For JAXA R&D

PPP - Models, Algorithms and Implementations (6-3)

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PPP - Models, Algorithms and Implementation

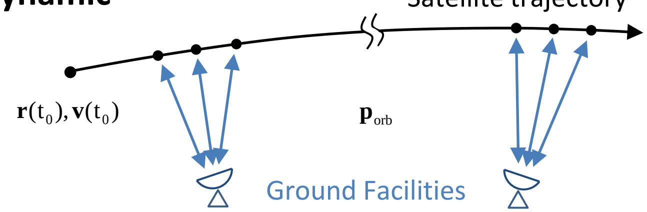
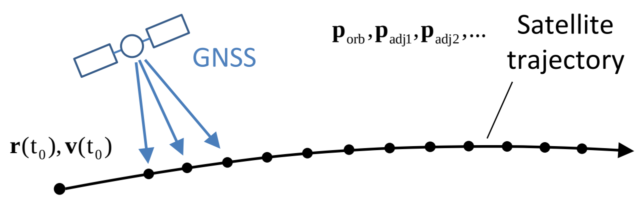
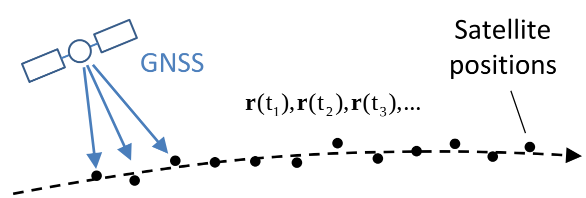
1. 2019-10-04 **PPP models**
   geometric range, ionosphere, troposphere, antenna PCV, earth tides, wind-up, relativity, biases, coordinates
2. 2019-10-18 **PPP algorithms**
   SPP, LSQ, GN, EKF, noise-model, RAIM/QC, LAPACK/BLAS
3. 2019-11-01 **PPP data handling**
   LC, interpolation, slip detection, RINEX, SP3, ANTEX, RTCM, CSSR
4. 2019-11-22 **PPP-AR**
   UPD/FCB, EWL/WL/NL, ILS, LAMBDA, TCAR, PAR, validation
5. 2019-12-06 **INS integration**
   INS sensors, Inertial navigation, INS integration
6. 2019-12-20 **POD of satellites**
   orbit element, orbit model, reduced-dynamic, ECI-ECEF transformation, precession/nutation, EOP

(1.5 h / session)

POD

(Precise Orbit Determination)

POD Types

POD Type	Measurement Data	Orbit Dynamics	Estimated Parameters	Typical Application
Dynamic 	Intermittent (Antenna Az/El, RARR, SLR, Delta DOR ...)	Yes	Initial States, Orbit parameters	Satellite Tracking and Navigation
Reduced Dynamic 	Continuous (GNSS)	Yes	Initial States, Orbit parameters, Adjustment parameters, GNSS parameters	Earth Sensor Calibration
Kinematic 	Continuous (GNSS)	No	Epoch-wise positions, GNSS parameters	Gravity Field Recovery

Dynamic POD

Dynamic POD with RARR (Two-way)

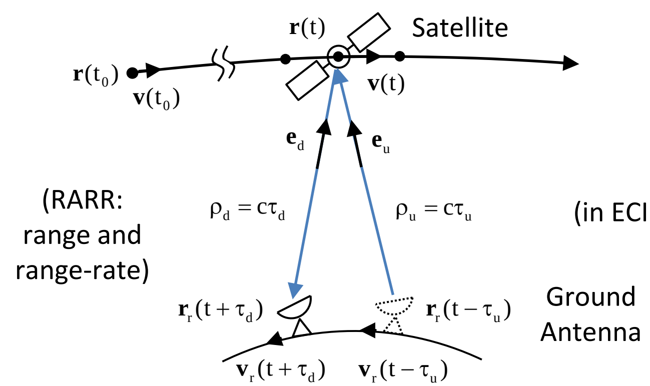
$$\rho_d = \left|\mathbf{r}(t) - \mathbf{r}_r(t + \rho_d/c)\right| \qquad \dot{\rho}_d = \mathbf{e}_d^T\left(\mathbf{v}(t) - \mathbf{v}_r(t + \rho_d/c)\right)$$

