For JAXA R&D

PPP - Models, Algorithms and Implementations (1)

Tokyo Univ. of Marine Science and Technology (TUMSAT)

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2019-10-04 @Tokyo, Japan

PPP - Models, Algorithms and Implementation

1. 2019-10-04 **PPP models**
   geometric range, ionosphere, troposphere, antenna PCV, earth tides, wind-up, relativity, biases, coordinates
2. 2019-10-18 **PPP algorithms**
   SPP, LSQ, GN, EKF, noise-model, RAIM/QC, LAPACK/BLAS
3. 2019-11-01 **PPP data handling**
   LC, interpolation, slip detection, RINEX, SP3, ANTEX, RTCM, CSSR
4. 2019-11-22 **PPP-AR**
   UPD/FCB, EWL/WL/NL, ILS, LAMBDA, TCAR, PAR, validation
5. 2019-12-06 **INS integration**
   INS sensors, Inertial navigation, INS integration
6. 2019-12-20 **POD of satellites**
   orbit element, orbit model, reduced-dynamic, ECI-ECEF transformation, precession/nutation, EOP

(1.5 h / session)

PPP Models

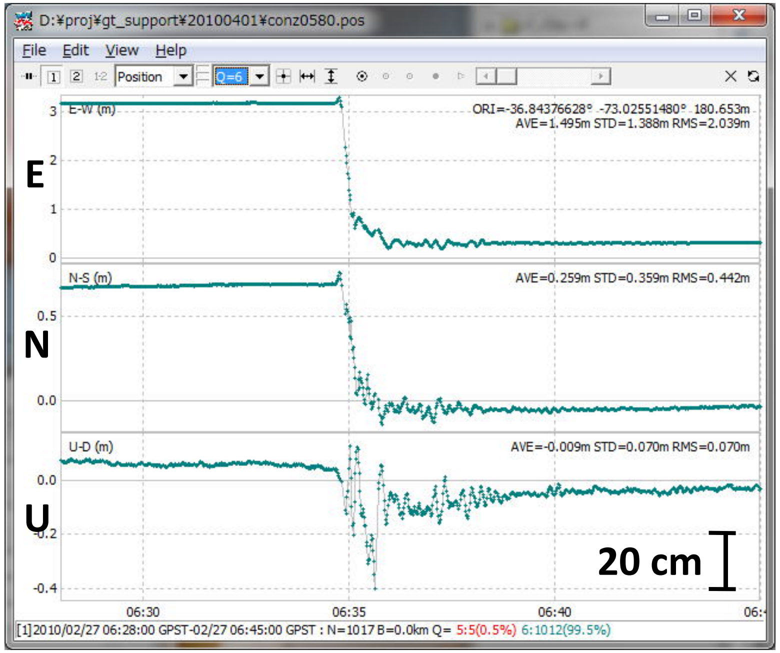
Acronyms

- c : Speed of light (m/s)
- P_r^s : Peudorange measurement (m)
- φ_r^s : Carrier phase measurement (cyc)
- Φ_r^s : Phase-range measurement (m)
- t_r : Signal reception time (s)
- t^s : Signal transmission time (s)
- ρ_r^s : Geometric range (m)
- $\mathbf{r}^s(t)$: Satellite position in ECEF (m)
- $\mathbf{v}^s(t)$: Satellite velocity in ECEF (m)
- $\mathbf{r}_r$: Receiver position in ECEF (m)
- $\mathbf{e}_r^s$: LOS vector in ECEF
- $\mathbf{e}_{r,enu}^s$: LOS vector in local coordinates
- ϕ_r : Latitude of receiver position (rad)
- λ_r : Longitude of receiver position (rad)
- h_r : Ellipsoidal height of receiver (m)
- H_r : Orthometric height of receiver (m)
- Az : Azimuth angle of satellite (rad)
- El : Elevation angle of satellite (rad)
- dt_r : Receiver clock bias (s)
- dT^s : Satellite clock bias (s)

- I_r^s : Ionospheric delay (m)
- T_r^s : Tropospheric delay (m)
- f_i : L_i carrier frequency (Hz)
- λ_i : L_i carrier wavelength (m)
- B_r^s : Carrier phase bias (m)
- N_r^s : Carrier phase ambiguity (cyc)
- ε_P : Code measurement error (m)
- ε_Φ : Phase measurement error (m)
- ω_e : Earth rotation velocity (rad/s)
- Z_t : Zenith total delay (m)
- Z_h : Zenith hydrostatic delay (m)
- Z_w : Zenith wet delay (m)
- $m_h(El)$: Hydrostatic mapping function
- $m_w(El)$: Wet mapping function
- $\mathbf{U}(t)$: ECEF to ECI transformation matrix
- $\mathbf{E}_r$: ECEF to local coordinates rotation matrix
- $\mathbf{E}^s$: ECEF to satellite body rotation matrix
- $\mathbf{R}_x(\theta)$: Coordinates rotation matrix around X
- $\mathbf{R}_y(\theta)$: Coordinates rotation matrix around Y
- $\mathbf{R}_z(\theta)$: Coordinates rotation matrix around Z

Typical Accuracy of PPP

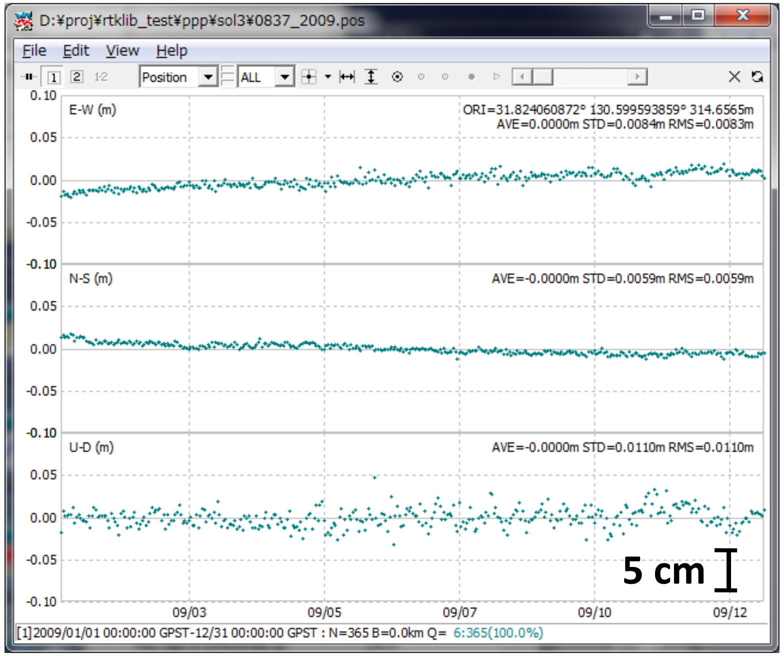
1 Hz Kinematic PPP



IGS CONZ, 2010/2/27 6:28-6:45 GPST

H-RMS: 1 cm, V-RMS: 2 cm (PP)
H-RMS: 3 cm, V-RMS: 6 cm (RT)

24 H Static PPP



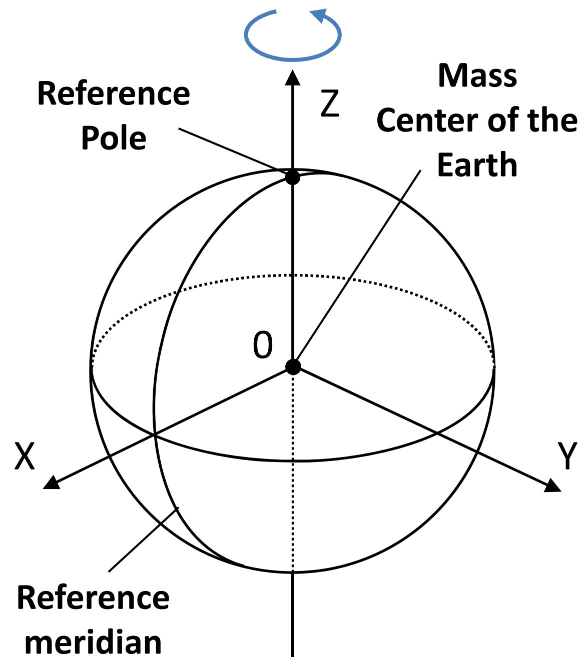
GEONET 0837 (Aira), 2009/1/1-2009/12/31

H-RMS: 3 mm, V-RMS: 6 mm (PP)

PPP Model Error < 1 mm

Coordinate Systems

- **ECEF (earth-centered earth-fixed)**
  - ITRF
  - WGS84 (GPS)
  - PZ90.14 (GLONASS)
  - JGS (QZSS)
  - JGD (Japan, GSI) ...
- **Latitude, Longitude and Height**
  - Latitude: geodetic, geocentric
  - Height: orthometric, ellipsoidal
- **ECI (earth-centered inertial)**
  - J2000.0 (equatorial coordinate system)
  - ICRF (international celestial reference frame)



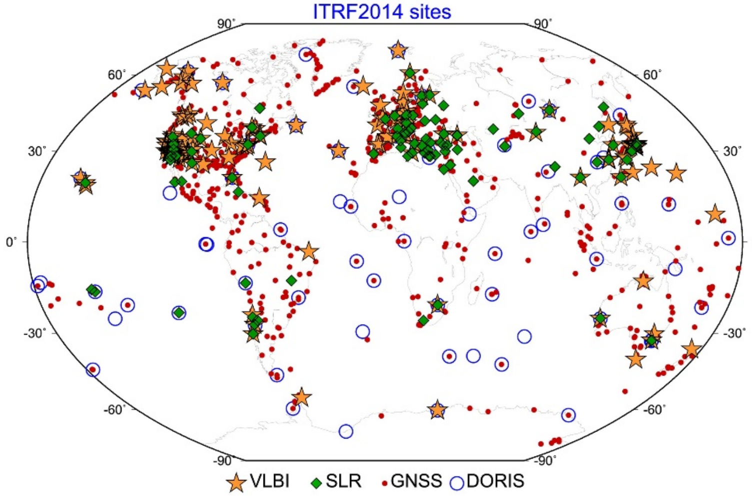
ECEF Definitions

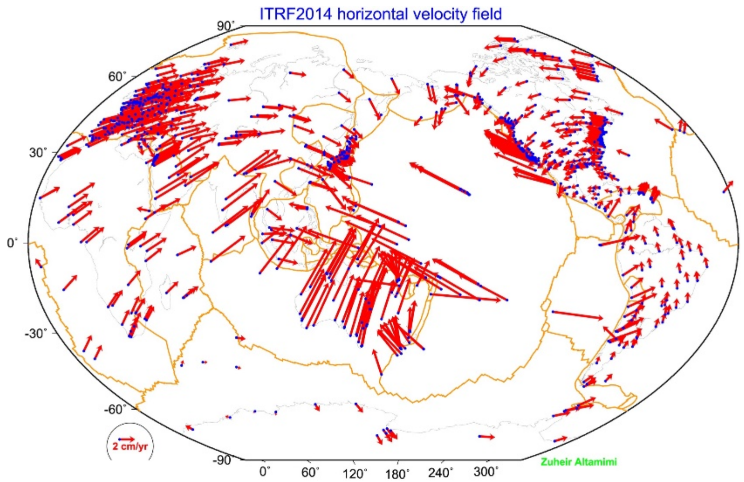
ITRF

- **International Terrestrial Reference Frame**
  - A "Realization" of ITRS developed and maintained by IERS
  - GNSS, VLBI, SLR, DORIS site position/velocity, EOP, PSD (SINEX)
  - ITRF2014, ITRF2008, ITRF2005, ITRF2000, ITRF97, ITRF96, ...

