For JAXA R&D

PPP - Models, Algorithms and Implementations (6-1)

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2020-02-28 @Tokyo, Japan

PPP - Models, Algorithms and Implementation

1. 2019-10-04 **PPP models**
   geometric range, ionosphere, troposphere, antenna PCV, earth tides, wind-up, relativity, biases, coordinates
2. 2019-10-18 **PPP algorithms**
   SPP, LSQ, GN, EKF, noise-model, RAIM/QC, LAPACK/BLAS
3. 2019-11-01 **PPP data handling**
   LC, interpolation, slip detection, RINEX, SP3, ANTEX, RTCM, CSSR
4. 2019-11-22 **PPP-AR**
   UPD/FCB, EWL/WL/NL, ILS, LAMBDA, TCAR, PAR, validation
5. 2019-12-06 **INS integration**
   INS sensors, Inertial navigation, INS integration
6. 2019-12-20 **POD of satellites**
   orbit element, orbit model, reduced-dynamic, ECI-ECEF transformation, precession/nutation, EOP

(1.5 h / session)

Inertial Coordinate System

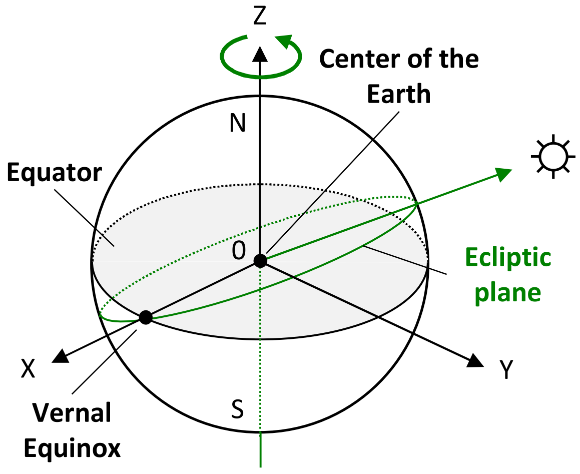
Inertial Coordinate System

Equatorial Coordinate System

- An ECI (earth centered inertial) coordinate system
- MEME in J2000.0 (epoch 2000-01-01 12:00 TT)
- MEME of date
- TETE of date
- TEME of date (TLE)

ICRF (international celestial reference frame)

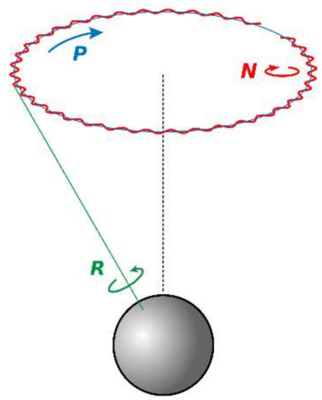
- A barycentric inertial coordinate system
- A realization of ICRS (international celestial reference system)
- Defined and maintained by IERS
- 4536 extragalactic radio sources in J2000.0 (ICRF3) [1]
- Axis stability < 0.01 mas, Position uncertainty < 0.1 mas,



Equatorial Coordinate System

TT: terrestrial time (= TAI + 32.184 s)
mas: milli-arcsec (~ 4.8 x 10^{-9} rad)
TLE: NORAD two line elements

MEME: mean equator mean equinox
TETE: true equator true equinox
TEME: true equator mean equinox



Precession (P) and Nutation (N) [2]

[1] https://www.iers.org/IERS/EN/DataProducts/ICRF/ICRF3/icrf3.html
[2] https://en.wikipedia.org/wiki/Nutation