$$\rho_u = \left|\mathbf{r}(t) - \mathbf{r}_r(t - \rho_u/c)\right| \qquad \dot{\rho}_u = \mathbf{e}_u^T\left(\mathbf{v}(t) - \mathbf{v}_r(t - \rho_u/c)\right)$$

$$\rho = (\rho_d + \rho_u)/2 \qquad \dot{\rho} = (\dot{\rho}_d + \dot{\rho}_u)/2$$

$$\mathbf{r}_r(t) = \mathbf{U}(t)^T\mathbf{r}_r', \ \mathbf{v}_r(t) = \dot{\mathbf{U}}(t)^T\mathbf{r}_r'$$

$$\mathbf{e}_d = \left(\mathbf{r}(t) - \mathbf{r}_r(t + \rho_d/c)\right)/\rho_d, \mathbf{e}_u = \left(\mathbf{r}(t) - \mathbf{r}_r(t - \rho_u/c)\right)/\rho_u$$



Parameter Estimator by LSE

$$\mathbf{x} = (\mathbf{r}(t_0)^T, \mathbf{v}(t_0)^T, C_D, C_R)^T, \quad \mathbf{y} = (\rho_1, \rho_2, \ldots, \rho_m, \dot{\rho}_1, \dot{\rho}_2, \ldots, \dot{\rho}_m)^T$$

$$\mathbf{h}(\mathbf{x}) = \begin{pmatrix} (\rho_{d,1} + \rho_{u,1})/2 \\ (\rho_{d,2} + \rho_{u,2})/2 \\ \vdots \\ (\rho_{d,m} + \rho_{u,m})/2 \\ (\dot{\rho}_{d,1} + \dot{\rho}_{u,1})/2 \\ (\dot{\rho}_{d,2} + \dot{\rho}_{u,2})/2 \\ \vdots \\ (\dot{\rho}_{d,m} + \dot{\rho}_{u,m})/2 \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} (\mathbf{e}_{d,1} + \mathbf{e}_{u,1})^T \mathbf{\Phi}_r(t_1, t_0)/2 & (\mathbf{e}_{d,1} + \mathbf{e}_{u,1})^T \mathbf{S}_r(t_1)/2 \\ (\mathbf{e}_{d,2} + \mathbf{e}_{u,2})^T \mathbf{\Phi}_r(t_2, t_0)/2 & (\mathbf{e}_{d,2} + \mathbf{e}_{u,2})^T \mathbf{S}_r(t_2)/2 \\ \vdots & \vdots \\ (\mathbf{e}_{d,m} + \mathbf{e}_{u,m})^T \mathbf{\Phi}_r(t_m, t_0)/2 & (\mathbf{e}_{d,m} + \mathbf{e}_{u,m})^T \mathbf{S}_r(t_m)/2 \\ (\mathbf{e}_{d,1} + \mathbf{e}_{u,1})^T \mathbf{\Phi}_v(t_1, t_0)/2 & (\mathbf{e}_{d,1} + \mathbf{e}_{u,1})^T \mathbf{S}_v(t_1)/2 \\ (\mathbf{e}_{d,2} + \mathbf{e}_{u,2})^T \mathbf{\Phi}_v(t_2, t_0)/2 & (\mathbf{e}_{d,2} + \mathbf{e}_{u,2})^T \mathbf{S}_v(t_2)/2 \\ \vdots & \vdots \\ (\mathbf{e}_{d,m} + \mathbf{e}_{u,m})^T \mathbf{\Phi}_v(t_m, t_0)/2 & (\mathbf{e}_{d,m} + \mathbf{e}_{u,m})^T \mathbf{S}_v(t_m)/2 \end{pmatrix}$$

$$\hat{\mathbf{x}}_0 = \mathbf{x}_0$$

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{W}(\mathbf{y} - \mathbf{h}(\hat{\mathbf{x}}_k))$$

$$\hat{\mathbf{x}} = \lim_{k\to\infty} \hat{\mathbf{x}}_k$$

Coarse/Initial OD [1] → $\hat{\mathbf{x}}_0$

→ **POD Solution**

ρ, ρ_d, ρ_u : Measured/downlink/uplink range (m)