VLBI: Very Long Baseline Interferometry
SLR: Satellite Laser Ranging
DORIS: Doppler Orbit determination and Radiopositioning Integrated on Satellite

ITRS: International Terrestrial Reference System
IERS: International Earth Rotation Service





Z.Altamimi et al., ITRF2014: A new release of the International Terrestrial Reference Frame modeling nonlinear station motion, JGR Solid Earth, 2016

Coordinates Transformation

Helmert Transformation (A to B)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_B = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + (1+D) \begin{pmatrix} 1 & -R_3 & R_2 \\ R_3 & 1 & -R_1 \\ -R_2 & R_1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_A$$

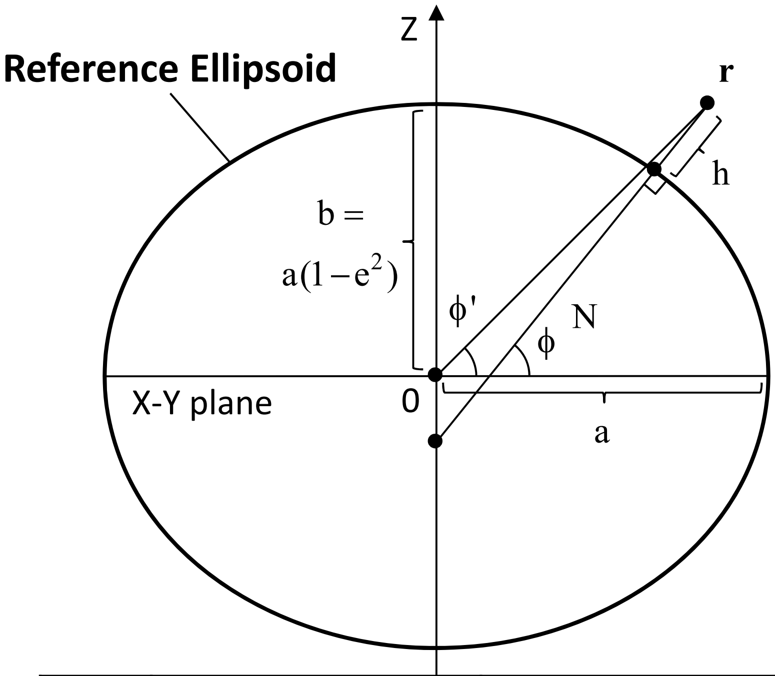
- T_1, T_2, T_3 : Translation along coordinate axis
- D : Scale factor
- R_1, R_2, R_3: Rotation around coordinate axis

Frame		T1 (mm)	T2 (mm)	T3 (mm)	D (10^{-9})	R1 (mas)	R2 (mas)	R3 (mas)
A	B							
ITRF2014	ITRF2008	1.6	1.9	2.4	-0.02	0.000	0.000	0.000
		-0.0/y	0.0/y	-0.1/y	0.03/y	0.00/y	0.00/y	0.00/y

(Epoch 2010.0)

http://itrf.ensg.ign.fr/ITRF_solutions/2014/tp_14-08.php

Latitude, Longitude and Height



	GRS80	WGS84
a	6378137 m	6378137 m
f	1/298.257222101	1/298.257223563
b	6335439.32708 m	6335439.32729 m

a : Semi-major axis length (m)
ϕ' : Geocentric latitude
ϕ : Geodetic latitude
f : Flattening
λ : Longitude
h : Ellipsoidal height

Lat/Lon/Hgt -> ECEF-XYZ

$$e^2 = f(2-f)$$

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

$$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ (N(1-e^2)+h)\sin\phi \end{pmatrix}$$

ECEF-XYZ -> Lat/Lon/Hgt

$$R = \sqrt{x^2+y^2},\ \phi_0 = 0$$

$$\phi_{i+1} = \arctan\left(\frac{z}{R} - \frac{ae^2\tan\phi_i}{R\sqrt{1+(1-e^2)\tan^2\phi_i}}\right)$$

$$\phi = \lim_{i\to\infty}\phi_i$$

$$\lambda = \mathrm{ATAN2}(y,x)$$

$$h = \frac{R}{\cos\phi} - \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

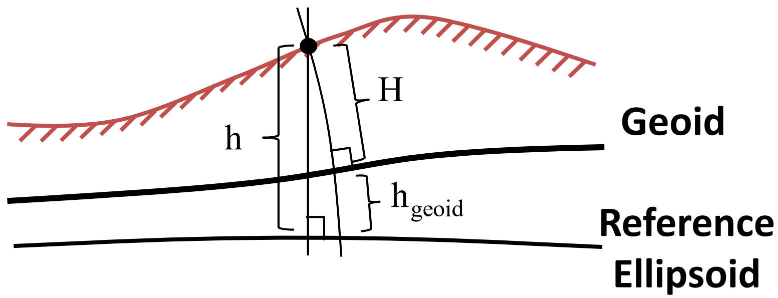
Heights

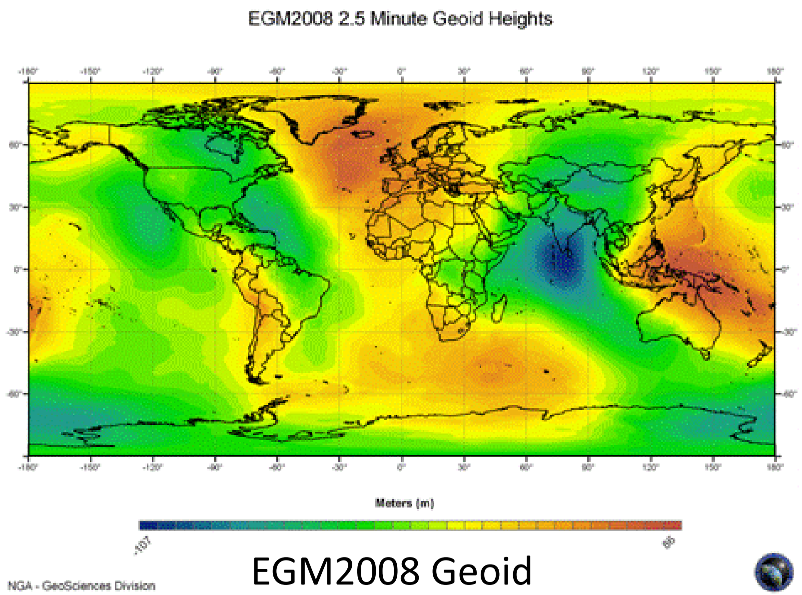
$$H \approx h - h_{geoid}(\phi, \lambda)$$

H : Orthometric height (m)

h : Ellipsoidal height (m)

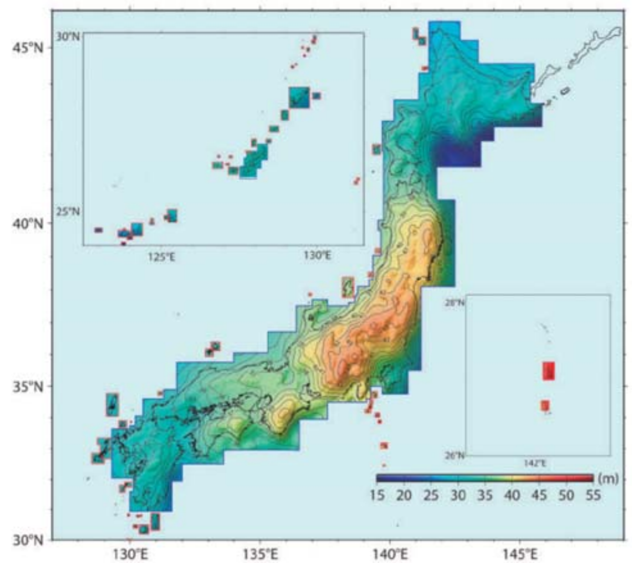
h_{geoid} : Geoid height (m)





EGM2008 Geoid

https://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html

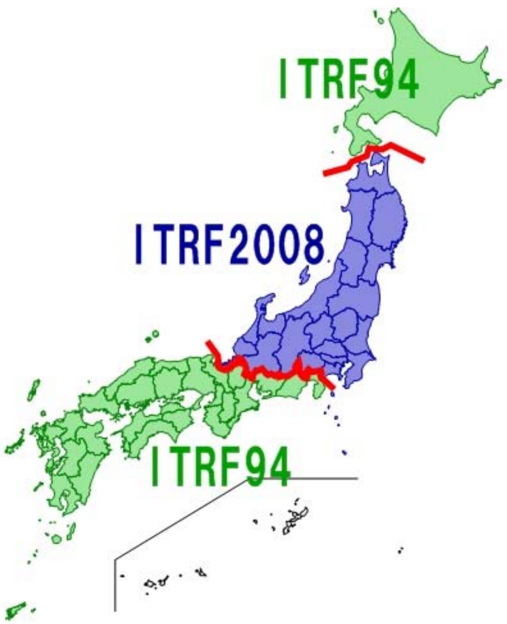


GSIGEO2011 (Ver.2) Geoid

https://www.gsi.go.jp/buturisokuchi/grageo_geoidseika.html

Japanese Geodetic Datum

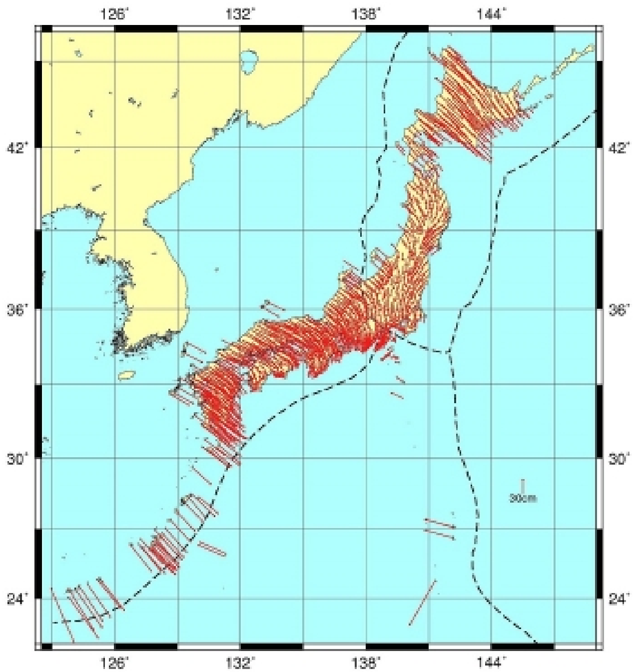
Tokyo -> JGD2000 -> JGD2011



JGD2011 (GSI)

