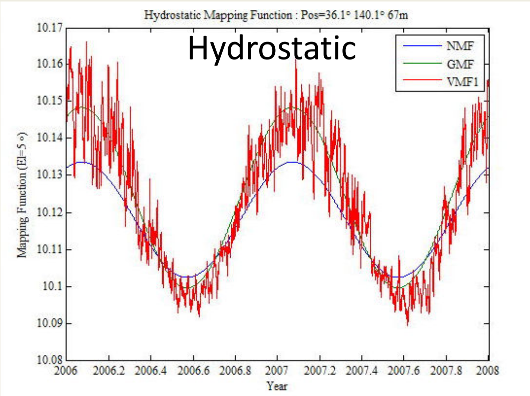
$R_v = 461, k_1 = 77.6,$
$k_2 = 71.98, k_3 = 3.754\times 10^5$
$m_v = 18.0152, m_d = 28.9644$

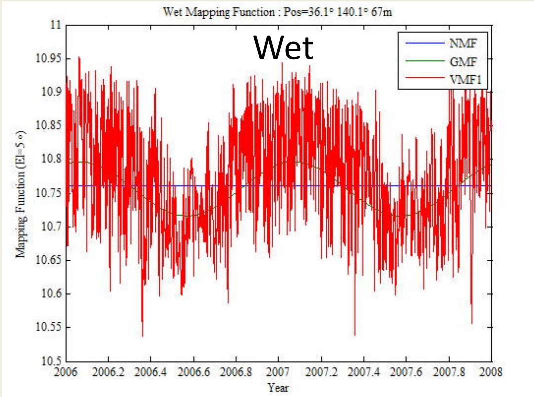
Mapping Function

$$m(El) = \frac{1 + \dfrac{a}{1 + \dfrac{b}{1 + c}}}{\sin(El) + \dfrac{a}{\sin(El) + \dfrac{b}{\sin(El) + c}}}$$

a, b, c : Mapping Function Coefficients

NMF, GMF, VMF1





(2006/1/1-2007/12/31, TSKB, El=5°)

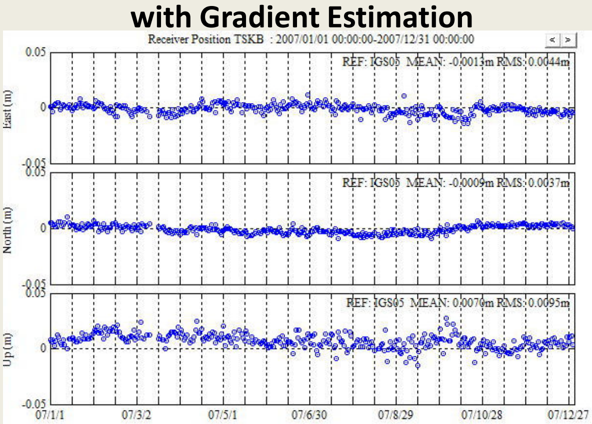
Tropospheric Gradient

Mapping Function with Horizontal Gradient:

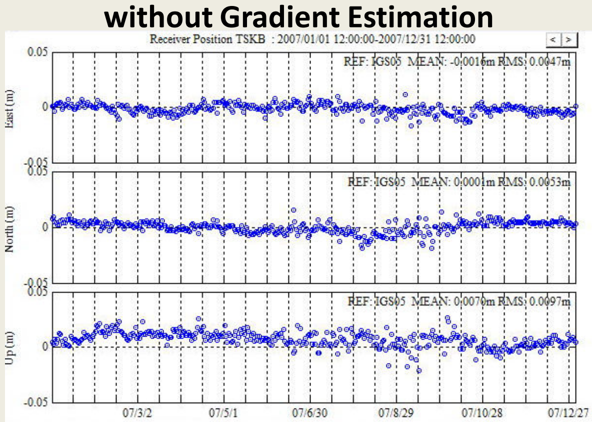
$$m(El, Az) = m_0(El) + m_0(El)\cot(El)\big(G_N \cos(Az) + G_E \sin(Az)\big)$$