Precession and Nutation

Precession/Nutation Models

IAU 1976/1980

IAU 2000A

IAU 2000A/2006

IAU 1976/1980 Precession/Nutation [1]

$$t = (TT - TT_{J2000.0}) \text{ in days} / 36525$$

Precession

$$\zeta_A = 2306''.2181t + 0''.30188t^2 + 0''.017998t^3$$

$$\theta_A = 2004''.3109t - 0''.42665t^2 - 0''.041833t^3$$

$$z_A = 2306''.2181t + 1''.09468t^2 + 0''.018203t^3$$

$$\varepsilon_A = 84381''.448 - 46''.8150t - 0''.00059t^2 + 0''.001813t^3$$

Nutation

$$\Delta\phi = \sum_{i=1}^{106} (A_i + A'_i T) \ \sin(\text{ARGUMENT}) \qquad \text{ARGUMENT} = \sum_{i=1}^{5} N_i F_i$$

$$\Delta\varepsilon = \sum_{i=1}^{106} (B_i + B'_i T) \ \cos(\text{ARGUMENT}) \qquad F = (l, l', F, D, \Omega)$$

GST

$$r = 1.002737909350795 + 5.9006 \times 10^{-11} T_u{}' - 5.9 \times 10^{-15} T_u{}'^2$$

$$GMST = GMST_{0hUT1} + r((UT1 - UTC) + UTC)$$

$$GST = GMST + \Delta\psi \cos\varepsilon_A + 0''.00264 \sin\Omega + 0''.000063 \sin 2\Omega$$

IAU 2000A, 2000A/2006

IAU SOFA (standard of fundamental astronomy) [2]

[1] D.D. McCarthy, IERS Technical note 21, IERS Conventions (1996), July, 1996

[2] http://www.iausofa.org

IAU 1980 Nutation

MULTIPLIERS OF					PERIOD	LONGITUDE (0″.0001)		OBLIQUITY (0″.0001)	
l	l'	F	D	Ω	(days)	A_i	A'_i	B_i	B'_i
0	0	0	0	1	-6798.4	-171996	-174.2	92025	8.9
0	0	2	-2	2	182.6	-13187	-1.6	5736	-3.1
0	0	2	0	2	13.7	-2274	-0.2	977	-0.5
0	0	0	0	2	-3399.2	2062	0.2	-895	0.5
0	-1	0	0	0	-365.3	-1426	3.4	54	-0.1
1	0	0	0	0	27.6	712	0.1	-7	0.0
0	1	2	-2	2	121.7	-517	1.2	224	-0.6
0	0	2	0	1	13.6	-386	-0.4	200	0.0
1	0	2	0	2	9.1	-301	0.0	129	-0.1
0	-1	2	-2	2	365.2	217	-0.5	-95	0.3
-1	0	0	2	0	31.8	158	0.0	-1	0.0
0	0	2	-2	1	177.8	129	0.1	-70	0.0
-1	0	2	0	2	27.1	123	0.0	-53	0.0
1	0	0	0	1	27.7	63	0.1	-33	0.0
0	0	0	2	0	14.8	63	0.0	-2	0.0
-1	0	2	2	2	9.6	-59	0.0	26	0.0
-1	0	0	0	1	-27.4	-58	-0.1	32	0.0
1	0	2	0	1	9.1	-51	0.0	27	0.0
-2	0	0	2	0	-205.9	-48	0.0	1	0.0
-2	0	2	0	1	1305.5	46	0.0	-24	0.0
0	0	2	2	2	7.1	-38	0.0	16	0.0
2	0	2	0	2	6.9	-31	0.0	13	0.0
2	0	0	0	0	13.8	29	0.0	-1	0.0
1	0	2	-2	2	23.9	29	0.0	-12	0.0
0	0	2	0	0	13.6	26	0.0	-1	0.0
0	0	2	-2	0	173.3	-22	0.0	0	0.0
-1	0	2	0	1	27.0	21	0.0	-10	0.0
0	2	0	0	0	182.6	17	-0.1	0	0.0
0	2	2	-2	2	91.3	-16	0.1	7	0.0

…

```
  ECEF-ECI TRANSF. ERROR (m) (35N,135E)
IAU2000-IAU1980:  0.00026 -0.00104  0.00110
IAU2006-IAU2000: -0.00005 -0.00069  0.00047
APPROX -IAU1980: -0.04507 -0.04516 -0.02099
W/O DSPI/DEPS  :  0.38679  0.11150  0.38844
```