http://club.informatix.co.jp/?p=998

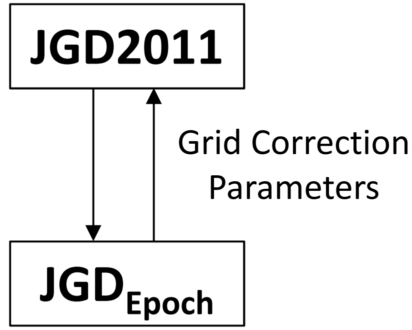
Displacement of Japanese Island

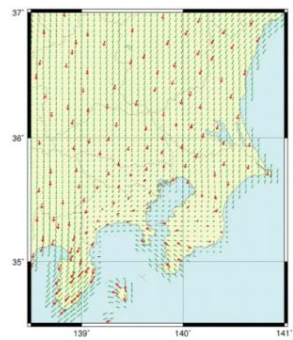


1997 -> 2009

https://www.gsi.go.jp/sokuchikijun/semidyna01.html

Semi-dynamic corrections





https://www.gsi.go.jp/sokuchikijun/semidyna03.html

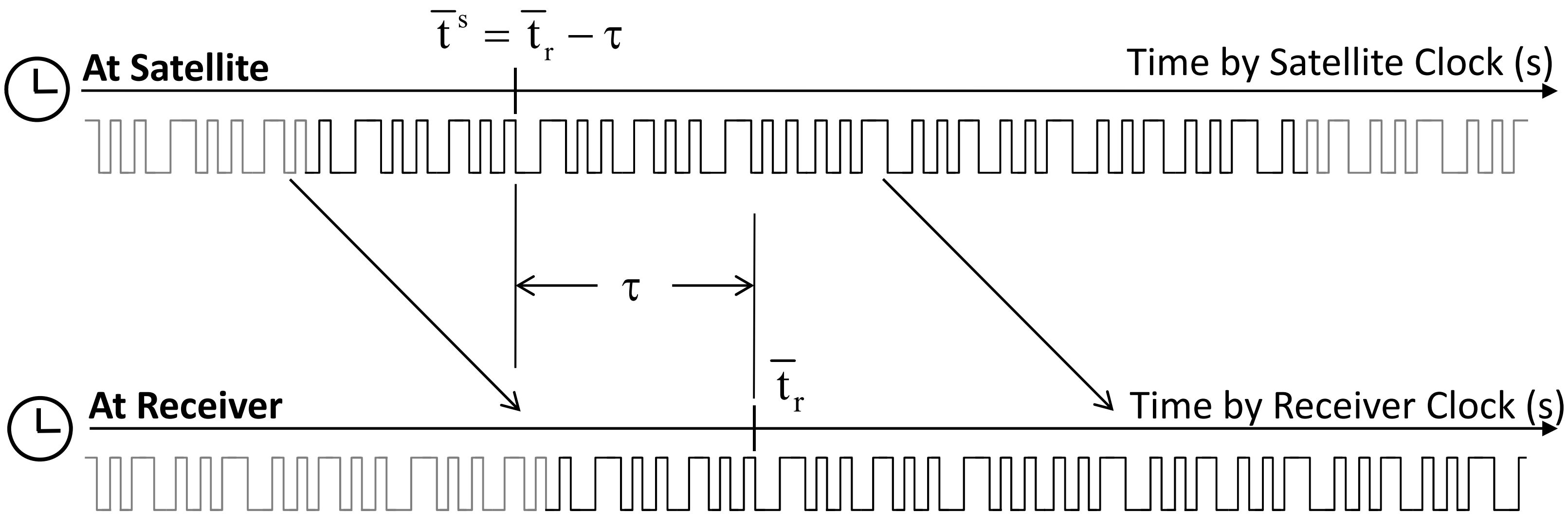
Pseudorange

Definition:

$$P_r^s \equiv c\tau = c(\bar{t}_r - \bar{t}^s)$$

(m)

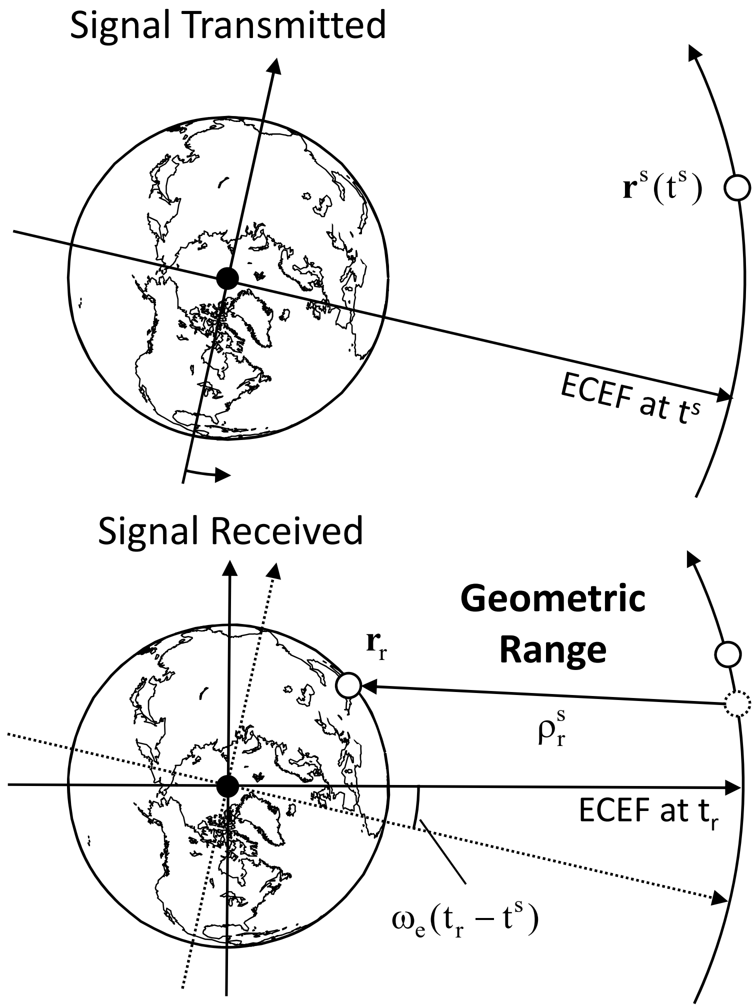
The pseudo-range (PR) is the distance from the receiver antenna to the satellite antenna including receiver and satellite clock offsets (and other biases, such as atmospheric delays) *(RINEX 2.10)*



$\bar{t}^s$ Signal transmission time by satellite clock (s)

$\bar{t}_r$ Signal reception time by receiver clock (s)

Geometric Range



Signal Transmission Time

$$t^s = \bar{t}_r - P_r^s / c - dT^s(t^s)$$

$$\rho_r^s = \left\| \mathbf{U}(t_r)\mathbf{r}_r - \mathbf{U}(t^s)\mathbf{r}^s(t^s) \right\| \qquad (1)$$

$$\rho_r^s \approx \left\| \mathbf{r}_r - \mathbf{R}_z(\omega_e(t_r - t^s))\mathbf{r}^s(t^s) \right\| \qquad (2)$$

$$\rho_r^s \approx \left\| \mathbf{r}_r - \mathbf{R}_z(\omega_e \rho_r^s / c)\mathbf{r}^s(t^s) \right\| \qquad (3)$$

$$\rho_r^s \approx \left\| \mathbf{r}_r - \mathbf{r}^s(t^s) \right\| + \frac{\omega_e(x^s y_r - y^s x_r)}{c} \qquad (4)$$

Sagnac Effect Correction

(m)

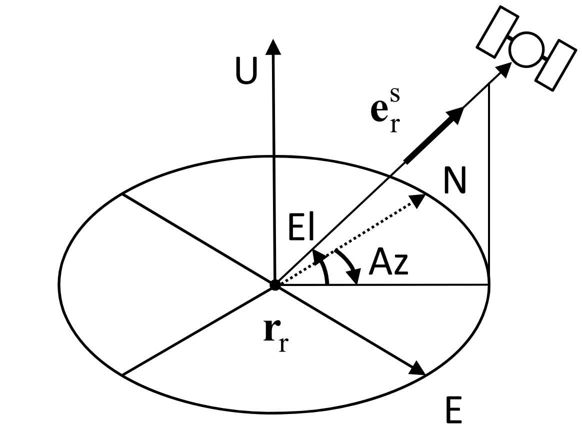
$$\mathbf{r}_r = (x_r, y_r, z_r)^T \quad \mathbf{r}^s(t) = (x^s, y^s, z^s)^T$$

LOS (Line-of-Sight) Vector

LOS Vector

$$\mathbf{e}_r^s \approx \frac{\mathbf{r}^s(t^s)-\mathbf{r}_r}{\left\|\mathbf{r}^s(t^s)-\mathbf{r}_r\right\|},\ \mathbf{e}_{r,enu}^s = \mathbf{E}_r\mathbf{e}_r^s = (e_e, e_n, e_u)^T$$

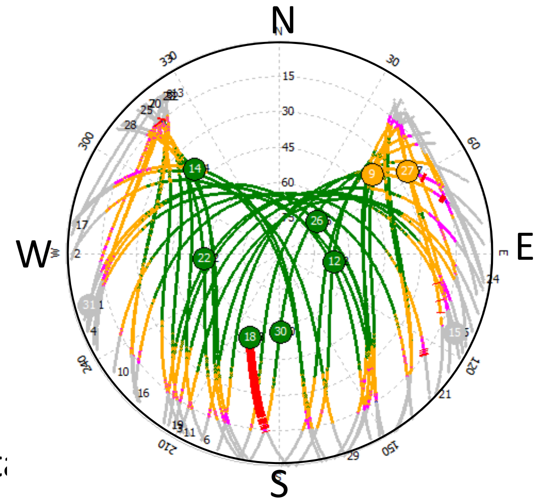
$$\mathbf{E}_r = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{pmatrix}$$



Azimuth and Elevation Angle (rad)

$$Az = ATAN2(e_e, e_n)$$
$$El = \arcsin(e_u)$$
$$(0 \le Az < 2\pi)$$

$ATAN2(y, x)$: 2-argument arct



Note: Standard C function **atan2**(x, y) raises domain error if x = 0.0 and y =0.0
Standard C function **asin**(x) raises domain error if x < -1.0 or x > 1.0

Ionospheric Delay

Ionospheric Delay Model (L-band)

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{Y_T{}^2}{2(1-X-iZ)} \pm \sqrt{\frac{Y_T{}^4}{4(1-X-iZ)^2} + Y_L{}^2}} \approx 1 - X = 1 - \frac{f_N^2}{f^2}$$

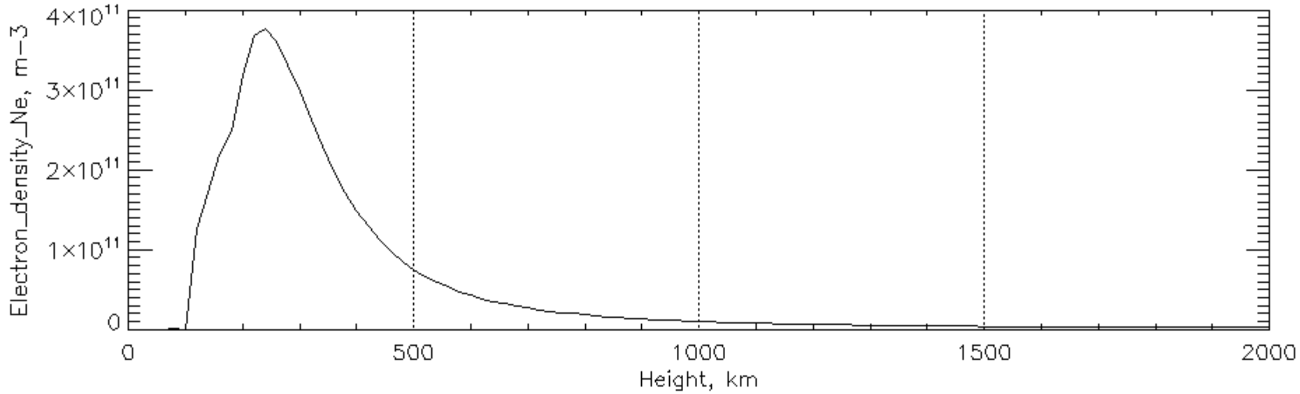
(Appleton-Hartree Formula)

$$n = \sqrt{1 - \frac{f_N^2}{f^2}} \approx 1 - \frac{f_N^2}{2f^2} = 1 - \frac{40.30N_e}{f^2} \qquad f_N = \sqrt{\frac{N_e e^2}{4\pi^2 \varepsilon_0 m_e}}$$

$$I_r^s \approx \int \frac{40.30N_e}{f^2} dl = \frac{40.30 \times 10^{16}\,\mathrm{TEC}}{f^2}$$ (m)