$\dot{\rho}, \dot{\rho}_d, \dot{\rho}_u$: Measured/downlink/uplink range-rate (m/s)

$\mathbf{r}(t)$: Satellite position at time t in ECI (m)

$\mathbf{v}(t)$: Satellite velocity at time t in ECI (m/s)

$\mathbf{r}_r(t)$: Gnd antenna position at time t in ECI (m/s)

$\mathbf{v}_r(t)$: Gnd antenna velocity at time t in ECI (m/s)

$\mathbf{r}_r'$: Gnd antenna position in ECEF (m)

$\mathbf{U}(t)$: ECI to ECEF transformation matrix

$\dot{\mathbf{U}}(t)$: Derivatives of ECI to ECEF trans. matrix

$\mathbf{W}$: Weighting matrix

$\mathbf{\Phi}(t, t_0) = \begin{pmatrix} \mathbf{\Phi}_r(t, t_0) \\ \mathbf{\Phi}_v(t, t_0) \end{pmatrix}$: State transition matrix from time t_0 to t

$\mathbf{S}(t) = \begin{pmatrix} \mathbf{S}_r(t) \\ \mathbf{S}_v(t) \end{pmatrix}$: Sensitivity matrix at time t

Notes: Ground equipment delay, satellite transponder delay and atmospheric effects are neglected.

[1] D. A. Vallado, Fundamentals of Astrodynamics and Applications (2nd edition), chap. 7, Microcosm Press, 2001

Reduced Dynamic POD

Reduced Dynamic POD with GNSS

DD- or PPP-processing (with or w/o AR)
Needs precise ephemeris for GNSS satellites
Depends upon orbit dynamics (geopotential, drag, ...)

Orbit Dynamics Model

$$\mathbf{a}(t) = \mathbf{a}_{geop} + \mathbf{a}_{body} + \mathbf{a}_{drag} + \mathbf{a}_{srp} + \mathbf{a}_{emp}$$

$\mathbf{a}_{emp} = R_k \mathbf{e}_{radial} + A_k \mathbf{e}_{aling} + C_k \mathbf{e}_{cross}$: Empirical acceleration

$$\left(\mathbf{e}_{radial} = \mathbf{r}(t)/|\mathbf{r}(t)|, \mathbf{e}_{cross} = (\mathbf{r}(t)\times\mathbf{v}(t))/|\mathbf{r}(t)\times\mathbf{v}(t)|, \mathbf{e}_{along} = \mathbf{e}_{cross}\times\mathbf{e}_{radial}\right)$$

R_k, A_k, C_k : Radial/along-track/cross-track accel. (m/s²) $(t_{k-1} \le t < t_k)$

Measurement Model of GNSS

$$P_r^s(t) = \rho_r^s(t) + c(dt_r(t) - dT^s(t^s)) + \varepsilon_P$$

$$\Phi_r^s(t) = \rho_r^s(t) + c(dt_r(t) - dT^s(t^s)) + B_r^s + \varepsilon_\phi$$

$$\rho_r^s(t) = \left|\mathbf{U}(t^s)\mathbf{r}^s(t^s) - \mathbf{r}(t)\right|, \ \mathbf{e}_r^s(t) = \left(\mathbf{U}(t^s)\mathbf{r}^s(t^s) - \mathbf{r}(t)\right)/\rho_r^s(t)$$