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 ECEF-ECI TRANSF. CPU TIME (Core i7 2600K)
APPROX                    :       1.91 us
IAU1976/1980              :       5.38 us
IAU2000A      (SOFA)      :     114.21 us
IAU2000A/2006 (SOFA)      :     110.22 us
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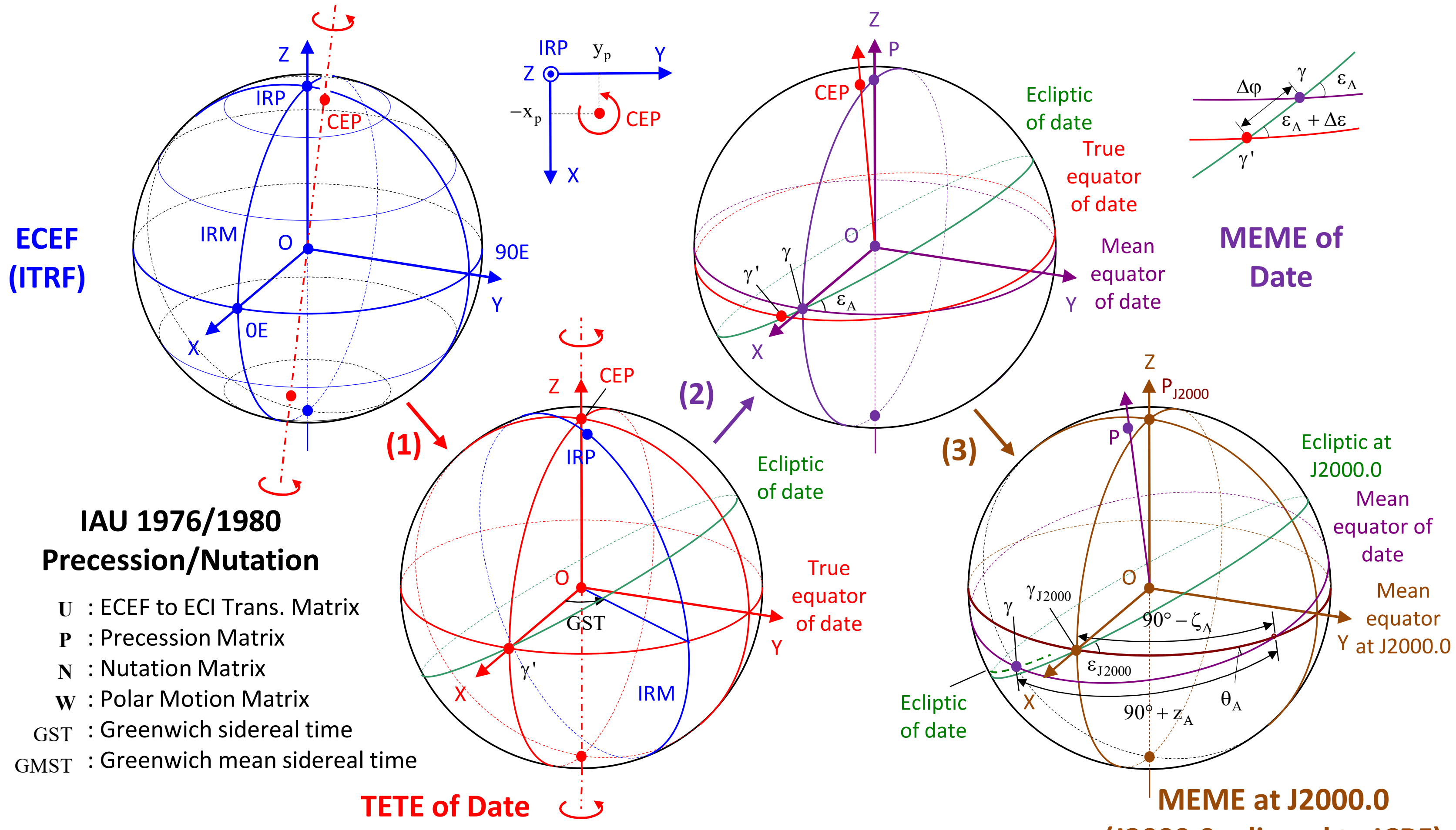
Coordinates Transformation