- f : Carrier frequency (Hz)
- f_N : Plasma frequency (Hz)
- N_e : Electron density (m^{-3})
- TEC : Total electron content (TECU)

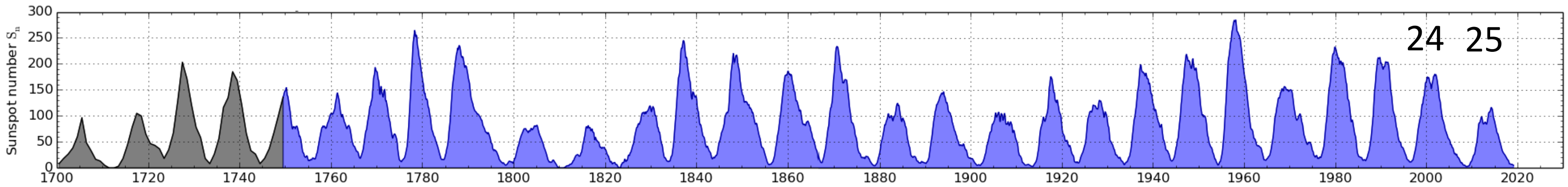
Electron Density (N_e)



IRI-2007 model: 2009/7/31 0:00 Tokyo (http://modelweb.gsfc.nasa.gov/models/iri.html)

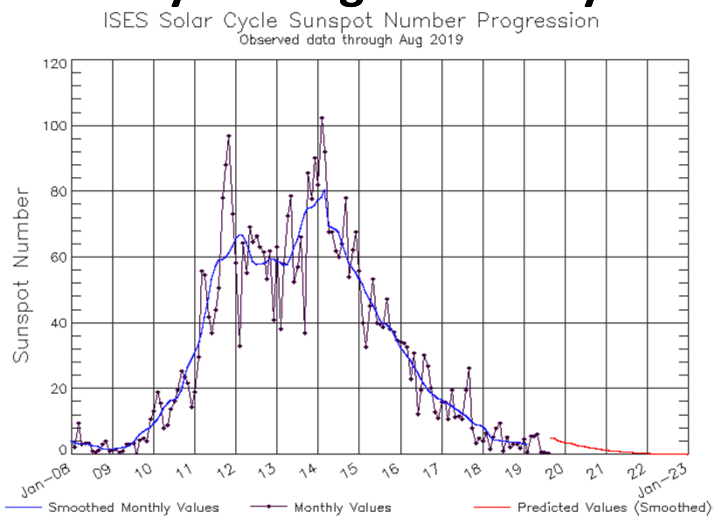
Solar Cycle

International Sunspot Number (ISN): 1700-2019



by SIDC (Solar Influences Data Analysis Center) in Belglum (http://sidc.oma.be)

Solar Cycle Progression: Cycle 25



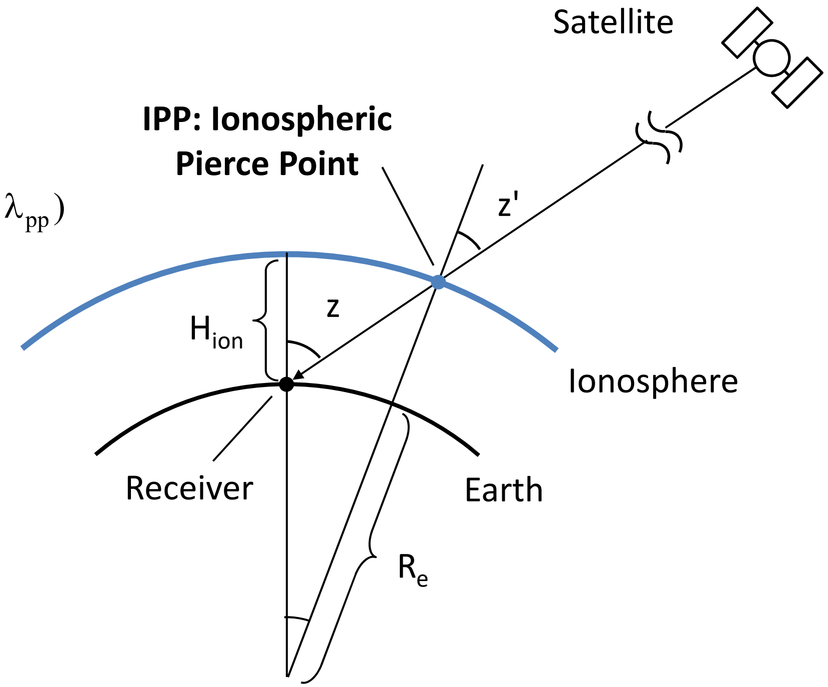
by NOAA SWPC (Space Weather Prediction Center)
(http://www.swpc.noaa.gov/products/solar-cycle-progression)

Single Layer Iono Model

Ionospheric Delay

$$I_r^s = \frac{40.30\times10^{16}}{f^2}\mathrm{TEC} \approx \frac{1}{\cos z'}\frac{40.30\times10^{16}}{f^2}\mathrm{VTEC}(t_r,\phi_{pp},\lambda_{pp})$$

(m)



IPP Position

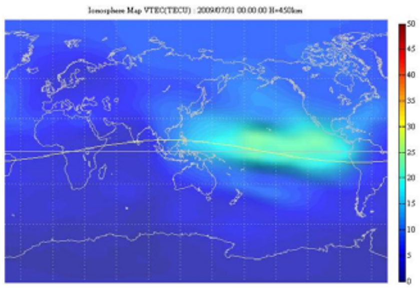
$$z = \pi/2 - El$$

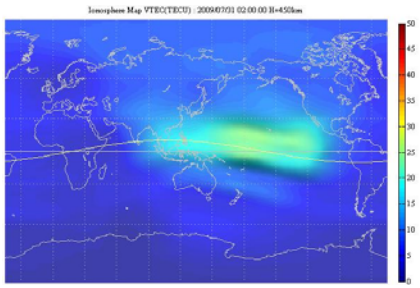
$$z' = \arcsin\left(\frac{R_e \sin z}{R_e + H_{ion}}\right), \alpha = z - z'$$

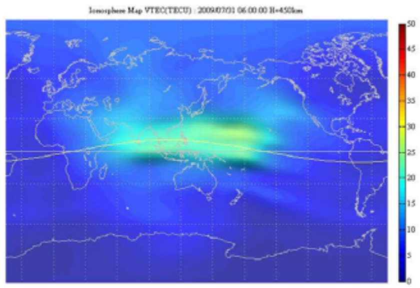
$$\phi_{pp} = \arcsin(\cos\alpha\sin\phi + \sin\alpha\cos\phi\cos Az)$$

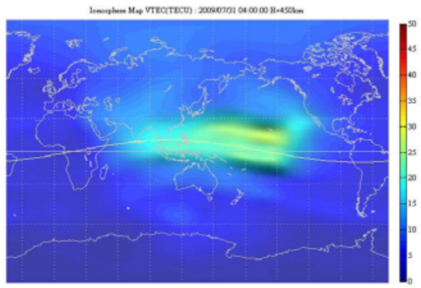
$$\lambda_{pp} = \begin{cases} \lambda + \pi - \arcsin\left(\dfrac{\sin\alpha\sin Az}{\phi_{pp}}\right) & \begin{array}{l}((\phi > 70^\circ \text{ and } \tan\alpha\cos Az > \tan(\pi/2-\phi)) \text{ or} \\ (\phi < -70^\circ \text{ and } -\tan\alpha\cos Az > \tan(\pi/2+\phi)))\end{array} \\ \lambda + \arcsin\left(\dfrac{\sin\alpha\sin Az}{\phi_{pp}}\right) & (\text{otherwise}) \end{cases}$$

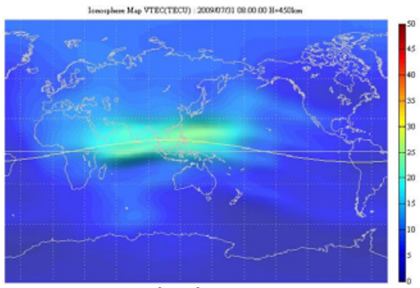
Ionospheric TEC Grid

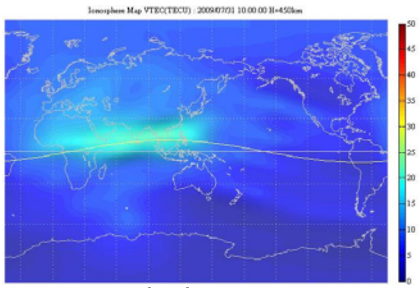

2009/7/31 0:00

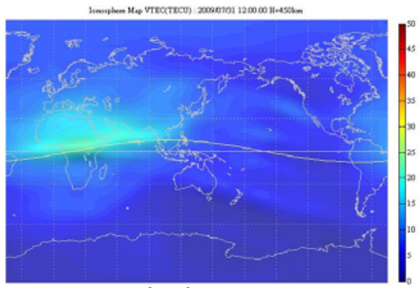

2009/7/31 2:00

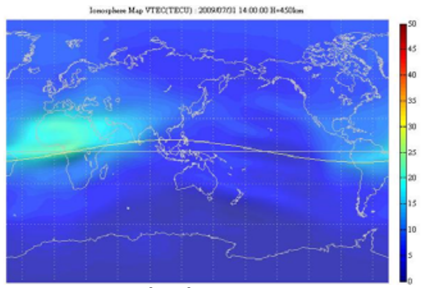

2009/7/31 4:00


2009/7/31 6:00


2009/7/31 8:00


2009/7/31 10:00


2009/7/31 12:00


2009/7/31 14:00

(IGS TEC Final, GPS Time)

$$\mathrm{VTEC}(t,\phi_{IPP},\lambda_{IPP}) = \frac{(t-t_i)\,\mathrm{VTEC}(t_i,\phi_{IPP},\lambda_{IPP}+\omega(t-t_i)) + (t_{i+1}-t)\,\mathrm{VTEC}(t_{i+1},\phi_{IPP},\lambda_{IPP}+\omega(t-t_{i+1}))}{t_{i+1}-t_i}$$

$$(t_i \le t < t_{i+1},\ \ \omega = 2\pi / 86400)$$

LC (Linear Combination)

$$LC = a\Phi_1 + b\Phi_2 + cP_1 + dP_2 (\Phi_1 = \lambda_1\varphi_1, \Phi_2 = \lambda_2\varphi_2)$$