G_N, G_E : North/East Gradient Parameters

PPP Solutions with Gradient Estimation



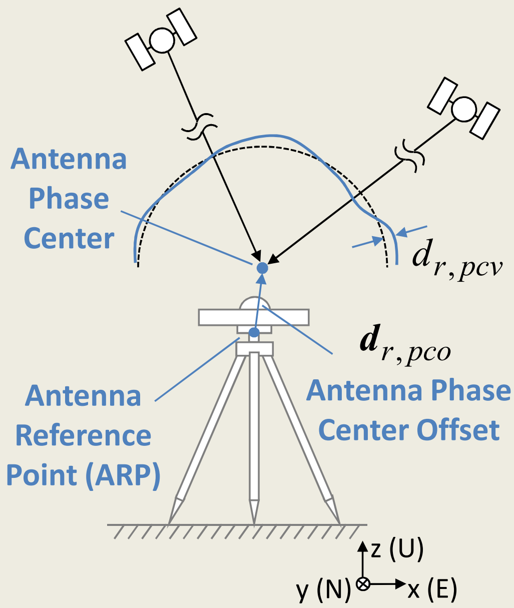
PPP Solutions without Gradient Estimation



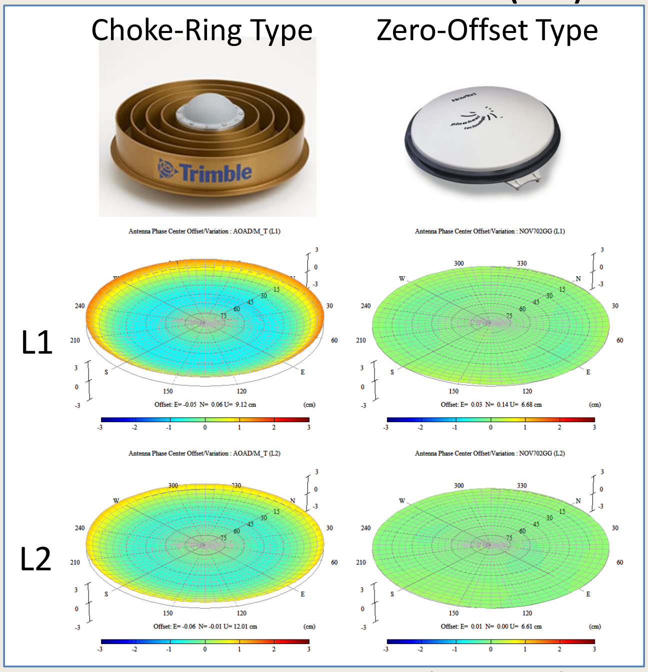
2007/1/1-12/31, 24H-Static PPP, TSKB

Antenna Phase Center 1

Receiver Antenna Phase Center:



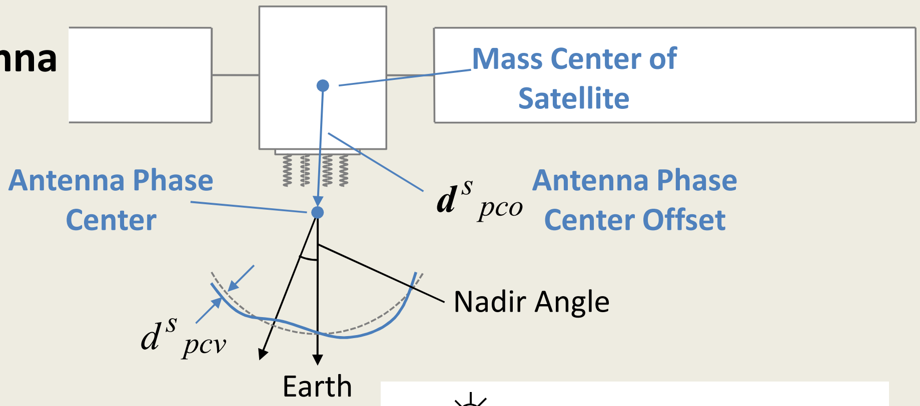
Antenna Phase Center Variation (PCV)



IGS Absolute Antenna Model (IGS05.PCV)

Antenna Phase Center 2

Satellite Antenna Phase Center:

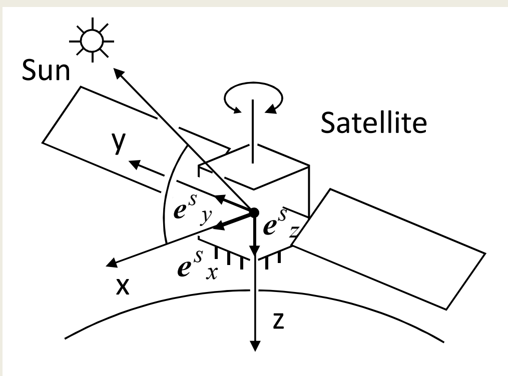


Satellite Coordinate to ECEF:

$$E_{sat \to ecef} = (\boldsymbol{e}^s{}_x, \boldsymbol{e}^s{}_y, \boldsymbol{e}^s{}_z)$$

$$\boldsymbol{e}^s{}_z = -\frac{\boldsymbol{r}^s}{\left\|\boldsymbol{r}^s\right\|}, \boldsymbol{e}_s = \frac{\boldsymbol{r}_{sun} - \boldsymbol{r}^s}{\left\|\boldsymbol{r}_{sun} - \boldsymbol{r}^s\right\|}$$

$$\boldsymbol{e}^s{}_y = \frac{\boldsymbol{e}^s{}_z \times \boldsymbol{e}^s{}_s}{\left\|\boldsymbol{e}^s{}_z \times \boldsymbol{e}_s\right\|}, \boldsymbol{e}^s{}_x = \boldsymbol{e}^s{}_y \times \boldsymbol{e}^s{}_z$$

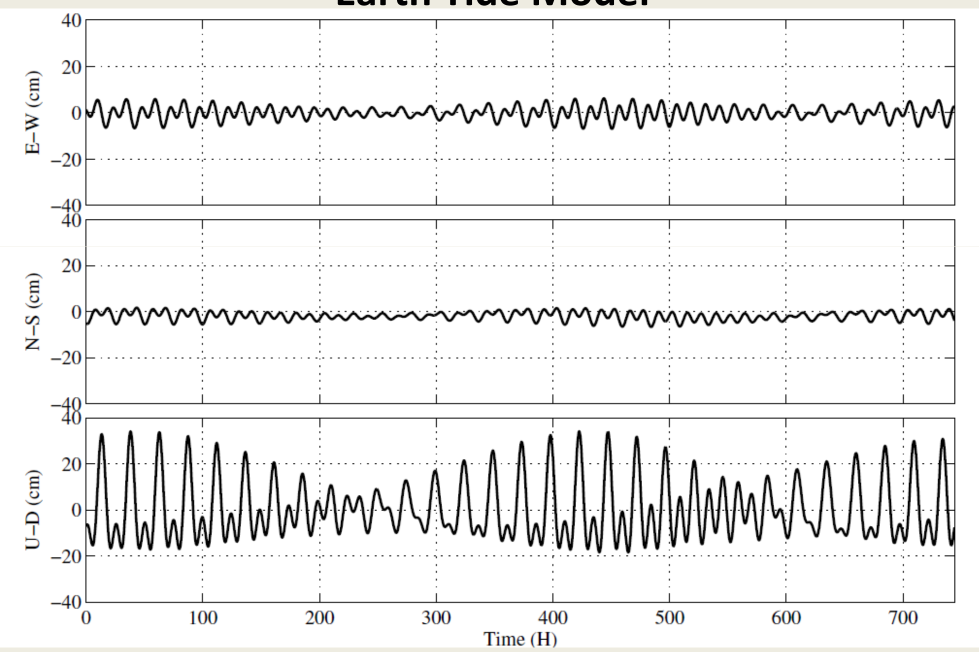


Site Displacement

- Displacement of Ground-Fixed Receiver
  - Solid Earth Tides
  - Ocean Loading
  - Pole Tide
  - Atmospheric Loading
- Tide Model
  - IERS Conventions 1996/2003
  - Ocean Loading: Schwiderski, GOT99.2/00.2, CSR 3.0/4.0, FES99/2004, NAO99.b
  - $M_2,S_2,N_2,K_2,K_1,O_1,P_1,Q_1,M_1,M_m,S_{sa}$