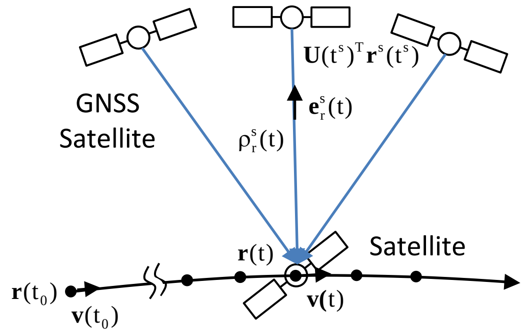
$$\frac{\partial \rho_r^s(t)}{\partial(\mathbf{r}(t_0)^T, \mathbf{v}(t_0)^T)^T} = -\mathbf{e}_r^s(t)^T \mathbf{\Phi}_r(t, t_0), \ \frac{\partial \rho_r^s(t)}{\partial \mathbf{p}} = -\mathbf{e}_r^s(t)^T \mathbf{S}(t)$$

Parameter Estimator by LSE

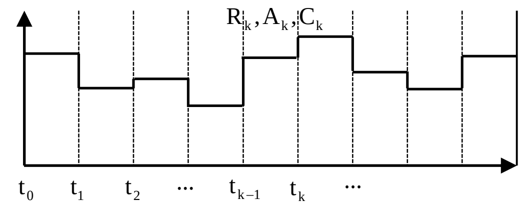
$$\mathbf{x} = (\mathbf{r}(t_0)^T, \mathbf{v}(t_0)^T, \mathbf{p}^T, \mathbf{t}^T, \mathbf{B}^T)^T \xrightarrow{\text{Non-linear LSE}} \hat{\mathbf{x}}, \mathbf{Q}_\mathbf{x}$$

$$\mathbf{p} = (C_d, C_R, R_1, A_1, C_1, ...)^T, \mathbf{t} = (dt_r(t_1), dt_r(t_2), ...)^T, \mathbf{B} = (B_r^1, B_r^2, B_r^3, ...)^T$$

Orbit parameters   Receiver clocks   Phase ambiguities



Piecewise Constant



$P_r^s(t)$	: Iono-free pseudorange (m)
$\Phi_r^s(t)$	: Iono-free phase range (m)
ρ_r^s	: Geometric range (m)
$dt_r(t)$	: Receiver clock bias (s)
$dT^s(t^s)$	: GNSS satellite position in ECEF (m)
$\mathbf{r}^s(t^s)$	: GNSS satellite clock bias (s)
B_r^s	: Carrier-phase ambiguity (m)
$\mathbf{e}_r^s(t)$	: LOS vector from receiver to GNSS satellite
$\mathbf{U}(t)$	: ECI to ECEF transformation matrix

Kinematic POD

Kinematic POD with GNSS

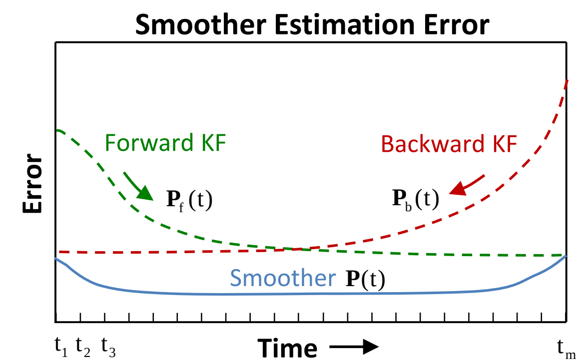
DD- or PPP-processing (with or w/o AR)
Solutions: epoch-wise satellite positions
Needs precise ephemeris for GNSS satellites
Independent to orbit dynamics (geopotential, drag, ...)