	LC	Coefficients				Wave Length (cm)	Iono Effect wrt L1	Typical Noise (cm)
		a	b	c	d			
L1	L1 Carrier-Phase	1	0	0	0	19.0	1.0	0.3
L2	L2 Carrier-Phase	0	1	0	0	24.4	1.6	0.3
LC/L3	Iono-Free Phase	C_1	C_2	0	0	-	0.0	0.9
LG/L4	Geometry-Free Phase	1	-1	0	0	-	0.6	0.4
WL	Wide-Lane Phase	λ_{WL}/λ_1	$-\lambda_{WL}/\lambda_2$	0	0	86.2	1.3	1.7
NL	Narrow-Lane Phase	λ_{NL}/λ_1	λ_{NL}/λ_2	0	0	10.7	1.3	1.7
MW	Melbourne-Wübbena	λ_{WL}/λ_1	$-\lambda_{WL}/\lambda_2$	λ_{NL}/λ_1	λ_{NL}/λ_2	86.2	0.0	21
MP1	L1-Multipath	$2C_2 - 1$	$-2C_2$	1	0	-	0.0	30
MP2	L2-Multipath	$-2C_1$	$2C_1 - 1$	0	1	-	0.0	30

$$C_1 = f_1^2/(f_1^2 - f_2^2),\ C_2 = -f_2^2/(f_1^2 - f_2^2),\ \lambda_{WL} = 1/(1/\lambda_1 - 1/\lambda_2),\ \lambda_{NL} = 1/(1/\lambda_1 + 1/\lambda_2)$$

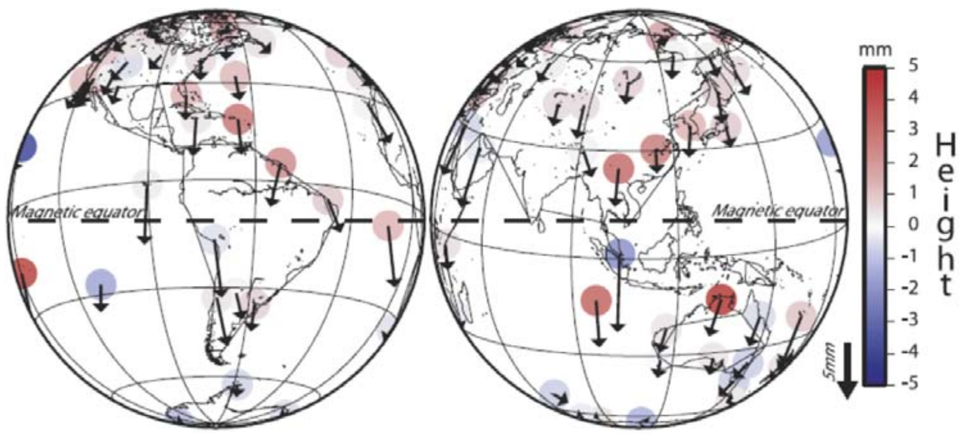
2nd Order Iono Effect

$$P_i = \rho + \frac{40.3 \times TEC}{f_i^2} + \frac{s}{f_i^3}$$

$$\Phi_i = \rho + \lambda_i N_i - \frac{40.3 \times TEC}{f_i^2} - \frac{1}{2}\frac{s}{f_i^3} \quad \text{(m)}$$

$$s = \int f_g f_N^2 \cos\theta_B dl = 7527c \int N_e B_0 \cos\theta_B dl$$

- ρ : Geometric range, clock and troposphere terms
- f_g : Gyro-frequency (~0.59 MHz)
- B_0 : Amplitude of the earth's magnetic field (T)
- θ_B : Angle between earth's magnetic field and direction of signal path (rad)



Global effect of the 2nd-order iono correction (April, 2002)

S. Kedar et al., The effect of the second order GPS ionospheric correction on receiver positions, Geophysical Research Letters, 2003

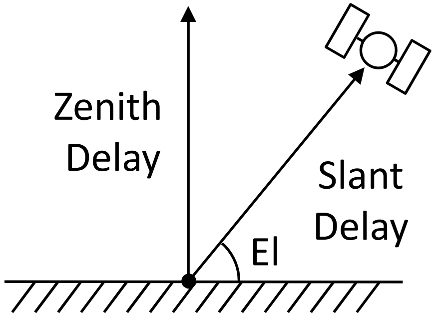
Tropospheric Delay

Tropospheric Delay

$$T_r^s = m_h(El)Z_h + m_w(El)Z_w \quad (1)$$

$$T_r^s = m_h(El)Z_h + m_w(El)(Z_t - Z_h) \quad (2)$$

(m)



Zenith hydrostatic delay

$$Z_h = \frac{0.0022768\,p}{1 - 0.00266\cos 2\phi - 2.8\times 10^{-7} H}$$

p : Total atmospheric pressure (hPa)

Troposphere Model by Standard Atmosphere

$$p = 1013.25\times(1 - 2.2557\times 10^{-5} h)^{5.2568}$$

$$T = 15.0 - 6.5\times 10^{-3} h + 273.15$$

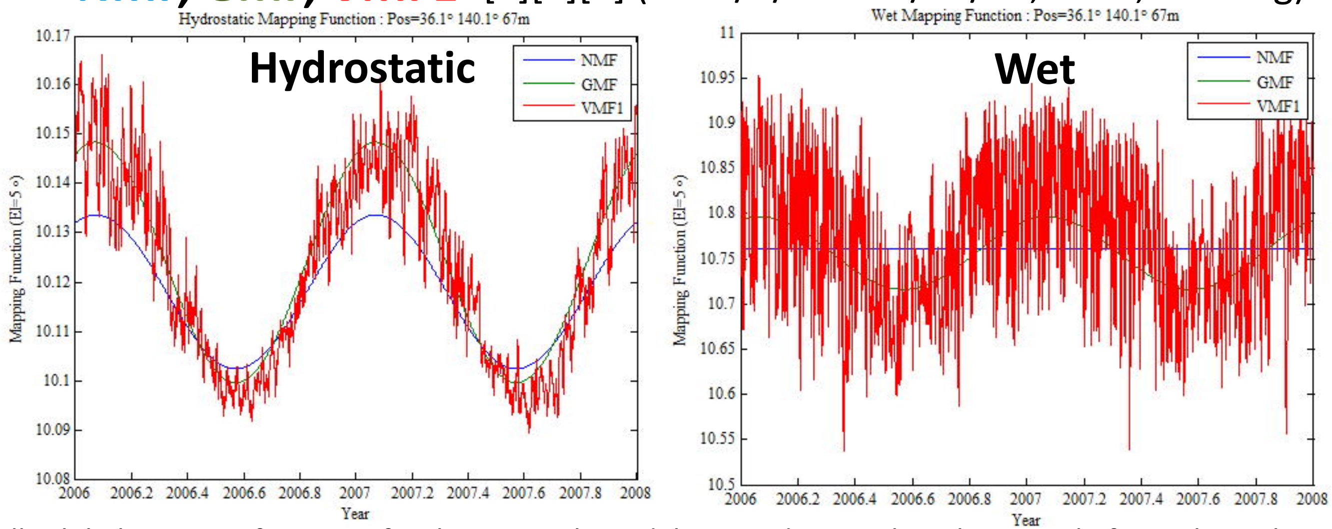
$$e = 6.108\times \exp\left\{\frac{17.15T - 4684.0}{T - 38.45}\right\}\times \frac{h_{rel}}{100}$$

Mapping Function

$$m_x(\mathrm{El}) = \frac{1 + \dfrac{a_x}{1 + \dfrac{b_x}{1 + c_x}}}{\sin(\mathrm{El}) + \dfrac{a_x}{\sin(\mathrm{El}) + \dfrac{b_x}{\sin(\mathrm{El}) + c_x}}}$$

a_x, b_x, c_x : Mapping Function Coefficients

NMF, **GMF**, **VMF1** [1][2][3] (2006/1/1-2007/12/31, TSKB, El=5deg)



[1] A.E.Niell, Global mapping functions for the atmosphere delay at radio wavelength, Journal of Geophysical Research, 1996

[2] J.Boehm, A.Niell, P.Tregoning and H.Shuh, Global Mapping Function (GMF): A new empirical mapping function base on numerical weather model data, G/formeophysical Res Lett, 33, L07304, 2006

[3] J.Boehm, R.Heinkelmann and H.Schuh, Short note: A global model of pressure and temperature for geodetic applications, Journal of Geodesy, 2007

Tropospheric Gradient

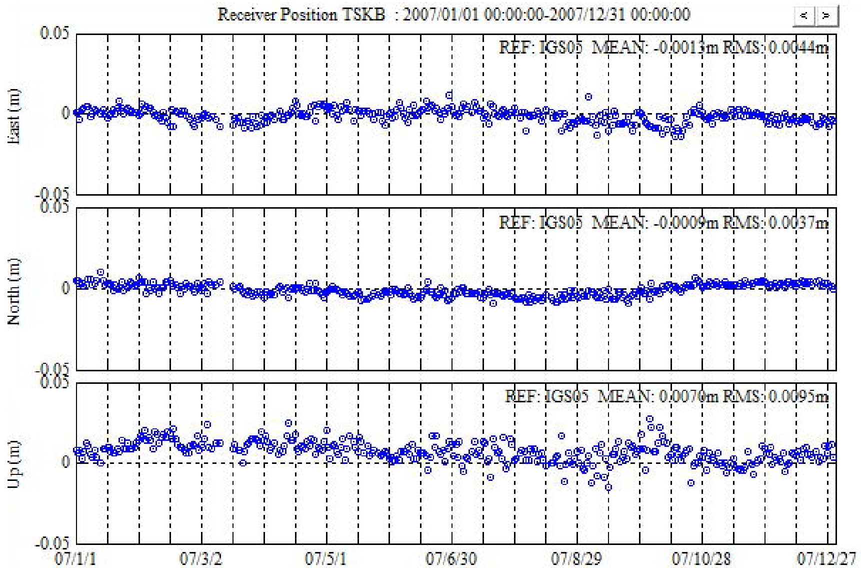
Tropospheric delay with Horizontal Gradient

$$T_r^s = m_h(El)\,Z_h + m_w(El)\,Z_w + m_g(El)\left(G_N\cos(Az) + G_E\sin(Az)\right) \qquad \text{(m)}$$

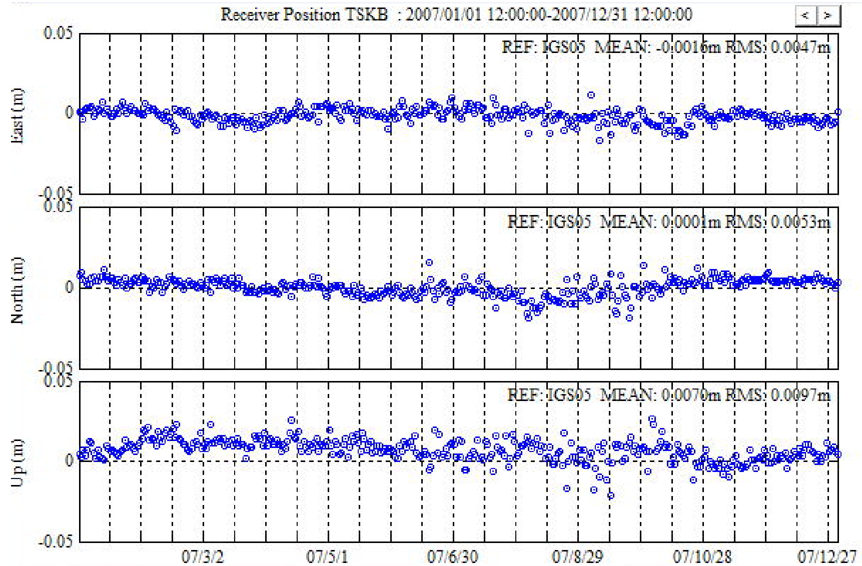
$m_g(El) = 1/(\sin El \tan El + 0.0032)$: Gradient mapping function (Chen and Herring, 1997)