IAU 1976/1980
Precession/Nutation

- $\mathbf{U}$: ECEF to ECI Trans. Matrix
- $\mathbf{P}$: Precession Matrix
- $\mathbf{N}$: Nutation Matrix
- $\mathbf{W}$: Polar Motion Matrix
- GST : Greenwich sidereal time
- GMST : Greenwich mean sidereal time

$$\mathbf{r}_{eci} = \mathbf{U}\,\mathbf{r}_{ecef} = \mathbf{P}\mathbf{N}\mathbf{R}_z(-\mathrm{GST})\mathbf{W}\,\mathbf{r}_{ecef}$$

$$= \underbrace{\mathbf{R}_z(\zeta_A)\mathbf{R}_y(-\theta_A)\mathbf{R}_z(z_A)}_{(3)}\underbrace{\mathbf{R}_x(-\varepsilon_A)\mathbf{R}_z(\Delta\phi)\mathbf{R}_z(\varepsilon_A+\Delta\varepsilon)}_{(2)}\underbrace{\mathbf{R}_z(-\mathrm{GST})}\underbrace{\mathbf{R}_x(y_P)\mathbf{R}_y(x_P)}_{(1)}\mathbf{r}_{ecef}$$

EOP

EOP (Earth Orientation Parameters)

Pole offset: Xp, Yp

Pole rate: Xp-dot, Yp-dot

Earth rotation angle: UT1-UTC

Earth rotation rate: LOD

Corrections for precession/nutation models

EOP Products

IERS Bulletin A: Weekly [1]

IERS Bulletin B: Monthly [1]

USNO EOP daily, weekly, long-term [2]

IGS ERP: final, rapid, ultra-rapid [3]

Tidal Variation of Earth Rotation

IERS Conventions 2010 Sec. 8 [5]

[1] https://www.iers.org/IERS/EN/Publications/Bulletins/bulletins.html

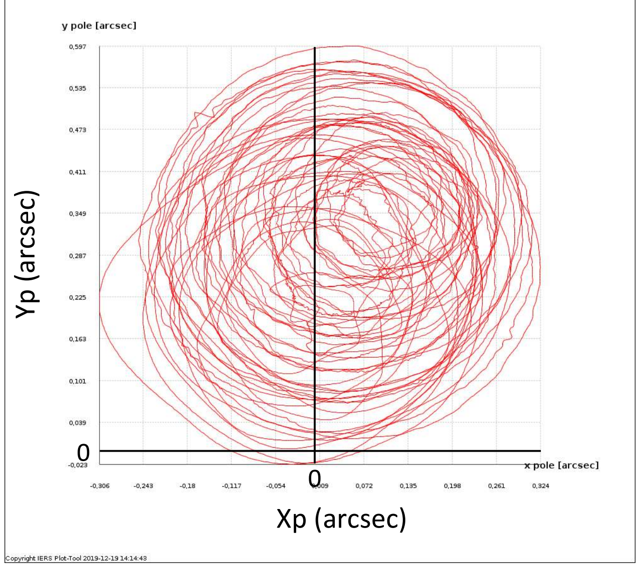
[2] https://www.usno.navy.mil/USNO/earth-orientation/eo-products

[3] http://www.igs.org/products

[4] IERS EOP C04 (IAU2000)

[5] G. Petit and B. Luzum (eds.), IERS Conventions (2010), 2010

Pole offset: Xp, Yp (1962/1/1-2009/8/11) [4]



Earth Rotation Angle: UT1-UTC [4]