Parameter Estimator Options

(1) Smoother with Forward and Backward KF

$$\mathbf{P}(t) = \left(\mathbf{P}_f(t)^{-1} + \mathbf{P}_b(t)^{-1}\right)^{-1}$$

$$\hat{\mathbf{x}}(t) = \mathbf{P}(t)\left(\mathbf{P}_f(t)^{-1}\hat{\mathbf{x}}_f(t) + \mathbf{P}_b(t)^{-1}\hat{\mathbf{x}}_b(t)\right) \quad (t = t_1, t_2, t_3, \ldots, t_m)$$



$\hat{\mathbf{x}}(t), \mathbf{P}(t)$: Smoother solutions and VC-matrix
$\hat{\mathbf{x}}_f(t), \mathbf{P}_f(t)$: Forward KF solutions and VC-matrix
$\hat{\mathbf{x}}_b(t), \mathbf{P}_b(t)$: Backward KF solutions and VC-matrix

(2) LSE with Block Elimination

$$\mathbf{x} = (\mathbf{x}_c^T, \mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_m^T)^T, \ \mathbf{y} = (\mathbf{y}_1^T, \mathbf{y}_2^T, \ldots, \mathbf{y}_m^T)^T$$

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{1c} & \mathbf{H}_{11} & & & \\ \mathbf{H}_{2c} & & \mathbf{H}_{22} & & \\ \vdots & & & \ddots & \\ \mathbf{H}_{mc} & & & & \mathbf{H}_{mm} \end{pmatrix}, \ \mathbf{W} = \begin{pmatrix} \mathbf{W}_1 & & & \\ & \mathbf{W}_2 & & \\ & & \ddots & \\ & & & \mathbf{W}_m \end{pmatrix} \Rightarrow$$

$$\mathbf{N}\hat{\mathbf{x}} = \mathbf{b}, \ \mathbf{N} = \mathbf{H}^T\mathbf{W}\mathbf{H} = \begin{pmatrix} \mathbf{N}_{cc} & \mathbf{N}_{1c}^T & \mathbf{N}_{2c}^T & \cdots & \mathbf{N}_{mc}^T \\ \mathbf{N}_{1c} & \mathbf{N}_{11} & & & \\ \mathbf{N}_{2c} & & \mathbf{N}_{22} & & \\ \vdots & & & \ddots & \\ \mathbf{N}_{mc} & & & & \mathbf{N}_{mm} \end{pmatrix}, \ \mathbf{b} = \mathbf{H}^T\mathbf{W}\mathbf{y} = \begin{pmatrix} \mathbf{b}_c \\ \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_m \end{pmatrix}$$

$$\mathbf{N}_{cc} = \sum_{k=1}^{m}\left(\mathbf{H}_{kc}^T\mathbf{W}_k\mathbf{H}_{kc}\right), \mathbf{b}_c = \sum_{k=1}^{m}\left(\mathbf{H}_{kk}^T\mathbf{W}_k\mathbf{y}_k\right), \ \mathbf{N}_{kc} = \mathbf{H}_{kk}^T\mathbf{W}_k\mathbf{H}_{kc}, \mathbf{b}_k = \mathbf{H}_{kc}^T\mathbf{W}_k\mathbf{y}_k$$

$$\hat{\mathbf{x}}_c = \left(\mathbf{N}_{cc} - \sum_{k=1}^{m}\left(\mathbf{N}_{kc}\mathbf{N}_{kk}^{-1}\mathbf{N}_{kc}^T\right)k\right)^{-1}\left(\mathbf{b}_c - \sum_{k=1}^{m}\left(\mathbf{N}_{kc}\mathbf{N}_{kk}^{-1}\mathbf{b}_k\right)\right)$$

$$\hat{\mathbf{x}}_k = \mathbf{N}_{kk}^{-1}(\mathbf{b}_k - \mathbf{N}_{kc}\hat{\mathbf{x}}_c) \quad (k = 1, 2, \ldots, m)$$

$\mathbf{x}_c$: Common parameters (ambiguities, ...)
$\mathbf{x}_k$: Epoch parameters at t_k (position, receiver clock)
$\mathbf{y}_k$: Measurement data at t_k (pseudorange, carrier-phase)