G_N, G_E : North/East Gradient Parameters

PPP Solutions with Gradient Estimation



PPP Solutions without Gradient Estimation

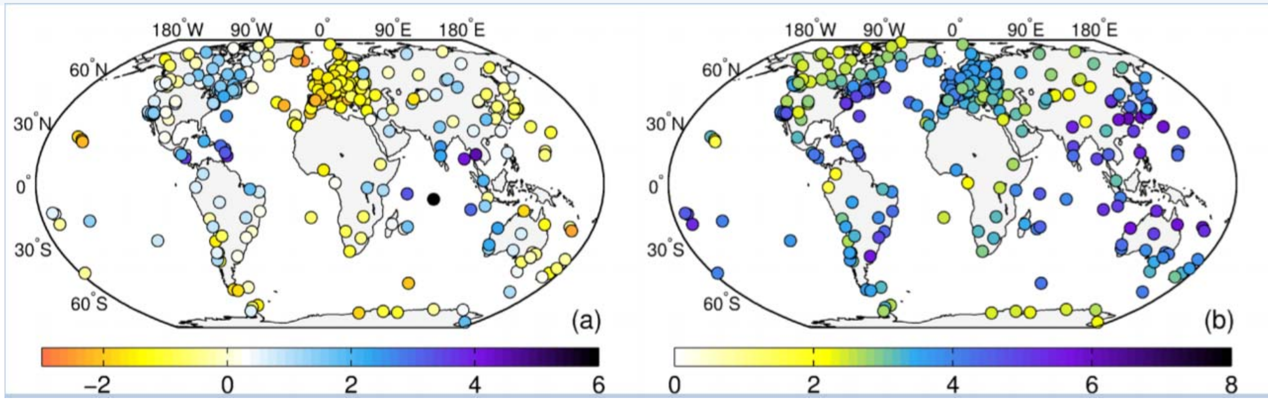


2007/1/1-12/31, 24H-Static PPP, TSKB

Empirical Tropospheric Model

Model ZTD Bias and RMS error (cm) (2012)

	mean bias	mean standard deviation	
RTCA-MOPS (1999)	-2.50 cm	4.55 cm	[1]
ESA (Martellucci 2012)	0.83 cm	3.82 cm	[2]
GPT2 (Lagler et al. 2013)	-0.28 cm	3.79 cm	[3]
GPT2w (Böhm et al. 2014)	-0.02 cm	3.61 cm	



GPT2w ZTD Bias and RMS error (cm)

J. Bohm et al., Development of an improved empirical model for slant delays in the troposphere (GPT2w), GPS Solutions, 2015

[1] RTCA/DO-229, Minimum operational standards for global positioning system/wide area augmentation system airborne equipment. 1999

[2] A. Martellucci, Galileo reference troposphere model for the user receiver. ESA-APPNG-REF/00621-AM v2.7, 2012

[3] K. Lagler et al., GPT2: empirical slant delay model for radio space geodetic techniques. Geophys Res Lett, 2013

Satellite Attitude Model

Nominal Yaw Attitude Model

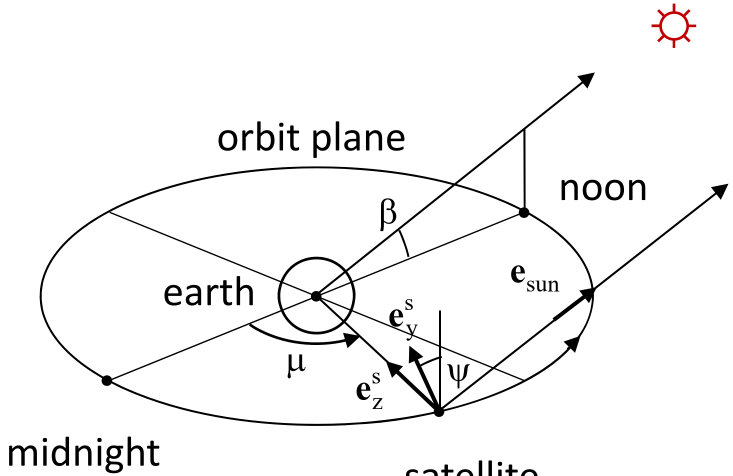
$$\mathbf{E}^s = (\mathbf{e}_x^{s\,T}, \mathbf{e}_y^{s\,T}, \mathbf{e}_z^{s\,T})^T$$

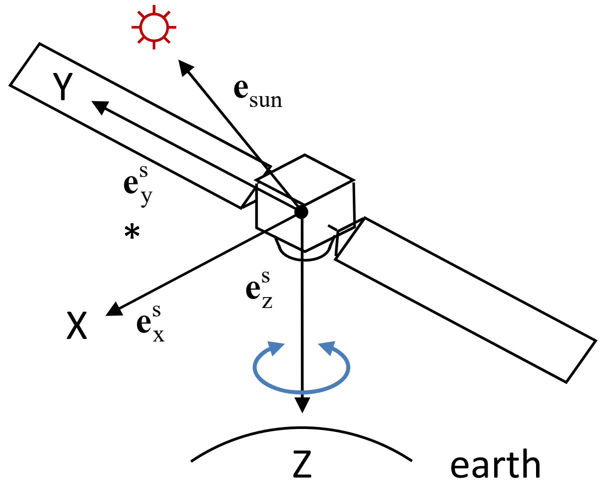
$$\mathbf{e}_z^s = -\frac{\mathbf{r}^s(t^s)}{\left\|\mathbf{r}^s(t^s)\right\|}, \mathbf{e}_{sun} = \frac{\mathbf{r}_{sun}(t^s) - \mathbf{r}^s(t^s)}{\left\|\mathbf{r}_{sun}(t^s) - \mathbf{r}^s(t^s)\right\|}$$

$$\mathbf{e}_y^s = \frac{\mathbf{e}_z^s \times \mathbf{e}_{sun}}{\left\|\mathbf{e}_z^s \times \mathbf{e}_{sun}\right\|}, \mathbf{e}_x^s = \mathbf{e}_y^s \times \mathbf{e}_z^s$$

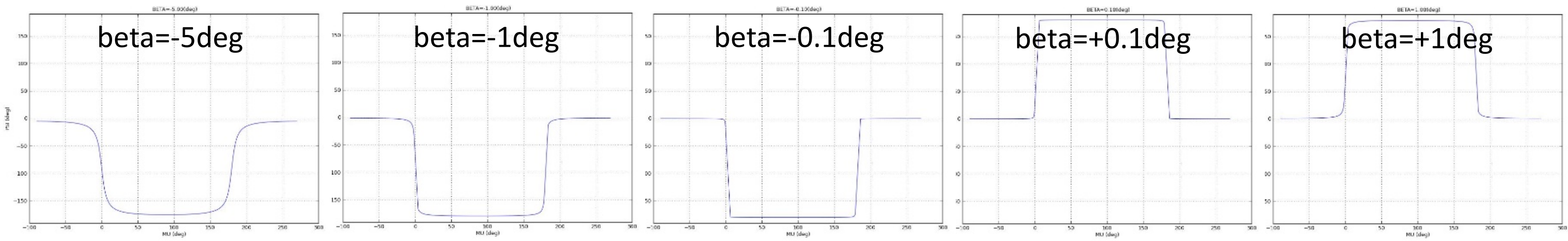
$\mathbf{r}_{sun}(t)$: Sun position in ECEF (m)
β : Beta angle of orbit plane (rad)
μ : Orbit angle from midnight (rad)
ψ : Yaw angle of satellite attitude (rad)

* Some satellites take opposite direction of Y-axis



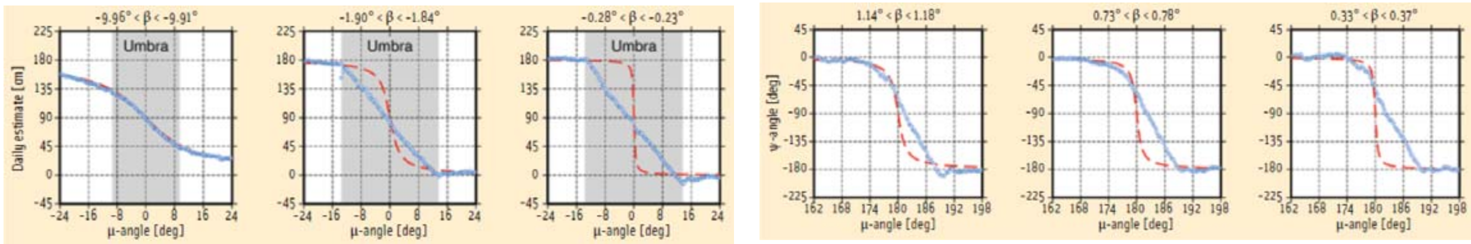


Yaw Attitude of Eclipsing Satellites



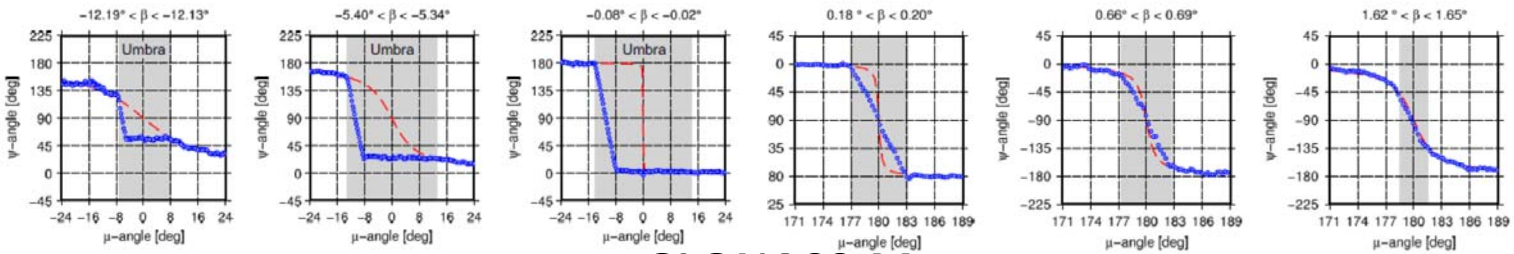
GPS Block IIR

J.Kouba, A simplified yaw-attitude model for eclipsing GPS satellites, GPS Solutions, 2009