Earth Tide

Earth Tide Model



IERS Conventions 1996 + NAO99.b, 2007/1/1-1/31, TSKB

Phase Wind-up Effect

- Relative rotation between satellite and receiver antennas effects to the measured phase of GPS RHCP (Right Hand Circular Polarization) signal.

$$d_{pw} = \lambda \left\{ sign(\boldsymbol{e}_r^s \cdot (\boldsymbol{D}^s \times \boldsymbol{D}_r)) \arccos \frac{\boldsymbol{D}^s \cdot \boldsymbol{D}_r}{\left\| \boldsymbol{D}^s \right\| \left\| \boldsymbol{D}_r \right\|} / 2\pi + N \right\}$$

$\boldsymbol{D}^s = \boldsymbol{e}_x^s - \boldsymbol{e}_u^s(\boldsymbol{e}_u^s \cdot \boldsymbol{e}_x^s) - \boldsymbol{e}_u^s \times \boldsymbol{e}_y^s$: Dipole Vector of Satellite Antenna

$\boldsymbol{D}_r = \boldsymbol{e}_{r,x} - \boldsymbol{e}_r^s(\boldsymbol{e}_r^s \cdot \boldsymbol{e}_{r,x}) + \boldsymbol{e}_r^s \times \boldsymbol{e}_{r,y}$: Dipole Vector of Receiver Antenna

$\boldsymbol{E}_{ecef \to enu} = (\boldsymbol{e}_{r,x}{}^T, \boldsymbol{e}_{r,y}{}^T, \boldsymbol{e}_{r,z}{}^T)^T$: ECEF to ENU Transformation Matrix

$\boldsymbol{e}_r^s$: LOS Vector from Receiver to Satellite Antenna

N : Integer Ambiguity

Relativistic Effects

- Satellite/Receiver:
  - Frequency Shift by Earth Gravity (General Rel.)
  - Frequency Shift by Sun/Moon Gravity (General Rel.)
  - Second-Order Doppler-Shift by Motion (Special Rel.)
- Signal Propagation:
  - Sagnac Correction (Rotating Coordinates)
  - Shapiro Time Delay Effect
  - Lense-Thirring Drag

Satellite Clock Bias/Rate Correction
+ Periodic Term:

$$d_{rel} = -\frac{2\boldsymbol{r}^s \cdot \boldsymbol{v}^s}{c^2}$$

Relative Positioning

DD: Double Difference

DD Carrier-Phase Model:

$$\Phi_{ub}^{ij} \equiv \lambda((\phi_u^i - \phi_b^i) - (\phi_u^j - \phi_b^j))$$

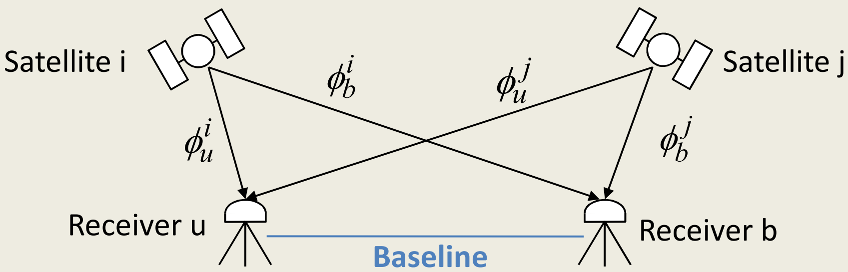
$$= \rho_{ub}^{ij} + c(dt_{ub}^{ij} - dT_{ub}^{ij}) - I_{ub}^{ij} + T_{ub}^{ij} + \lambda B_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_{\Phi}$$

$$dt_{ub}^{ij} = dt_u^{ij} - dt_b^{ij} = 0,\ dT_{ub}^{ij} = dT_{ub}^i - dT_{ub}^j \approx 0$$

$$B_{ub}^{ij} = (\phi_{u,0} - \phi_0^i + N_u^i) - (\phi_{b,0} - \phi_0^i + N_b^i) - (\phi_{u,0} - \phi_0^j + N_u^j) + (\phi_{b,0} - \phi_0^j + N_b^j) = N_{ub}^{ij}$$