(3) GRAPHIC [1]

$$\rho_g = \left(P_{L1} + \Phi_{L1}\right)/2 = \rho_r^s + c(dt_r - dT^s) + B_r^s/2 + \varepsilon_P/2$$: iono-free LC of L1 pseudorange and carrier-phase

[1] O. Montenbruck, Kinematic GPS positioning of LEO satellites using ionosphere-free single frequency measurements, Aerospace Science and Technology, 2003

GNSS SSV, Lunar and Deep Space

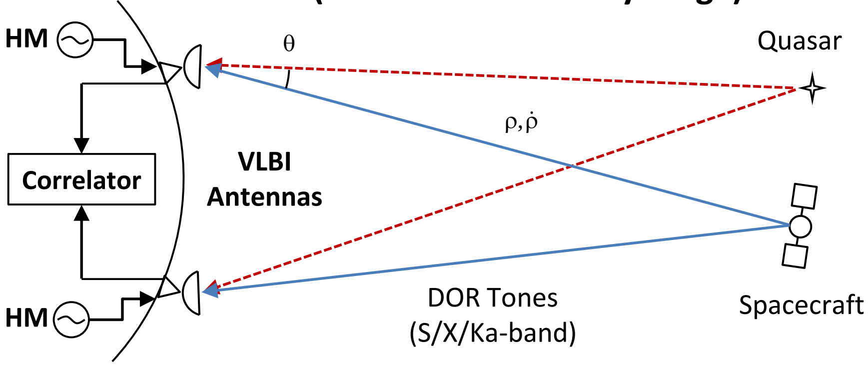
GNSS SSV (Space Service Volume)

MEO, GEO and HEO satellites
Altitude: 3,000 - 36,000 km
RT navigation, post-maneuver recovery, ...
Dynamic POD with GNSS sidelobe signals
Interoperable GNSS SSV: ICG WG-B

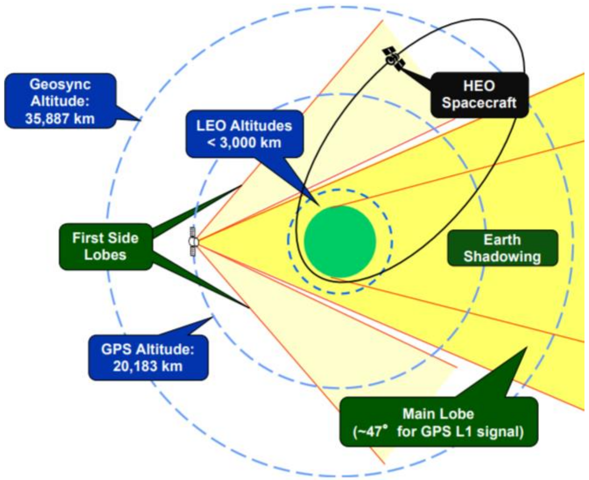
Beyond GNSS SSV

Spacecrafts for lunar or interplanetary mission
Dynamic POD with
- Range, range-rate : RARR
- Angle measurement : Delta DOR

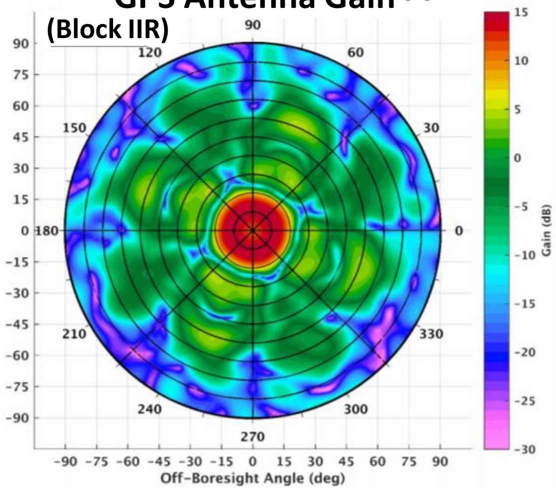
Delta DOR (differential one-way range)