GPS Block IIF

F. Dilssner, GPS IIF-1 satellite antenna phase center and attitude modeling, Inside GNSS, 2010



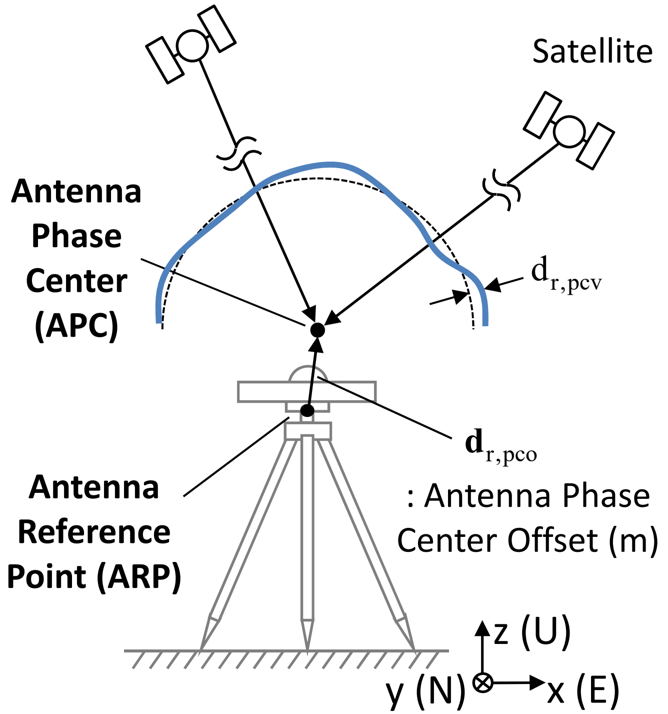
GLONASS-M

F. Dilssner, The GLONASS-M satellite yaw-attitude model, Advances in Space Research, 2010

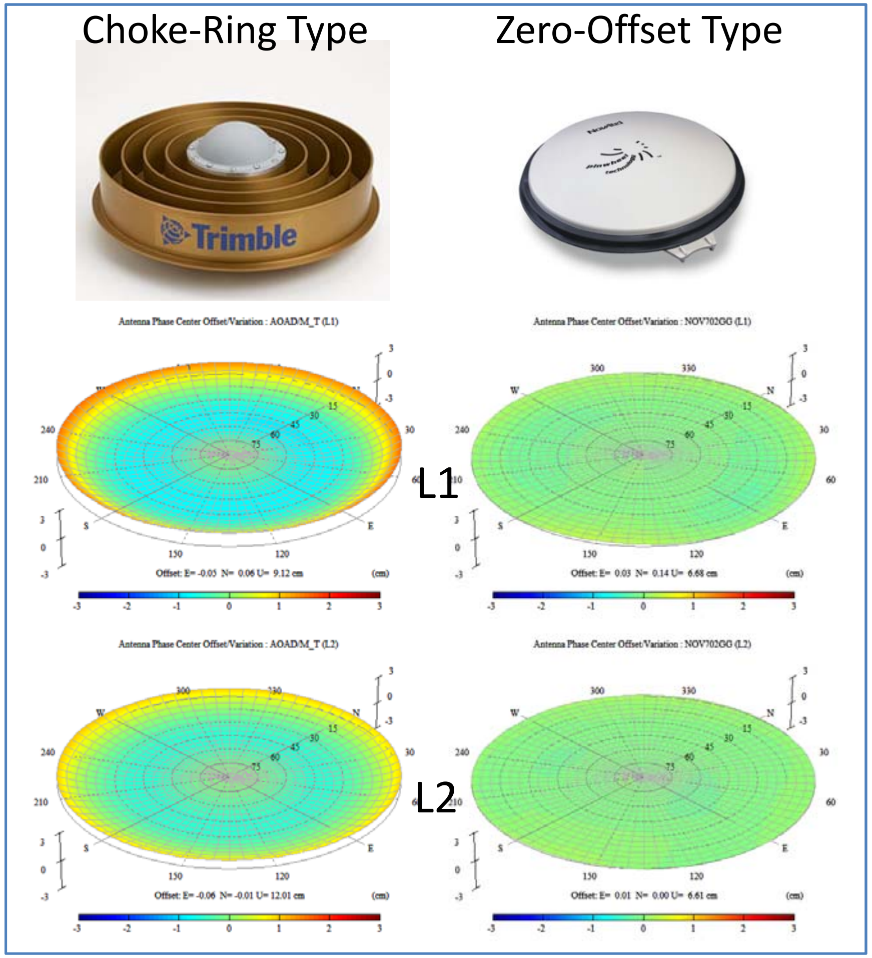
Antenna Phase Center 1

Receiver Antenna Phase Center Corrections

$$d_{r,ant} = -\mathbf{d}_{r,pco}{}^{T}\mathbf{e}^{s}_{r,enu} + d_{r,pcv}(El, Az) \qquad \text{(m)}$$



$d_{r,pcv}$: Antenna Phase Center Variation (PCV)



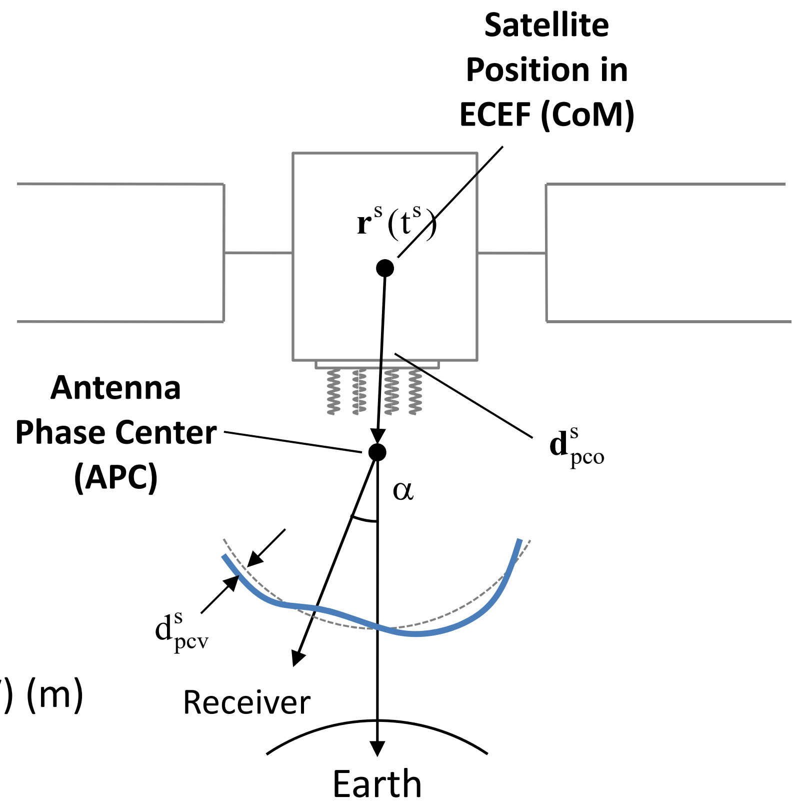
IGS Absolute Antenna Model (IGS05.PCV)

Antenna Phase Center 2

Satellite Antenna Phase Center Corrections

$$d_{ant}^{s} = \left(\mathbf{E}^{sT}\mathbf{d}_{pco}^{s}\right)^{T}\mathbf{e}_{r}^{s} + d_{pcv}^{s}(\alpha)$$

$$\alpha = \arccos\left(\frac{\mathbf{e}_{r}^{sT}\mathbf{r}^{s}(t^{s})}{\left\|\mathbf{r}^{s}(t^{s})\right\|}\right)$$



$\mathbf{d}_{pco}^{s}$: Antenna Phase Center Offset (m)

d_{pcv}^{s} : Antenna Phase Center Variation (PCV) (m)

α : Off-nadir Angle (rad)

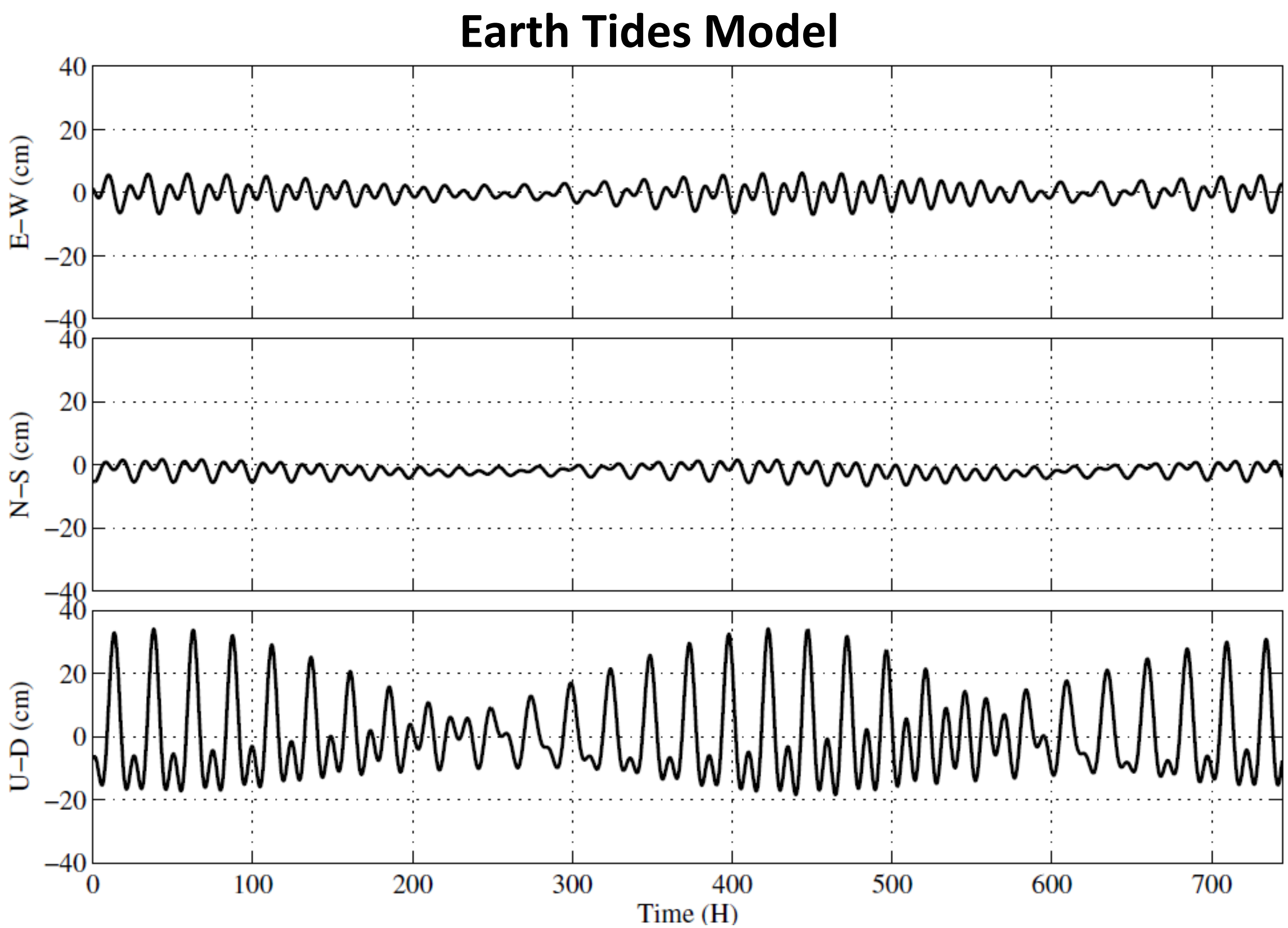
Site Displacement

$$d\mathbf{r}_{tide} = \mathbf{E}_r^T (d\mathbf{r}_{solid}(t_r, \mathbf{r}_r) + d\mathbf{r}_{otl}(t_r, \mathbf{r}_r) + d\mathbf{r}_{pole}(t_r, \mathbf{r}_r) + d\mathbf{r}_{atmos}(t_r, \mathbf{r}_r)) \qquad \text{(m)}$$

- **Displacement of Ground-Fixed Receiver**
  - Solid Earth Tide $d\mathbf{r}_{solid}(t_r, \mathbf{r}_r)$
  - Ocean Tide Loading (OTL) $d\mathbf{r}_{otl}(t_r, \mathbf{r}_r)$
  - Pole Tide $d\mathbf{r}_{pole}(t_r, \mathbf{r}_r)$
  - Atmospheric Loading $d\mathbf{r}_{atmos}(t_r, \mathbf{r}_r)$
- **Tide Models**
  - IERS Conventions 1996/2003/2010
  - Ocean Loading: Schwiderski, GOT99.2/00.2, CSR 3.0/4.0, FES99/2004, NAO99.b
  - M_2,S_2,N_2,K_2,K_1,O_1,P_1,Q_1,M_1,M_m,S_{sa} (11 constituents)
- **IERS Subroutines**
  - DEHANTTIDEINEL.F

http://iers-conventions.obspm.fr/content/chapter7/software/dehanttideinel/DEHANTTIDEINEL.F