$I_{ub}^{ij} = I_{ub}^i - I_{ub}^j \approx 0, T_{ub}^{ij} = T_{ub}^i - T_{ub}^j \approx 0, d_{ub}^{ij} = d_{ub}^i - d_{ub}^j \approx 0$ **(Short Baseline)**

$$\Phi_{ub}^{ij} = \rho_{ub}^{ij} + \lambda N_{ub}^{ij} + \varepsilon_{\Phi}$$



Relative Positioning

Nonlinear-LSE:

Parameter Vector:

$$x = (r_u{}^T, N_{ub}^{s_2 s_1}, N_{ub}^{s_3 s_1}, \ldots, N_{ub}^{s_m s_1})^T$$

Measurement Vector:

$$y = (y_{t_1}{}^T, y_{t_1}{}^T, \ldots, y_{t_n}{}^T)^T$$

Meas Model, Design Matrix:

$$h(x) = \left(h_{t_1}(x)^T, h_{t_2}(x)^T, \ldots, h_{t_n}(x)^T\right)^T$$

$$H = \left(H_{t_1}{}^T, H_{t_2}{}^T, \ldots, H_{t_n}{}^T\right)^T$$

Meas Error Covariance:

$$R = blkdiag\left(R_{t_1}, R_{t_2}, \ldots, R_{t_n}\right)$$

Solution (Static/Float):

$$\hat{x} = x_0 + (H^T R^{-1} H)^{-1} H^T R^{-1} (y - h(x_0))$$

$$y_{t_k} = (\Phi_{ub,t_k}^{s_2 s_1}, \Phi_{ub,t_k}^{s_3 s_1}, \ldots, \Phi_{ub,t_k}^{s_m s_1})^T$$

$$h_{t_k}(x) = \begin{pmatrix} \rho_{u,t_k}^{s_2 s_1} - \rho_{b,t_k}^{s_2 s_1} + \lambda N_{ub}^{s_2 s_1} \\ \rho_{u,t_k}^{s_3 s_1} - \rho_{b,t_k}^{s_3 s_1} + \lambda N_{ub}^{s_m s_1} \\ \vdots \\ \rho_{u,t_k}^{s_m s_1} - \rho_{b,t_k}^{s_m s_1} + \lambda N_{ub}^{s_m s_1} \end{pmatrix}$$

$$H_{t_k} = \begin{pmatrix} -e_{u,t_k}^{s_2 s_1\,T} & \lambda & 0 & \cdots & 0 \\ -e_{u,t_k}^{s_3 s_1\,T} & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -e_{u,t_k}^{s_m s_1\,T} & 0 & 0 & \cdots & \lambda \end{pmatrix}$$

$$R_{t_k} = \begin{pmatrix} 4\sigma_\Phi{}^2 & 2\sigma_\Phi{}^2 & \cdots & 2\sigma_\Phi{}^2 \\ 2\sigma_\Phi{}^2 & 4\sigma_\Phi{}^2 & \cdots & 2\sigma_\Phi{}^2 \\ \vdots & \vdots & \ddots & \vdots \\ 2\sigma_\Phi{}^2 & 2\sigma_\Phi{}^2 & \cdots & 4\sigma_\Phi{}^2 \end{pmatrix}$$

r_b : Fixed Base-Station Position

Integer Ambiguity Resolution

- Many Proposals for AR
  - Simple Integer Rounding
  - Wide-Lane + Narrow-Lane
  - Coordinate Domain Search
  - Ambiguity Domain Search
  - Shrink Search Space with Constraints
- Examples of AR Algorithms
  - AFM, FARA, LSAST, LAMBDA, ARCE, HB-L^3, Modified Cholesy Decomposition, Null Space, FAST, OMEGA, ...