GNSS SSV [1]

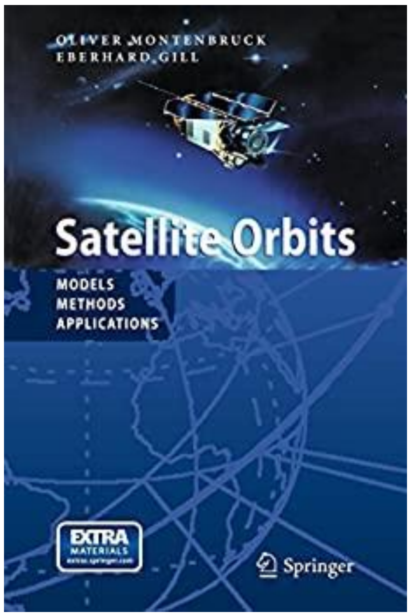


GPS Antenna Gain [1]

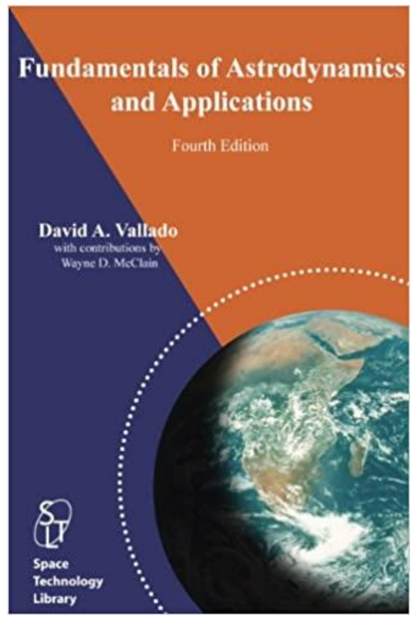


[1] J. J. K. Parker, Interoperable GNSS Space Service Volume (SSV), 22nd PNT Advisory Board Meeting, 2018

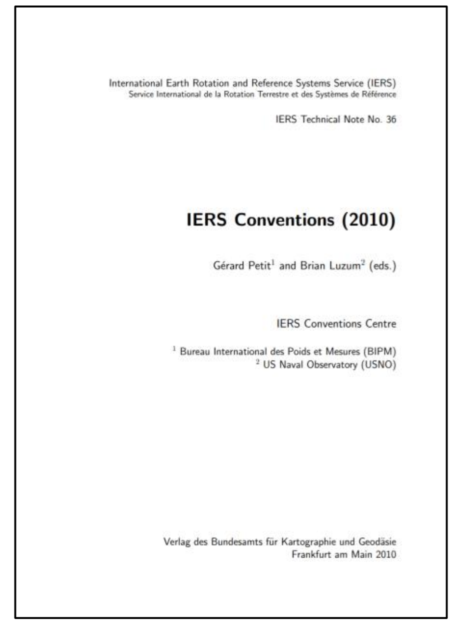
References



O. Montenbruck and E. Gill,
Satellite Orbits - Models,
Methods, and Applications,
Springer, 2000



D. A. Vallado,
Fundamentals of Astrodynamics
and Applications (4th edition),
Microcosm Press, 2013


International Earth Rotation and Reference Systems Service (IERS)
Service International de la Rotation Terrestre et des Systèmes de Référence

IERS Technical Note No. 36

IERS Conventions (2010)

Gérard Petit[1] and Brian Luzum[2] (eds.)

IERS Conventions Centre

[1] Bureau International des Poids et Mesures (BIPM)
[2] US Naval Observatory (USNO)

Verlag des Bundesamts für Kartographie und Geodäsie
Frankfurt am Main 2010

G. Petit and B. Luzum (eds.),
IERS Technical Note No. 36 -
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（https://www.iers.org/IERS/EN/
Publications/TechnicalNotes/tn36.html)