Earth Tides



IERS Conventions 1996 + NAO99.b, 2007/1/1-1/31, TSKB

Phase Wind-up

- Relative rotation between satellite and receiver antennas effect to the measured phase of RHCP signal.

$$d_{pw} = \mathrm{sign}(\mathbf{e}_r^{sT}(\mathbf{D}^s \times \mathbf{D}_r))\arccos\left(\frac{\mathbf{D}^s \cdot \mathbf{D}_r}{\|\mathbf{D}^s\|\|\mathbf{D}_r\|}\right) / 2\pi + N \qquad \text{(cyc)}$$

$$\mathbf{E}^s = (\mathbf{e}_x^{s\,T}, \mathbf{e}_y^{s\,T}, \mathbf{e}_z^{s\,T})^T$$

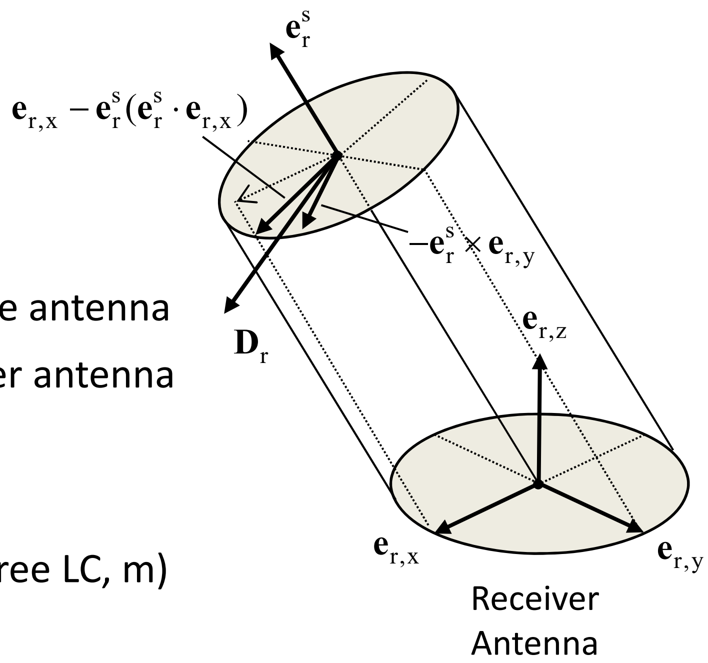
$$\mathbf{E}_r = (\mathbf{e}_{r,x}{}^T, \mathbf{e}_{r,y}{}^T, \mathbf{e}_{r,z}{}^T)^T$$

$\mathbf{D}^s = \mathbf{e}_x^s - \mathbf{e}_r^s(\mathbf{e}_r^s \cdot \mathbf{e}_x^s) + \mathbf{e}_r^s \times \mathbf{e}_y^s$: Dipole vector of satellite antenna

$\mathbf{D}_r = \mathbf{e}_{r,x} - \mathbf{e}_r^s(\mathbf{e}_r^s \cdot \mathbf{e}_{r,x}) - \mathbf{e}_r^s \times \mathbf{e}_{r,y}$: Dipole vector of receiver antenna

N : Integer ambiguity

$$\delta_{pw,LC} = C_1\lambda_1 d_{pw} + C_2\lambda_2 d_{pw} = \frac{c}{f_1 + f_2} d_{pw} = \lambda_{NL} d_{pw} \quad \text{(Iono-free LC, m)}$$



J. T. Wu et al., Effects of antenna orientation on GPS carrier phase, Manuscripta Geodaetica, 1993

Relativistic Effects

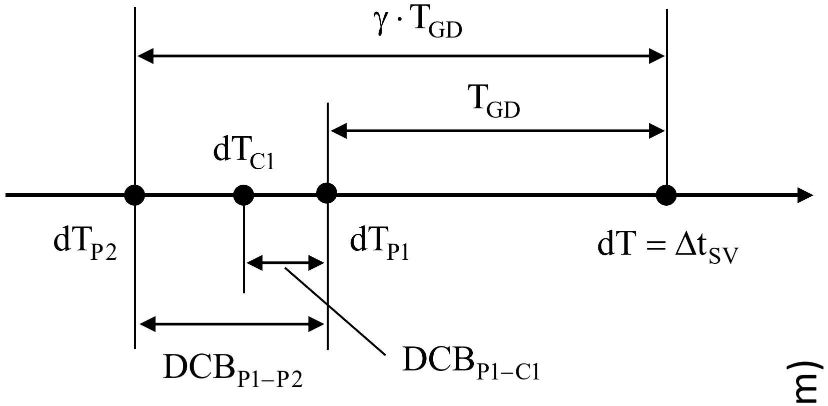
- **Satellite/Receiver:**
  - Frequency Shift by Earth Gravity (General Rel.)
  - Frequency Shift by Sun/Moon Gravity (General Rel.)
  - Second-Order Doppler-Shift by Motion (Special Rel.)
- **Signal Propagation:**
  - Sagnac Correction (Rotating Coordinates) (ignored)
  - Shapiro Time Delay Effect (ignored)
  - Lense-Thirring Drag (ignored)

Satellite Clock Bias/Rate Correction + Periodic Term:

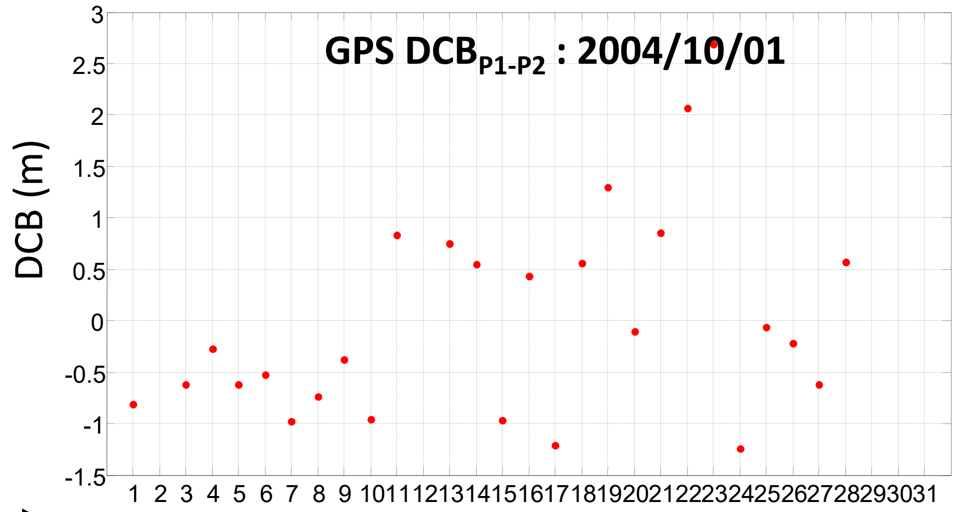
$$d_{rel} = -\frac{2\mathbf{r}^s(t^s)\cdot\mathbf{v}^s_{eci}}{c^2} \quad \text{(m)} \qquad \mathbf{v}^s_{eci} = \mathbf{v}^s(t^s) + \begin{pmatrix} -\omega_e y^s \\ \omega_e x^s \\ 0 \end{pmatrix} \quad (*) \quad \text{(m/s)}$$

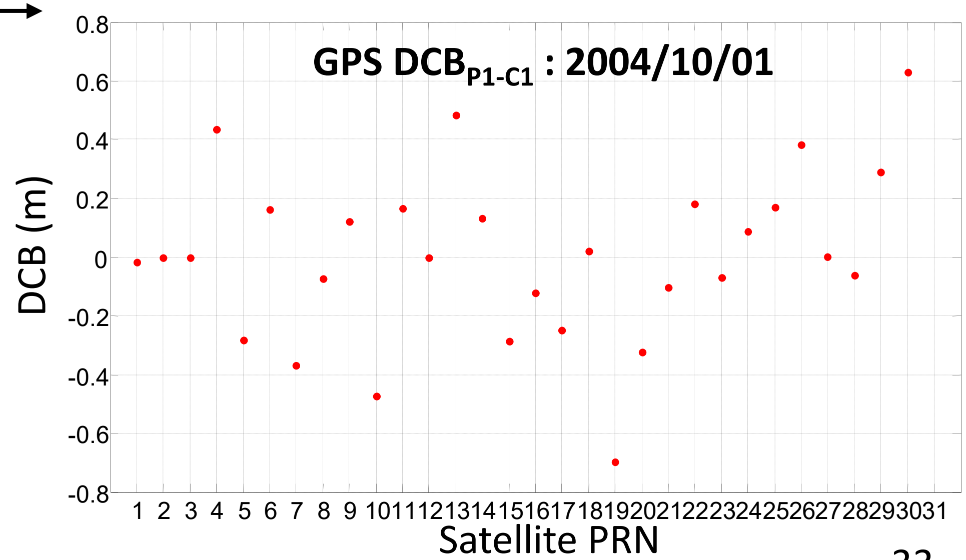
* Depends on satellite clock convention

Code Biases



$$\gamma = (f_1 / f_2)^2$$
$$= 1.647$$





Pseudorange/Carrier-phase Model

$$
\begin{aligned}
P_r^s &\equiv c\tau \\
&= c(\bar{t}_r - \bar{t}^s) \\
&= c((t_r + dt_r) - (t^s + dT^s(t^s))) + \varepsilon_P \\
&= c(t_r - t^s) + c(dt_r - dT^s(t^s)) + \varepsilon_P \\
&= (\rho_r^s + I_r^s + T_r^s) + c(dt_r - dT^s(t^s)) + \varepsilon_P \\
&= \rho_r^s + c(dt_r - dT^s(t^s)) + I_r^s + T_r^s + d_r^s + \varepsilon_P
\end{aligned}
$$

(1) (2) (3) (4) (5)

$$
\begin{aligned}
\phi_r^s &= \phi_r(t_r) - \phi^s(t^s) + N_r^s + \varepsilon_\phi \\
&= (f(t_r + dt_r - t_0) + \phi_{r,0}) - (f(t^s + dT^s(t^s) - t_0) + \phi_0^s) + N_r^s + \varepsilon_\phi \\
&= \frac{c}{\lambda}(t_r - t^s) + \frac{c}{\lambda}(dt_r - dT^s(t^s)) + (\phi_{r,0} - \phi_0^s + N_r^s) + \varepsilon_\phi
\end{aligned}
$$

$$
\begin{aligned}
\Phi_r^s \equiv \lambda\phi_r^s &= c(t_r - t^s) + c(dt_r - dT^s(t^s)) + \lambda(\phi_{r,0} - \phi_0^s + N_r^s) + \lambda\varepsilon_\Phi \\
&= \rho_r^s + c(dt_r - dT^s(t^s)) - I_r^s + T_r^s + d_r^s + \lambda d_{pw} + \lambda B_r^s + \varepsilon_\Phi
\end{aligned}
$$

$$d_r^s = d_{r,ant} + d_{ant}^s - \mathbf{d}_{disp}{}^T \mathbf{e}_{r,enu}^s + d_{rel}$$

$$B_r^s = \phi_{r,0} - \phi_0^s + N_r^s$$

$\phi_{r,0}$: Receiver initial phase (cyc)

ϕ_0^s : Satellite initial phase (cyc)