ILS: Integer Least Square

Problem:

$$x = (a^T, b^T)^T,\ H = (A, B)$$
$$y = Hx + v = Aa + Bb + v$$
$$\breve{x} = \underset{a \in Z^n, b \in R^m}{\arg\min}\ (y - Hx)^T Q_y^{-1} (y - Hx)$$

Strategy:

(1) Conventional LSE

$$\hat{x} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = Q_x H^T Q_y^{-1} y,\ Q_x = \begin{pmatrix} Q_a & Q_{ab} \\ Q_{ba} & Q_b \end{pmatrix} = (H^T Q_y H)^{-1}$$

(2) Search Integer Vector with Minimum Squared Residuals

$$\breve{a} = \underset{a \in Z^n}{\arg\min} (\hat{a} - a)^T Q_a^{-1} (\hat{a} - a)$$

(3) Improve solution

$$\breve{b} = \hat{b} - Q_{ba} Q_a^{-1} (\hat{a} - \breve{a})$$

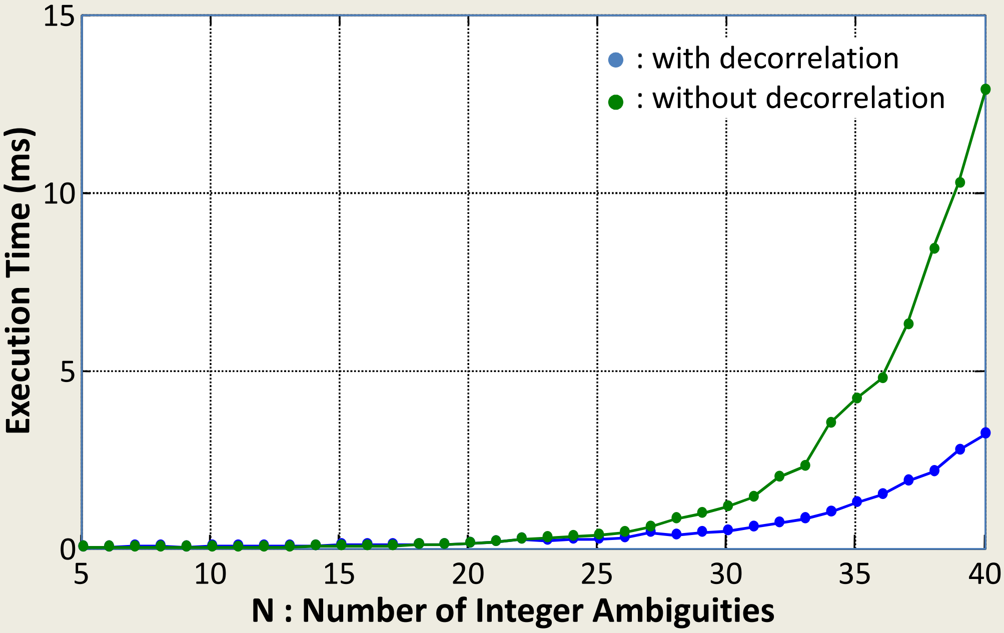
LAMBDA

Teunissen, P.J.G. (1995)
The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. *Journal of Geodesy,* Vol. 70, No. 1-2, pp. 65-82.

- ILS Estimation with:
  - Shrink Integer Search Space with "Decorrelation"
  - Efficient Tree Search Strategy
  - Similar to Closest Point Search with LLL Lattice Basis Reduction Algorithm

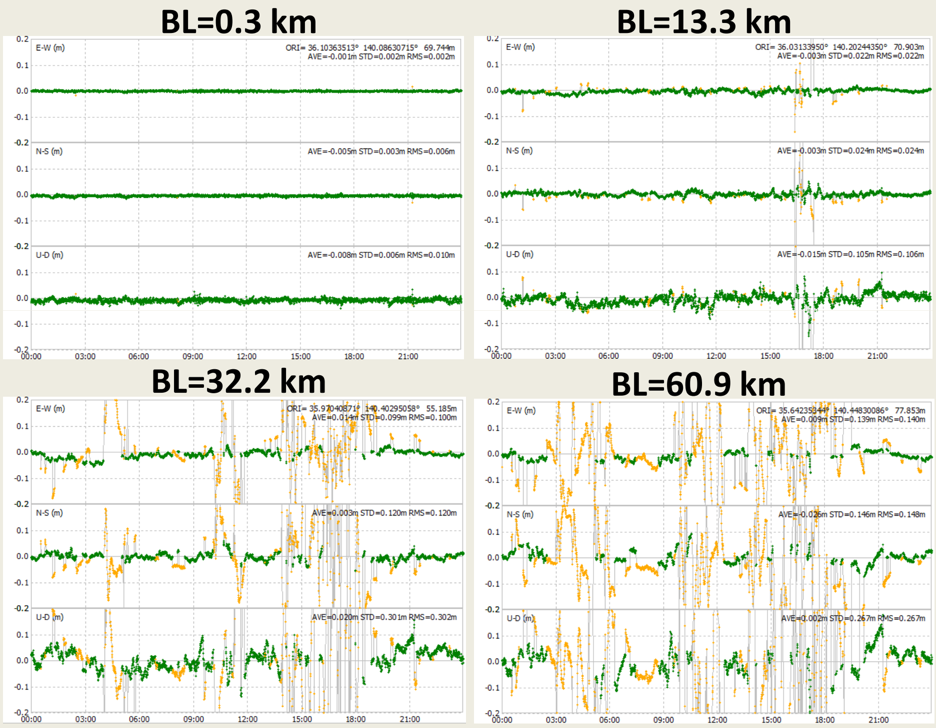
$$\breve{a} = \underset{a \in Z^n}{\arg\min} (\hat{a} - a)^T Q_a^{-1} (\hat{a} - a) \Leftrightarrow \begin{array}{c} \hat{z} = Z^T \hat{a},\ Q_z = Z^T Q_a Z \\ \breve{z} = \underset{z \in Z^n}{\arg\min} (\hat{z} - z)^T Q_z^{-1} (\hat{z} - z) \\ \breve{a} = Z^{-T} \breve{z} \end{array}$$

Performance of LAMBDA



(Pentium 4 3.2GHz, Intel C/C++ 8.0)

Effect of Baseline Length



RMS Error:
E: 0.2cm
N: 0.6cm
U: 1.0cm
Fix Ratio:
99.9%

RMS Error:
E: 2.2cm
N: 2.4cm
U: 10.6cm
Fix Ratio:
94.2%

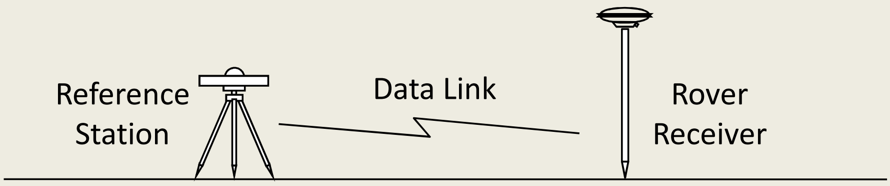
RMS Error:
E: 10.0cm
N: 12.0cm
U: 30.2cm
Fix Ratio:
64.3%

RMS Error:
E: 14.0cm
N: 14.8cm
U: 26.7cm
Fix Ratio:
44.4%

(24 hr Kinematic ●: Fixed Solution ●: Float Solution)

RTK-GPS

- Technique with Relative Positioning
  - Real-time Position of the Rover Antenna
  - Transmit Reference Station Data to Rover via Data Link
  - OTF (On-the-Fly) Integer Ambiguity Resolution
  - Typical Accuracy: 1 cm + 1ppm x BL RMS (Horizontal)
  - Applications:
    Land Survey, Construction Machine Control, Precision Agriculture etc.



NRTK: Network RTK

- Extension of RTK-GPS
  - RTK-GPS with Single Receiver
  - Use Networked Reference Stations
  - Generate Correction Messages with Interpolation of Sparse Station Data
  - Provide Correction Messages via Mobile-Phone Network
  - VRS, FKP, MAC, RTCM 2.3, RTCM 3.1
  - NTRIP: Networked Transport of RTCM via Internet Protocol
- Commercial Services

Precise Point Positioning (PPP)

Precise Point Positioning (PPP)

- Typical Strategy
  - ZD (Zero-Difference) Measurement Equations
  - Precise Ephemeris: IGS or Others
  - Ionosphere: Eliminated by Dual-Freq LC
  - Troposphere: Estimated ZTD/ZWD + Mapping Function
  - Antenna Model, Earth-Tide, Phase Wind-Up,...
- Integer Ambiguity Resolution
  - Float Estimation
  - Some Research propose PPP-AR with Satellite H/W Bias Estimation

Applications

Exercise 2

PPP with GT 0.6.4